總　　說
REVIEW

베나드 對流現象

崔　昌　均

韓國科學技術研究所 化學工程研究室

Bénard Convection

C. K. Choi

Chemical Engineering Process Laboratory Korea Institute of Science and Technology
Seoul 132, *Korea*

요　　약

最初 停止狀態에 있는 流體를 밑에서 加熱할 때, 基本狀態가 不安定하게되어 생기는 自然對流現象은 오랜동안 相當한 關心을 지닌 研究對象이 되어 왔다. 通常的으로 베나드對流라고 불리워지는 이러한 現象은 主로 加熱로 因한 比重差異 또는 溫度기울기로 因한 表面張力 變化로 일어난다는 것이 알려져 왔다. 本總說에서는 重點的으로 規則的인 形態로부터 漸進的으로 不規則的인 流動現象을 보여주는 베나드 對流現象을 檢討한 후, 附加的으로 흐르는 流體를 加熱함으로써 생기는 二次的인 流動現象과 가스吸收 또는 電場下에서 일어나는 對流現象을 紹介한다.

Abstract

Natural convection in an initially stagnant horizontal layer of fluid heated from below has received considerable attention since it is a simple example of convective motion generated by the instability of a basic state. Traditionally, it is called Bénard convection. The principal convection mechanisms have been identified: buoyancy forces resulting from thermally induced density gratients and/or interfacial forces due to surface tension variations produced by temperature gradients. The facts and the problems of Bénard convection in which turbulence gradually emerges from an ordered pattern are discussed here. Additionally, thermal convection with flowing films and convection induced by gas absorption and electric fields are introduced.

1. Introduction

The convective flow with a thermal orgin is one of the most interesting phenomena that nature has hidden from our view. This kind of natural convection occurs in a lake, inside the viscous mantle of our planet, or even in a coffee cup, but it is rarely discernable to the naked eye.

Bénard convection exists in its simplest form when a thin horizontal layer is heated from below so that the warm fluid near the bottom tends to float up buoyantly and displace the denser fluid above. In a range exceeding some adverse temperature (concentration) gradient the increase of heat transfer due to convection that occurs has been recognized to be of practical importance. For example, in heat transfer between the ground and the lower atmosphere, and in heat transfer through a pipe or duct, etc. Such convection also occurs often in distillation, absorption, and extraction in which heat or mass transfer occurs. In addition to its practical applications, Bénard convection provides one of the most ideal methods of studying turbulence as turbulent patterns emerge gradually followed by several discrete changes in flow pattern.

Since more than 400 contributions have been made to this field, the present review focuses on convective motion with a thermal origin. Additionally, some relavent topics are presented. Convection in the atmosphere and astrophysics, and the geophysical implications of convection are not covered here. As the linear theory is the solid basis for all convection work, we stick closely to results pertaining to this theory as far as concerned. Also, the review is mostly confined to convection in thin horizontal layers.

2. Buoyancy-driven convection

2.1 Basic considerations

Buoyancy-driven convection in a fluid heated from below can be regarded as a typical example of a gradual transition from laminar to turbulent flow. In contrast to the flow pattern exhibited by plane parallel shear flows, a horizontal film heated from below shows transitions from one type of stationary convection to another. When the initially motionless thin film is heated from below, the layer will become unstable as the buoyancy force is increased sufficiently to overcome the stabilizing effect of viscosity and thermal conduction.

Early work consisted chiefly of casual or experimental observations concerning peculiar fluid motion which were either buoyancy-driven, surface-tension-driven or driven by combined effects. In 1900, Bénard (4) published the first systematic experimental results in a thin horizontal layer bounded by rigid-free boundaries. The principal results are: first, a certain critical adverse temperature gradient must be exceeded before instability sets in; and second, the motion which ensues on surpassing the critical value has a stationary cellular character. In 1916, Lord Rayleigh (75) laid the corner-stone for theoretical interpretation by analyzing the response of a liquid layer with an unstable density distribution to infinitesimal disturbances. Since then refined analyses and experiments of Bénard convection have been developed.

