氣體의 熱傳導度 測定에 대한 溫度 跳躍의 影響

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Effect of Temperature Jump on Measuring the
Thermal Conductivity of Gases

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요　약

가는 熱線과 그를 둘러싸는 低壓 가스와의 接觸面에서 發生하는 溫度跳躍現象의 가스의 熱傳導度測定 資料를 處理하는데에 對한 影響을 理論的인 見地에서 檢討하고, 이影響에 對한 補正法을 案出하여 알칼리金屬蒸氣의 熱傳導度에 관한 測定資料處理에 適用함으로서 低壓가스의 周波數應答法에 依한 熱傳導度 測定法을 完成하였다.

Abstract

A method of correcting for the effect of temperature jump at the interface of the fine wire and the surrounding gas was developed from the theoretical point of view to be used for processing the data obtained by the frequency response technique for measuring the thermal conductivity of gases at low pressures. The method was applied to the data obtained with the alkali metal vapors.

Introduction

A new method of measuring the thermal conductivity of gases, known as the Frequency Response Technique, was first contemplated by Lee et al [1], and the work was carried on by Tarmy et al [2,3], and further developed by Peterson et al [4] during a period of time. The measurement so far carried out based on this principle was concerned mainly

with the gases at nearly one atmospheric pressure or higher. Recently , Lee [5, 6] devised a method of handling low pressure gases incorporated with the same general principle of measurement. The method seemed to be fine judging both from the theoretical conjecture and from the internal consistency met by the method in the data processing step.

The measurement of gas thermal conductivity at low pressures was impelled by the difficulty encountered in taking electrical measurements with saturated alkali metal vapors. The condensation of the vapor and short-circuiting the lead wire by the liquid metal was the major soure of touble. It was, therefore, necessitated to employ superheated vapors of relatively low presures ranging from 0.08 to 0.2 atmospheres. According to the theory, the finest possible hot wire diameter was desirable to work within the linear range of the calibration curve. At the time of the previous work, the thinnest tungsten wire available was 0.0003 cm in diameter. The mean free path of the vapor molecules under this circumstance, particularly in contrast with the wire diameter, raises a serious problem in deeming the wire temperature as the gas temperature at the wire-gas interface. Designating the wall and gas temperatures by t_w and t, respectively, and the temperature jump, by Δt_j, one may relate these quantities by

$$t_w \sim t = \Delta t_j \qquad (1)$$

This situation will naturally necessitate a new boundary condition at the wire-gas interface for analysing the theory mathematically. The inner boundary condition for the cylindrical system, in absence of a temperature jump, can be written by taking a heat balance at $r=r_1$. Namely,

$$M \frac{\partial t}{\partial \theta} = I_0^2 \bar{R} \sin^2 \omega\theta + Ak \frac{\partial t}{\partial r} - Ah(t-t_2) \qquad (2)$$

where M is the heat capacity of the wire, A is its surface area, $\bar{R}$ is the time average of the fluctuating wire resistance, t_2 is the temperature of the outer shell, and the rest bears the usual meanings.

However, this can not be true when there is a temperature jump. The equation holding in this case

can be written by

$$M \frac{\partial t_w}{\partial \theta} = I_0^2 \bar{R} \sin^2 \omega\theta + Ak \frac{\partial (t_w - \Delta t_j)}{\partial r}$$
$$- Ah(t_w - t_2) \qquad (3)$$

or,

$$M \frac{\partial t_w}{\partial \theta} = I_2^0 \bar{R} \sin^2 \omega\theta + Ak^0 \frac{\partial t_w}{\partial r} - Ah(t_w - t_2) \quad (4)$$

where

$$k^0 = k \left\{ 1 - \left(\frac{\partial \Delta t_j}{\partial r} \right) \Big/ \left(\frac{\partial t_w}{\partial r} \right) \right\}$$

In the previous work, Lee approximated k^0 with the following equation derived from the kinetic theory of gases:

$$\frac{k^0}{k} = 1 + \frac{1}{r_1 \ln(r_2/r_1)} \frac{2-a}{a} \frac{k}{c_v}$$
$$\frac{\sqrt{2\pi RT}}{\gamma + 1} \frac{1}{p} \qquad (5)$$

He finally used eq. (4) as the inner boundary condition dropping out the subscript w from t.