In the theoretical analyses all nonlinear terms in the continuity, momentum and heat equations are neglected. This linearized stability analysis is conducted under the Boussinesq approximation, i.e., the density "ρ" is treated as a constant except in the external force of gravity "g", It is further approximated that the variations of kinematic viscosity "ν" thermal diffusivity "α" and thermal expansion coefficient "β" have negligible dependence on variations of temperature. The classical problem of thermal convection in Bénard convection is to analyze the motion of fluid confined between two horizontal planes uniformly heated from below and cooled from above with constant temperature difference "ΔT" and vertical thickness "d". These boundaries are either rigid or free. No stress on a free surface excludes surface tension effects, while no slip applies to a rigid boundary. Additionally, no temperature disturbance at the boundaries is assumed, implying perfect conduction. Usually no lateral boundaries are considered because the liquid layer is assumed to be of infinite horizontal extent. With this model, it is found from the proper dimensionless governing equations that the only relavant parameters are the Rayleigh number $Ra=\beta g d^3 \Delta T/\alpha\nu$ and the Prandtl number $Pr=\nu/\alpha$. To determine the stability of the stationary film to infinitesimal disturbances, the locus of the disturbances must be traced continuously after they appear. If disturbances decay with time, the system is said to be stable; if they grow, the system is unstable. There is also the possibility of oscillatory motion, i.e., overstability. However, overstability cannot occur in buoyancy-driven convection on a stagnant film as shown by Pellew and Southwell (72). Based on the principle of exchange of stabilities, i.e., the threshold of instability is marked by a stationary motion with the characteristic wave number "a_c" corresponding to the critical minimal Rayleigh number "Ra_c", Low (55), Pellew and Southwell (72), and Reid and Harris (76) obtained solutions for free-free, rigid-free and rigid-rigid boundaries. Detailed reviews on the classical hydrodynamic stability were provided by Lin (54), Chandrasekhar (13) and Berg, et al. (5).

It is known that the above linearized theory corresponding to a primary constant temperature gradient indeed predicts, to a good approximation, the stability of a layer and the size of resulting convection cells. However, if the experiment is performed with rapid heating exceeding Ra_c, the time to reach the onset of convective motion becomes a dominant question. Soberman (85), and Nielsen and Sabersky (66) reported experimental results such that the rapid heating can increase Ra_c required to initiate thermal convection. Recently, this stability problem for nonlinear temperature profiles in the unperturbed state has been investigated.

The earliest study pertaining to time-dependent heating was done by means of the extant marginal stability concept by Morton (64) for the special case wherein the deviations from a linear density gradient are small. Lick (53) proposed that the undisturbed nonlinear temperature profile may be approximated by two linear segments, where the area under actual and approximate curves is the same by using the same temperature limits. Through this approximation Lick (53) and Currie (17) analyzed the disturbances in a finite fluid. This method seems to have merit, but the criterion corresponding to the intersection point requires further study. This kind of quasi-stationary approach using the principle of exchange of stabilities has been critically attacked for the application to the case of a step change of surface temperature, i.e., very rapid heating from below or cooling from above, by Foster (26) and Gresho and Sani (34). They claimed that the quasistatic model is a poor one owing to the effect of the rapid change of the undisturbed temperature profile upon onset of instabilities, except when the temperature profile is very nearly linear. Differing from the marginal state concept, they suggested redefining the critical values as occurring at the time when the first motion is discernable. Assuming some initial disturbance the critical state was determined at the time when the fastest growing disturbance has grown by several orders of magnitude. The validity of this concept requires further justification because the initial conditions of disturbances and the means to predict the first observable convective motion are not easily described in each experiment. For a semi-infinite fluid initially with a constant temperature profile it is evident that there exists no definitive initial disturbance owing to the absence of the driving force. But this approach may produce some useful information to understand thermal convection by generating the propagation of disturbances from proper initial conditions.