This procedure amounts to saying that the wall and gas temperatures, t_w and t, can be represented by a single notation t just by imagining a fictitious thermal conductivity k^0 for the interfacial layer while keeping k for the remainder of the gas.

The present work was undertaken on the basis that the above treatment lacks a strict theoretical justification, or, at the most, is only an approximation. It was intended here to finalize the method of handling the temperatue jump problem in conjunction with the already established trequency response technique.

Theoretical

Suppose that the wire, mounted in a cylindrical shell, is heated by a d.c. current for a moment, and the temperature at the outer shell is maintained constant. If the wire temperature is maintained at somewhat a higher temperature than the outer shell, the heat input into the wire will be conducted away through the gas into the outer shell establishing a temperature profile as shown in Fig. 1.

In a highly idealized mathematical model, radiation may be assumed to be absent. In the figure, one will notice that t_w and t_g represent the wall and gas temperatures at the interface, and a temperature, t_k, is seen at the point where the wall line and the extension of the main body gas temperature profile intersect. For the purpose of theoretical analysis, t_k, instead of t_g, must be taken for the gas temperature t defined above for eq. (1). Thus, we may write, borrowing the expression of Kennard (7), as follows:

$$t_w \sim t_k = d \frac{\partial t}{\partial r} = \Delta t_J \qquad (6)$$

where $(\partial t/\partial r)$ is the temperature gradient of the gas in the neighbourhood of the wire, and d is the temperature jump distance given by (7)

$$d = \frac{2-a}{a} \frac{k}{c_v} \frac{\sqrt{2\pi RT}}{\gamma+1} \frac{1}{P}$$

or,   (7)

$$= \frac{2-a}{a} \frac{4c}{\gamma+1} \frac{k}{\eta c_v} L$$

where L is the mean free path of the gas molecules, c is a positive number ranging from 0.491 to 0.499, and a is the accommodation coefficient.
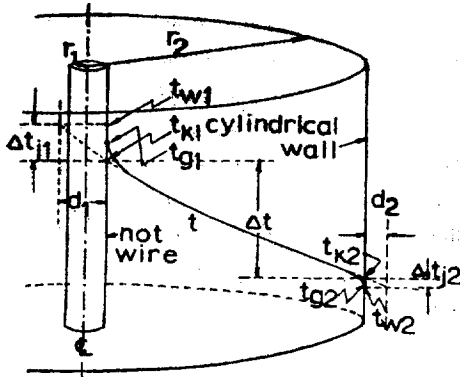

Fig. 1 Cylindrical System with Coaxially Mounted Wire

Definition of the Temperature Jump Heat Transfer Coefficient

If in Fig. 1, one defines the temperature jump at the wire and the shell by $\Delta t_{j1} = t_{w1} - t_{k1}$, and $\Delta t_{j2} = t_{k2} - t_{w2}$, respectively, and $\Delta t = t_{k1} - t_{k2}$, the rate of heat conduction is

$$q = 2\pi r_1 kl(\Delta t_{j1}/d_1)$$

$$= 2\pi r_2 kl(\Delta t_{j2}/d_2) \qquad (8)$$

$$= 2\pi kl \cdot \Delta t/ln(r_2/r_1)$$

Now, define k' such that

$$q = 2\pi k' l \; \Delta T/ln(r_2/r_1) \qquad (9)$$

where $\Delta T = t_{w1} - t_{w2} = \Delta t_{j1} + \Delta t + \Delta t_{j2}$, then, from eqs. (8) and (9),

$$q = \frac{2\pi kl}{ln \frac{r_2}{r_1} + \frac{d_1}{r_1} + \frac{d_2}{r_2}} \Delta T \qquad (10)$$

and from eqs. (9) and (10),

$$\frac{k}{k'} = 1 + \frac{d_1}{r_1 \, ln \frac{r_2}{r_1}} + \frac{d_2}{r_2 \, ln \frac{r_2}{r_1}} \qquad (11)$$

If one neglect the third term of the right side of the above equation because of $d_2 \ll r_2 ln(r_2/r_1)$, and replace k/k' with k^0/k, one obtains the expression given by eq. (5).

Let us now define h_j, the temperature jump heat transfer coefficient, as follows:

$$q = h_j A_1 \Delta t_{j1} \qquad (12)$$

Combining eqs. (10) and (12), one obtain

$$h_j = \frac{k}{r_1 \left(ln \frac{r_2}{r_1} + \frac{d_1}{r_1} + \frac{d_2}{r_1} \right)} \left(\frac{\Delta t_{j1} + \Delta t + \Delta t_{j2}}{\Delta t_{j1}} \right)$$

$$(13)$$

In the above equation, $d_2/r_2 \ll d_1/r_1$, that d_2/r_2 can be neglected. This implies that Δt_{j2} is also negligible because the gas molecules attain effectively an equilibrium temperature with the outer shell by colliding innumerable times with the shell during the time a molecule near the fine wire makes just one collision. This amounts to saying that the accommodation coefficient at the outer shell is, in effect, increased. Therefore, eq. (13) can be approximated by

$$h_j = \frac{k}{r_1 \left(ln \frac{r_2}{r_1} + \frac{d_1}{r_1} \right)} \cdot \left(1 + \frac{\Delta t}{\Delta t_{j1}} \right) \qquad (14)$$

Since at the steady state, at the interface $r = r_1$,

$$q = -kA_1 \frac{\partial t}{\partial r} = -kA_{av}\left(\frac{\Delta t}{r_2 - r_1} \right) \qquad (15)$$

where $A_1 = 2\pi r_1 1$ and A_{av} is the geometrical mean of A_1 and $A_2 = 2\pi r_2 1$, it follows that

$$\frac{\partial t}{\partial r} = \frac{\Delta t}{r_1 ln(r_2/r_1)} \qquad (16)$$

Therefore, from eq. (6), one obtains

$$\frac{\Delta t}{\Delta t_{j1}} = \frac{r_1}{d_1} ln \frac{r_2}{r_1} \qquad (17)$$

Substituting eq. (17) into eq. (14), one finally obtains

$$h_j = \frac{k}{d_1} \qquad (18)$$

Formulation of the Boundary Value Problem with Temperature Jump taken into Consideration

Suppose, now, that the fine wire is heated by a sinusoidal a. c. current, $i = I_0 sin\omega\theta$. Then, the wire temperature will fluctuate with two times the frequency of the current. The heat input into the wire is now considered to be transferred by both conduction and radiation. When the steady state is attained, there will be a temperature wave at each point in the gas travelling towards the outer shell with decreasing amplitudes, dying out completely as it reaches the outer shell.

The differential equation that applies within the gas region $r_1 < r < r_2$ is

$$\frac{\partial t}{\partial \theta} = \alpha \left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} \right) \qquad (19)$$

where $\alpha = k/c\rho$ is thermal diffusivity. The inner boundary condition is now written with a great discretion using different notations for the wall and gas temperatures. Namely, at $r = r_1$,

$$M \frac{\partial t_w}{\partial \theta} = I_0^2 \bar{R} sin^2 \omega\theta + kA \frac{\partial t}{\partial r} - hA(t_w - t_2) \qquad (20)$$

where $M = c'\rho' \pi r_1^2 l$, and A is the same as A_1, and h is the radiation heat transfer coefficient which can be estimated from the data of emissivity of the fine wire.

The heat capacity of the outer shell is made so large that the shell temperature is practically constant. Stated in the mathematical language this will constitute the outer boundary condition,

$$t = t_2 \quad at \quad r = r_2 \qquad (21)$$

Another relation, linking t_w with t, will be required for the complete solution of the problem. This relation can be obtained by equating the heat transfer with the temperature jump as its driving force and the second term of the right hand of eq. (20). Thus,

$$h_j A(t_w - t) = - kA \frac{\partial t}{\partial r} \quad at \quad r = r_1 \qquad (22)$$

Using the relation given by eq. (18), eq. (22) can be rewritten as follows:

$$t_w - t = - d_1 \frac{\partial t}{\partial r} \quad at \quad r = r_1 \qquad (23)$$

One should keep in mind that t corresponds to t_{kj} shown in Fig. 1, and the temperature profile used here is $t_{w1} - t_{k1} - t - t_{k2}$ rather than $t_{w1} - t_{g1} - t - t_{g2} - t_{wt}$.