2. 2 Finite amplitude convection

The stationary motions above Ra_c are not uniquely established by linear theory. In order to obtain the cell structure, flow direction and amplitude of the final steady state configuration in the supercritical range nonlinear theory must be introduced. In this field, the augmentation of heat transport is represented by the Nusselt number, Nu, which is usually defined as the ratio of the actual heat transport to the purely diffusive flux through a linear temperature gradient between two boundaries. This parameter must be a function of Pr and Ra.

First, it is necessary to understand the real physical features through observations. Bénard's observations (4) focused attention on the hexagonal cells which for a time were considered to be the only convectine pattern on a horizontal plane plate. In 1913 Dauzere (18) showed irregular rolls and patterns with singular circular cells in a field of irregular hexagonal cells, which were actually the first evidence of the ring-cell. Graham (33) observed the interesting phenomena in a layer of air heated from below that under steady conditions the fluid was descending in the cell centers as opposed to ascending motion in those of liquids. Graham argued that the variation of viscosity "μ" ($\partial\mu/\partial T>0$ in gases and $\partial\mu/\partial T<0$ in liquids) was the cause for the reversal of the direction of flow. In other words, the convective motions start in the region where viscosity is lowest, i.e., at the bottom in liquids and at the top in gases. Graham's assumption was verified by Tippelskirch (90). Another set of experiments was concerned with the heat transfer through a fluid layer. Schmidt and Milverton (79) discovered that the onset of convection is accompanied by a sudden increase in the heat transfer through a liquid. After the onset of convection heat transfer is supplemented by convection in addition to thermal conduction. Therefore, a definite break in the heat transfer curve should occur at the critical state. Using this concept they obtained the first quantitative measurement of the critical Rayleigh number ($Ra_c=1,770\pm140$ for rigid-rigid boundaries). In the further investigation Malkus (57) reported the change in slope of heat transfer at discrete points, implying that the cellular convection undergoes a series of discrete transitions to more complicated flows as Ra is increased.

In one of the earliest investigations on finite amplitude convection Malkus and Veronis (58) obtained steady-state solutions by expanding the variables in terms of the amplitude parameter $(Ra/Ra_c-1)^{1/2}$ valid only for small departures from Ra_c. They suggested a criterion for the realized solution on the basis of maximal vertical heat transport. This theoretical analysis of supercritical Bénard convection was the first effort to develop nonlinear stability theory and has had a very strong influence on later work. For the free boundaries their results led to square or rectangular cells with the increase of the preferred horizontal wave number as Ra was increased. The initial heat flux showed the linear dependence on Ra. Using this kind of perturbation solution based on parametric expansions very near Ra_c, Schlüter, et al. (78) and Palm, et al. (70) concluded that only the two-dimensional convection in the form of rolls is a stationary solution among all possible motions in the Boussinesq approximation. This nonlinear stability theory was summarized by Segel (82).

In agreement with analytical predictions of Schlüter, et al. (78), Silveston (83), and Somerscales and Dougherty (86) reported experimental results. They showed that two-dimensional rolls prevail near Ra_c except those hexagonal cells in a fluid layer with a sufficiently great variation of physical properties due to temperature change. From various experimental results it is concluded that the planform of the convective motion of a Boussinesq liquid on a uniformly heated plane and under a uniformly cooled lid is determined by the geometry of the walls. In a circular container circular concentric rolls appear (47), while in a rectangular container convective motion sets in with the form of rolls whose axes are parallel to the shorter side of the container (24). Also, it has been observed (47, 86) that convective motions initiate at the walls under the subcritical condition and the critical state showing the change in the heat transfer curve is reached when the motion covers the whole plate. In particular, the critical Rayleigh number depends upon the aspect ratio of a fluid layer. It has been found that an experiment with the aspect ratio larger than 20 is a reasonable approximation to an infinite layer, but the strictly straight parallel rolls predicted by theory cannot occur in the physically realized system. Therefore, it is evident that lateral walls are necessarily a finite disturbance in a liquid layer.