Soluton to the Temperature Jump Boundary Value Problem

Temperature of the Gas

Table 1. Dimensionless Groups

Group Name	Symbol New	Old	Formula
Conductivity Raito	CR	Z	$\dfrac{2\omega c'\rho' r_1^2}{k}$
Dimension Ratio	RD		r_2/r_1
Diffusion Ratio		X	$(2\omega/\alpha)^{\frac{1}{2}} r_1$
Interface Ratio		E	hd_1/k, $RR \cdot D$
Radiation Ratio	RR	U	hr_1/k
Jump Ratio	D		d_1/r_1
Capacity Density Ratio	CDR		$c'\rho'/c\rho$
Amplitude Ratio	ARG, ARW	or	$\sqrt{g_D^2 + b_D^2 \cdot \frac{1}{4}CR}$ eq. (69)

The complete solution consists of the transient and stationay state part of the solution. However, only the later part is required in this case. It was

found in this solution that the same trial function as was previously used (1) leads effectively to the final solution. The form of trial function is

$$t(r, \theta) = f(r) + g(r)\cos 2\omega\theta + b(r)\sin 2\omega\theta \quad (24)$$

Using the dimensionless groups defined in Table1. the temperature of the gas at the wire-gas interface can be written as follows:

$$t_1 = t_2 + \left(\frac{I_2^0 \bar{R}}{4\pi kl}\right)\frac{lnDR}{1+RR(D+lnDR)}$$
$$+ \frac{I_2^0 \bar{R}(ARG)}{2\omega c'\rho'r_1^2 1}\cos(2\omega\theta+\phi) \quad (25)$$

where ϕ, the phase angle, is a function of g_D, b_D, and CR, or alternatively a function of CR, CDR, DR, RR and D.

Temperature of the Wire

The temperatures of the gas and wire are linked together by eq. (23), and the wire temperature can finally be written as follows:

$$t_w = t_2 + \left(\frac{I_0^2 \bar{R}}{4\pi kl}\right)\frac{lnDR+D}{1+RR(D+lnDR)}$$
$$+ \frac{I_0^2 \bar{R}(ARW)}{2\omega c'\rho'r_1^2 1}\cos(2\omega\theta+\phi_w) \quad (26)$$

Defining the amplitude ratio, ARW, and the phase angle ϕ_w, both based on the wire temperature where

$$ARW = \tfrac{1}{4}CR \sqrt{(g_D - d_1 g_0'\beta)^2 + (b_D - d_1 b_0'\beta)^2} \quad (27)$$

and ϕ_w, g_0' and b_0' are also functions of the same variables as those for ϕ.

Theoretical Calibration Curves

From the experimental point of view, the oscillating temperature of the fine wire can not probably be registered without being converted into the form of IR-drop. For this purpose, a fine wire of high temperature coefficient of resistance, such as of tungsten or platinum, is thought to be helpful. Assume that the resistance varies linearly with the temperature, and let $d\bar{R}/dt$ be the temperature coefficient

of the wire resistance at the average wire temperature. Then, the instantaneous wire resistance at an arbitrary temperature t_w is as follows:

$$R = R_2 + \frac{d\bar{R}}{dt}(t_w - t_2) \quad (28)$$

where R_2 is the wire resistance at $t=t_2$. The IR-drop, e, across the wire is now obtained by multiplying R by i, the heating current, and substituting eq. (26) into eq. (28). Thus,

$$e = \left[I_0 R_2 + \frac{I_0^3 \bar{R}}{4\pi kl}\frac{lnDR+D}{1+RR(D+lnDR)}\cdot\frac{d\bar{R}}{dt}\right]\sin\omega\theta$$
$$-\left[\frac{I_0^3 \bar{R}(ARW)}{4\pi\omega c'\rho'r_1^2}\cdot\frac{d\bar{R}}{dt}\right]\sin(\omega\theta+\phi_w)$$
$$+\left[\frac{I_0^3 \bar{R}(ARW)}{4\pi\omega c'\rho'r_1^2 1}\cdot\frac{d\bar{R}}{dt}\right]\sin(3\omega\theta+\phi_w) \quad (29)$$

Eq. (29) suggests that the IR-drop consists of a fundamental harmonic wave of a large amplitude in phase with the current, another fundamental harmonic wave out of phase by ϕ_w from the current, and a third harmonic wave out of phase by ϕ_w. The amplitudes of the latter two harmonics are identical, but, are order of only one millionth of the main fundamental harmonics in magnitude. If the frequency sensitive Wienbridge, in which the measuring cell is placed as an arm, is actuated by the sinusoidal current $i=I_0\sin\omega\theta$, the balancing of the bridge will automatically eliminate the small and large fundamental harmonics leaving the third harmonic component alone in the bridge output. The signal, though small, can be registered in a VTVM after amplifications in several steps. From the calibrated reading, it is possible to calculate ARW. The value of ϕ_w can also be calculated from the readings of the variable capacitor which is a component of the bridge. The capacitor is adjusted to obtain a sharpest balance of the bridge. However, the capacitor readings are, in general, not as accurate as the resistance readings. We will, therefore, leave out the phase angle from our discussion.

Having determined our policy of measurement, let us go back to the theory to recall that ARW was

expressed in terms of

$$ARW = \text{func}(CR, CDR, DR, RR, D) \qquad (30)$$

The basic principle of the measuring method can be established by paying attention to the relation ARW vs. CR, while regarding CDR, DR, RR, and D as parameters. The relation plotted on a log-log graph paper for several different sets of parameters are called the "Theoretical Calibration Curves", although, actually, a computer program rather than drawn curves will be more useful. Fig. 2 shows some of such curves drawn when there are no radiation and temperature jump.
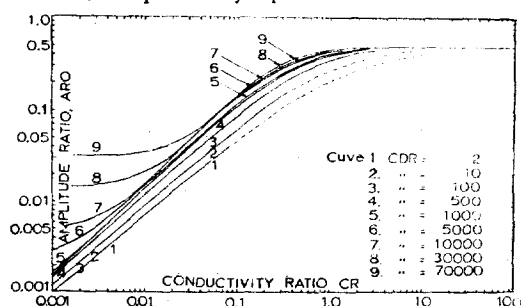

Fig. 2 Theoretical Calibration Curves for DR=1000, RR= 0, and D=0

When there is temperature jump, but no radiation (for simplicity), both ARW and ARG must be considered against CR. Such relations are shown in Fig. 3. It is evident although ARW approaches asymptotically to 0.5 as CR is increased, ARG always approaches to somewhat lower value merging in one curve as the temperature jump dies away. In any case, the effect of temperature jump on the ARW or ARG vs. CR curves is quite evident when the temperature jump can not be overlooked.
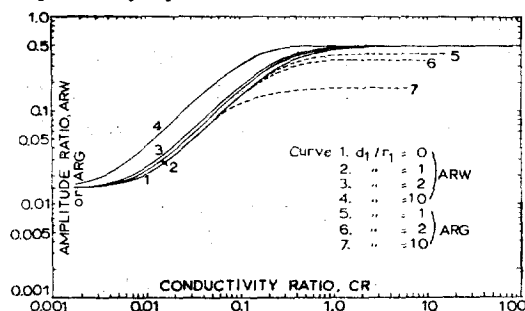

Fig. 3 Theoretical Calibration Curves for DR=1000, RR= 0, and CDR=50,000

Now, suppose that we have got the value of ARW calculated from the experimentally found amplitude, then one can pick up the corresponding value of CR from the theoretical calibration curve, or, using the computer program. The desired value of thermal conductivity can then be calculated from the known frequency value of heating current and the value of wire radius.

The method, just described, was named the "Absolute Method of Measurement". This method can be used only when(1) the mathematical model is real, particularly, when the heating current takes an exact sinusoidal wave form, (2) the hot wire radius is known with a sufficient accuracy, and (3) the physical properties of the gas and the hot wire material are known accurately. However, these requirements are impossible to be completely fulfilled. For this reason, this method shall not be followed further.