On a subsequent steady two-dimensional roll analysis Herring (36) introduced a model which greatly simplifies the analysis using mean field equations averaged over horizontal planes neglecting self-interactions of nonlinear terms. Also, Howard (40) proposed the Euler equations of the upper bounding approach to thermal convection. Deardorff (20), Fromm (28), Plows (74) and Moore, et al. (61) solved this problem numerically by using high speed computing techniques. Generally, these results have shown approximately 1/3-power dependence of Nu on Ra. Using Galerkin's scheme, Busse (9) found for two rigid boundaries at the high Pr that two-dimensional rolls are stable in a large range of $1,708<Ra<22,600$, but stable only in a small range of wave numbers near the critical value. Busse and Whitehead (10) confirmed the above results with experiments and further reported that nonlinear instability mechanisms induced by the lateral walls have been shown to change the wave number.

The great controversy on the magnitude of stationary wave numbers in the supercritical steady state has taken place only recently. All the theoretical analyses of stationary two-dimensional rolls have predicted without exception that the wave number would increase with the increase of Ra under the maximal vertical heat transport hypothesis of Malkus (57). Contrary to these predictions, Silveston (83), Rossby (77), Krishnamurti (48), Willis, et al. (96), Farhadieh and Tankin (24), and Koshmieder and Pallas (47) have observed experimentally that the wave number decreases with the increase of Ra. They have claimed that the maximal vertical heat transport governing all the conventional theoretical analyses is an unrealistic hypothesis. There is obviously no single analytical result to predict a wave number decreasing with the increase of Ra for a stationary motion. This essential step for the subsequent theoretical develop ment remains an unsolved one.

Recently Koshmieder (46) reported briefly that after sudden heating a wave number initially higher than the critical one appears and decreases slowly to steady lower limits if the suddenly applied supercritical condition is maintained long enough. Also, the effect of initial conditions and lateral boundaries on convection were analyzed by Foster (27). He found that for a particular Ra the number and size of the convection cells formed depend upon the aspect ratio and initial conditions, and if there are no lateral boundaries, the lateral edges of cells tend to tilt and Nu increases slightly. Caldwell (11) reported observations of the temperature drop as a function of heat flow on curved density profiles, showing slope reversal in the heating curve, oscillations with time, history dependence and an increase in Ra_c, as the curvature of the density profile is increased. However, it is noted that there is as yet neither absolutely unambiguous experimental evidence nor a definitive analytical explanation to correlate physical variations with new effects except those of Tippelskirch (90).

2.3 Turbulent thermal convection

In a horizontal layer several distinct transitions appear before flow with thermal stratification becomes turbulent. For two rigid boundaries the first transition from no-motion to a steady two-dimensional roll or a steady hexagonal cell appears independently of Pr at around $Ra_c=1,708$, which is consistent with linear stability theory for a linear temperature profile. The second transition from a steady two-dimensional roll to a steady three-dimensional roll occurs around 12 Ra_c with no definite dependence on Pr for $Pr \geq 7$. It has been shown by Willis and Deardorff (95), and Krishnamurti (48) that this transition is associated with the discrete change of heat flux curve consistent with the second transition observed by Malkus (57). The stability of two-dimensional rolls with respect to three-dimensional disturbances was analyzed by Clever and Busse (15), finding that the oscillatory instability is caused by the momentum advection terms. Krishnamurti (49, 50) reported detailed observations to the effect that for $Pr>50$ the steady three-dimensional roll persists for 12 $Ra_c<Ra<$ 55,000 and above this limit the flow becomes time-dependent with a slow tilting of the cell boundaries and a faster oscillation showing the nature of hot or cold spots advected with the original cellular motion.

As the oscillations increase in number and frequency, the flow becomes more distorted and turbulent. The transition to turbulence does not depend only on Ra but also on Pr. For low Pr (e.g., for mercury) no steady flows have been observed above Ra_c. For large Pr steady convection persists to rather large Ra. One may speculate whether oscillatory instabilities can be expected without the presence of the momentum advection terms in the limit of infinite Pr. No definite answer to this question has been given yet. Rossby (77) suggested the transition occurs at $Ra \simeq 14,000\ Pr^{0.6}$ for $Pr>1$, but this requires further investigation.