Frequency Response Technique as the Relative Method of Measurement

Consider any single curve in either Fig. 2 or 3. Regardless what substance—gas or liquid—is being tested, there is always a single value of ARW for a given value of CR, due to the virtue of the dimensionless groups used for the correlation. For a given fine wire, the value of CR is fixed if the ratio of frequency to thermal conductivity is unchanged. Alternately, if the gas and wire is fixed, ARW will depend only on the frequency of the heating current. Therefore, ARW can be regarded as a frequency response. Let us define what we shall call the "Frequency Factor" as follows:

$$GP = CR \cdot k \cdot 10^5 = 2\omega c' \rho' r_1^2 \cdot 10^5 \qquad (31)$$

We will now consider ARW as a function of GP with other dimensionless variables and k retained as the parameters. Namely,

$$ARW = \text{func}(GP, CDR, DR, RR, D, k) \qquad (32)$$

If the ARW vs. GP curves are drawn experimentally keeping other dimensionless parameters constant for two different gases whose thermal conduc-

tivities are k_x and k_r, the shapes of the two curves will appear as shown in Fig. 4. If the two gases had the same thermal conductivies, the two curves would have been superposable. Consider a value of ARW at any level in Fig. 2 or 3, then it follows that

$$CR_x = CR_r \qquad (33)$$

Now, shifting our attention to Fig. 4, and recalling the definition given by eq. (31), we can write as follows:
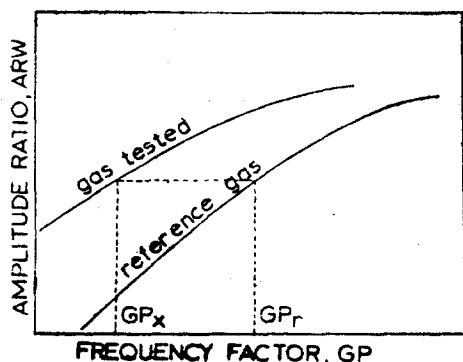

Fig. 4 Principle of Calculating Thermal Conductivity by the Relative Method

$$\frac{GP_x}{k_x} = \frac{GP_r}{k_r} \qquad (34)$$

or,

$$k_x = \frac{f_x}{f_r} k_r \qquad (35)$$

The above relation states that, if gas r is chosen as the reference gas, whose thermal conductity is well known, it is possible to calculate the thermal conductivity of the unknown gas x in terms of k_r, the thermal conductivity of the reference gas, by multiplying it by the ratio of the frequencies corresponding to GP_x and GP_r.

This method was named as the "Relative Method of Measuremrnt", or, as the "Frequency Response Technique" because of the philosophy involved. This method does not require the model to be strictly real, and the wire radius to be known accurately. The method, however, does not produce data of any higher accuracy than those of the reference gas, regardless how carefully the measurements are taken. The produced data are subject to revision whenever

the data of the reference gas are improved.

New Method of Correcting for the Temperature Jump Effect

The frequency response curve for the reference gas is constructed from the data taken at the normal temperature and pressure using a wire of fixed radius. So, the value of CDR and DR are fixed. Since, in the measurement, the wire and the shell temperatures are made different by less than $10°C$, the radiative heat transfer may be considered negligible letting $RR=0$. Since at the atmospheric pressure, the mean free path of the molecules is small, the temperature jump effect may become almost negligible. Thus, $D=0$.

On the other hand, the frequency response curve for a gas of unknown thermal conductivity is constructed from the data taken frequently at a low pressure and a high temperature. So, the effects of temperature jump and radiation intervene in the measurement of ARW, and so do likewise the effect of CDR by so much depending on the extent to which the CDR is different from that for the reference gas, and the effect of the axial conduction at the both ends of the wire.

These effects must be corrected for before the constructed frequency response curve for the test gas can be compared with that for the reference gas. The methods of correcting for the effects of radiation (RR), capacity density ratio(CDR), and the end conduction are well described in the previous work(5). In this work, we are primarily interested in the method of correcting for the effect of temperature jump.

For this purpose, we need to calculate the jump ratio D and the value of ARW as well as ARO using the theoretical formulas. It is possible, then, to define the following:

$$TJC = \frac{ARW - ARO}{ARO} \qquad (36)$$

where TJC, whose meaning is selfexplanatory, is the abbreviation for temperature jump correction term, and ARO is the amplitude ratio when $D=0$,

but, the other parameters are the same as for ARW.

Having defined the TJC, the next step is to calculate the corrected amplitude ratio ARO_e as per the following formula from the amplitude ratio ARW_e determined experimentally.

$$ARO_e = \frac{ARW_e}{1 + \frac{ARW - ARQ}{ARO}} \qquad (37)$$

After all corrections, including the one just mentioned, are made, one can immediately follow the procedure described in the proceding section to grind out the final value of thermal conductivity. The present method of correction is based purely on the theory. Although the theory based on the model may not apply exactly to the actual case, consideration has been paid in defining eq. (37) so that the result undergone this correction is still correct.

Of all the information needed for calculating the TJC, The valve of accommodation coefficient is most poorly available. It is not even easy to determine when it is not available.

The previously shown Fig. 4 is, in fact, the plot of ARO_e against GP.

Application of the New Method of Correction to the Experimental data Application to the Low Pressure Argon data

In order to test the validity of the new method for correcting for the temperature jump effect, the method was first applied to the low pressure data on
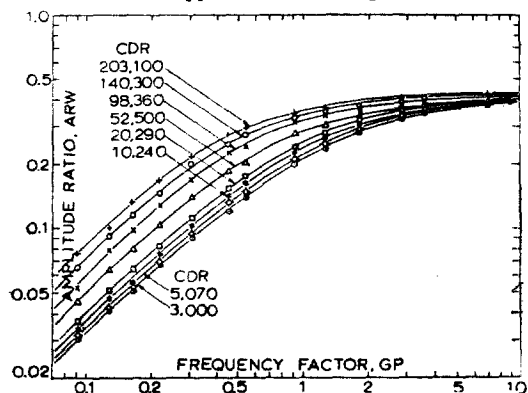

Fig. 5 Experimental Frequency Response Curves for Low Pressure Argon Runs Prior to the Temperature Jump Correction

argon gas. The data were summarily shown in Fig. 5.

The wide spreading of eight curves from one another is due to the difference in both CDR and temperature jump effect. It will be seen how closely the spreadings are narrowed down by applying the temperature jump correction in Fig. 6.
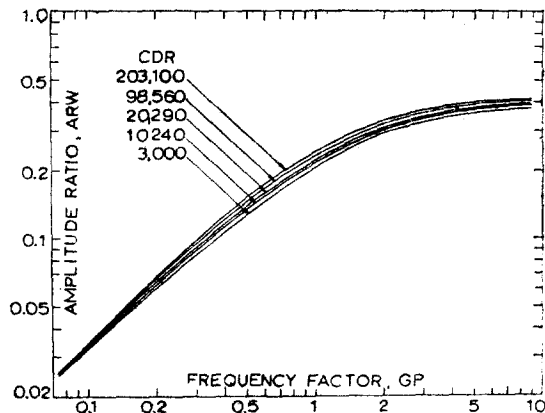

Fig. 6 Experimental Frequency Respone Curves for Low Pressure Argon Runs after the Temperature Jump Correction

In thid figure only five corrected curves are shown in order to avoid overcrowing. We may not be able to exactly superimpose the corrected experimental curves upon the theoretical curves in Fig. 2 which were constructed on the mathematical model. However, we can take the close resemblence between the two groups of curves as an ample evidence for the correctness of the method. Fig. 6 can also be effectively used for formulating the method of correcting for the effect of CDR difference from that of the reference gas.

In measuring the thermal conductivity, low pressure gases were not intentionally sought. There is of course no sense in measuring thermal conductivity at reduced presures because the pressure does not appreciably affect the thermal conductivity. The low pressure measurement was rather compelled by the difficulty in obtaining metal vapors at nearly one atmospheric pressure. The pressure actually encounted here in this experiment ranges from 0.08 to 0.2 atmospheres.

Here, by all means, the cases where pressure is so low that heat is transferred by the bombardments of

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unobstructed molecules is not considered.

Because of the fineness of the wire used, which is 0.0003 cm in diameter, the pressure range encounted here already belong to the case where the mean free path of the gas becomes comparable to the wire diameter.

Application to Cesium and Rubidium Vapors

Figs. 7 and 8 show the working curves for cesium and rubidium vapors along with the reference curves obtained using argon.
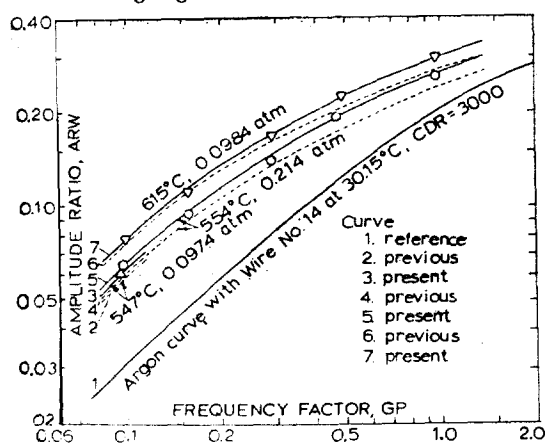