Various heat flux measurements have been made in this field. Silveston (83), Globe and Dropkin (31), Deardorff and Willis (22), Rossby (77) and Hollands, et al. (37) studied the range $0.02<Pr<10^4$, which shows some Pr dependence even at large Pr. Townsend (92) and Deardorff and Willis (22) made detailed measurements on mean and fluctuating quantities at various vertical distances. These observations on turbulent convection make it clear that heat transport from a heated boundary is intermittant rather than steady. Warm fluid slowly accumulates in narrow vertical regions and then breaks away either as a thermal which rises fairly rapidly into the interior of the layer, or an unsteady plume which wanders around the surface. Howard (39) proposed the following convection model centered on the idea of thermals. The thickness of the conduction boundary layer is built up until the Rayleigh numder based on ΔT and its conductive thickness, δ, Ra_δ reaches the order of

1, 000. Thus, for most of the time heat transport near the boundary is one of conduction, followed by a comparatively short time interval locally restored to the original uniform state after the removal of buoyant fluid as a thermal. This model received strong support from the detailed experimental results and also the numerical simulation of Elder (23). The agreement with observations has been found to be reasonable to a certain degree even though the model is not rigorous.

3. Surface-Tension-Driven Convection

In 1956 Block (8) discovered that the hexagonal cells of Bénard's original experiment are almost entirely due to surface tension variation in an experiment where the liquid is cooled from below with the upper boundary free. He concluded that the cells in this stably stratified layer must have been caused by variations of surface tension with temperature change. This result casts doubt upon the conclusions for the case of instabilities in some of the previous performed studies which considered only the buoyancy-driven convection with a free upper boundary.

In 1958 Pearson (71) established the basis of theoretical analysis such that Bénard's cells can be induced by only surface tension gradients on the free surface in the absence of the gravity force; the onset of convection is determined by the Marangoni number, $Ma=-\frac{d\Delta T}{\mu k}\left(\frac{\partial\Omega}{\partial T}\right)$, where Ω is the surface tension. This parameter indicates that convective motions in a very shallow fluid layer with a free boundary can be caused by surface tension variations on a free surface rather than vertical density variations. Scriven and Sterning (81) extended Pearson's work by including only the effects of surface viscosity and surface displacements without buoyancy effects. They found that in surface-tension-driven convection there is an upflow beneath depressions and a downflow beneath elevations on the free surface. Nield (65) applied Pearson's variable surface tension condition to a fluid layer subject to buoyancy force. He concluded that when the mechanisms reinforce one another they are closely coupled and mutually influence the stability limit. In two-fluid horizontal layers, Smith (87) added surface waves and found instability for heating in either direction. Zeren and Reynolds (97) showed that heating from above increases Ma_c and stabilizes certain wavelength disturbances, whereas heating from below lowers Ma_c and adds a buoyancy-driven instability mode. An unusual experiment related to this field was carried out on board the Apollo 14 and 17 spaceships and developed by Grodzka and Bannister (35). In almost zero gravity(less than 10^{-6} g) they found that there are, as expected, convective motions caused by surface tension gradients in a plane liquid layer heated below with a free upper surface, while heat flow in enclosed fluids occurs mainly by diffusive heat conduction.