Curve
1. reference
2. previous
3. present
4. previous
5. present
6. previous
7. present

Fig. 7 Morking Curves for Cesium Vapor based on CDR= 3000

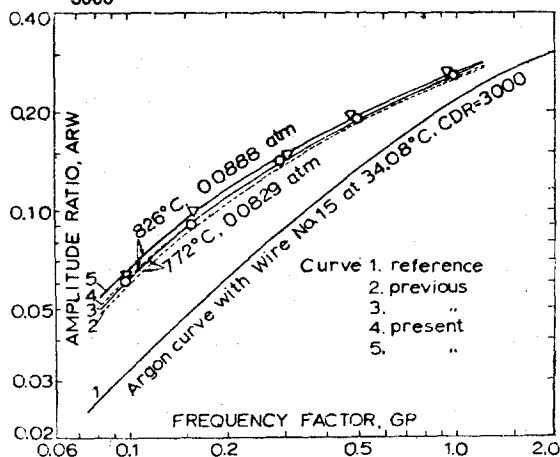Curve 1. reference
2. previous
3. "
4. present
5. "

Fig. 8 Working Curves for Rubidium Vapor based on CDR =3000

If the test gas curve comes on the upper side of the reference curve, the thermal conductivity of the gas is lower than that of argon, and vice versa.

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Conclusion and Evaluation

The thermal conductivity values must theoretically be the same regardless at which level of ordinate the calculating procedure is carried out. This is unfortunately not so in actuality. The calculation must be carried out in the region of ordinate where the curves become almost linear or parallel to each other. The thermal conductivity values calculated in this manner are shown in Table 2. The comparison is made with the old values(5, 6) which were produced by an approximate mathod of correcting for the temperature jump effect. The difference seems to be trivial. But the present method has a much more sound logical backing.

Table. 2 Thermal Conductivity of Cesium and Rubidium Vapors in cal/sec. cm. °C

Metal	Temp. °C	Press. atm	k, old $\times 10^4$	k, new $\times 10^4$
Cs	615	0.214	0.1587	0.1515
Cs	554	0.0984	0.2060	0.1838
Cs	547	0.0974	0.2120	0.1950
Rb	826	0.0829	0.2158	0.1980
Rb	772	0.0882	0.2280	0.2160

The extent of temperature jump correction applied to the amplitude ratio ranges approximately from 3 to 19%, and this value is about 5 to 25% smaller than the amount of correction applied previously.

The auter has previously published the theoretically calculated values of the thermal conductivity of alkali metal vapors. The final experimental values here are in reasonaly good agreement with the theoretical values. There are some work on measuring the thermal conductivity of sodium and potassium vapors in the Soviet Union, and of cesium and rubidium vapors in the United States. The accuracy claimed by the Soviet authors are 20% and their radiative part of heat transfer in the measuring cell is said to have amounted to as much as four times that of conduction. The workcarried out in the United States deviates considerably from the authors' theoretically calculated values, perhaps, because their data were processed under the assumption that the vapour

is monoatomic.

The value obtained here for cesium and rubidium vapors are correct within 6%, and they fall essentially within the region predicted by the theory.

Finally the author wishes to acknowledge the Ministry of Science and Technology for having provided the fund for this work.

Nomenclature

a; Accommodation coefficient

c; Specific heat at constant pressure of the test gas

c'; Specific heat of the fine wire

d; Temperature jump distance

h, h_j; Heat transfer coefficients

k; Thermal conductivity of test gas

p; Pressure of the test gas

t; Instantaneous gas temperature

t_w; Instantaneous fine wire temperature

I_0; Amplitude of the heating current

$\bar{R}$; Fine wire resistance at the average wire temperature

ARG; Amplitude ratio based on the gas temperature

ARW; Amplitude ratio based on the wire temperature

γ; Ratio of the specific heats

θ; Time

ρ, ρ'; Densities of the gas and fine wire

ϕ, ϕ_w; Phase shift angles based on the gas and wire temperatures

ω; Angular velocity, $2\pi f$

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