4. Thermal Convection with Flowing Films

The stability problem of horizontal parallel flow has been generally recognized to have the same criteria for thermal convection as those of an initially motionless film. Strictly speaking, there exists two kinds of instability. The first one is purely thermal in origin; it sets in at Ra_c independently of shear and takes a longitudinal roll mode. The other leads to Tollmien-Schlichting waves which characterize a shear flow instability over a solid bounday. In a plane Couette flow Kuo (51), Deardorff (21), Gallagher and Mercer (30), Ingersoll (45) and Davies-Jones (19) excluded the latter effect due to intrinsic stability of the flow itself such that the longitudinal roll occurs independently of the presence of shear flow. The trend of their results suggests that the most preferred unstable mode is always a longitudinal roll, independently of the Reynolds number, *Re*, if there are no lateral boundaries. In a poiseuille flow Gage and Reid (29) investigated the interaction between two mechanisms by introducing the Richardson number $Ri=-Ra/(64PrRe^2)$ that for $Pr=1$ there exists a unique value $Ri_*=-0.92\times10^{-6}$ which marks the abrupt transition from one type of instability to the other. They concluded that for $Ri<Ri_*$ Squire's theorem, i.e., two-dimensional disturbances are more unstable than three-dimensional ones, is not valid and the instability is purely thermal in origin. The smallness of this value emphasizes the dominant role of the density gradient. Görtler (32) suggested that a buoyancy force component directed normal to a wall would cause a longitudinal roll disturbance on a forced convection main flow analogous to that due to the stabilizing effect of the centrifugal forces associated with the concavity of the wall.

Longitudinal rolls were observed by Chandra (12) and Hung and Davis (41) in a plane Couette flow, by Mori and Uchida (63), Akiyama, et al. (1), and Ostrach and Kamotani (68) in a plane Poiseuille flow, and by Sparrow and Husar (88) in a slightly inclined plate. In a thermal entrance region with the externally imposed velocity a question arises about the axial position from where the natural convcetion sets in. This stability problem is analogous to the transient behavior in an initially stagnant film heated rapidly from below and was analyzed by Hung and Davis (41) for plane Couette flow, and by Hwang and Cheng (42) for plane Poise uille flow. Only the former one was compared with observations, showing that the conventional marginal state concept would be valid only in the region with nearly linear temperature profiles in the unperturbed state. All these analyses were modifications of the marginal state concept made by "freezing" the temperature profile at each axial position. Nonlinear aspects of the supercritical range were investigated numerically by Ogura and Yagahashi (67), and Hwang and Cheng (43). For further justification more refined experiments are required.

In view of the practical importance a great deal of experimental work has been done on the combined convection for a fluid flowing along a heated horizontal pipe. By the method of small perturbations Velte (93) analyzed the fluid motion. A thorough review of overall heat transfer characteristics was made by Bergles and Simonds (6), and Morcos and Bergles (62). The data indicate that the developing length for a secondary flow becomes much shorter than that required in the absence of natural convection, resulting in the gradual increase of thermal enhancement as the Rayleigh number is increased.

5. Other Related Topics

Several studies have been published on the convective motion created by solute concentrations instead of temperature differences. A film of liquid absorbing gas at the free surface was investigated by Plevan and Quinn (73), Mahler and Schecter (56), and Blair and Quinn (7), which is analogous to transient cooling from above with Bénard convection.

Liquid motion is often observed when an electric field is applied to a diaelectric liquid. It is well recognized that the flow of an electrical current in a diaelectric liquid can be accompanied by some convective motion. The characteristics of this motion are determined by the medium properties and geometry of the electrode and magnitude of the current flow. Such effects were studied by Avsec and Luntz (3), Melcher (59), Stuetzen (89), Ostroumov (69), and Atten and Gosse (2). The convective motion of liquids, which accompanies the current flow, disturbs the steady-state current resulting in deviations from Ohm's law in measurement of the specific conductivity of some liquids. Schneider and Watson (80) observed the same phenomena by bombarding the free surface of a thin film by an electronic beam. Evidence of convective instabilities was also observed, where highly charged thermoplastic layers are softened, by Cressman (16). He proposed that a wrinkling type of deformation occurs due to the surface tension reduced by the presence of surface charge.

The mechanism of such an instability can be seen by analogy to be the classical Bénard problem. The potential energy released by inverting a charged layer, will cause the convection. Melcher (60) presented a review relating to the basic electrohydrodynamic phenomena. These studies show that the onset of instability is associated with some critical voltage. Experimentally the stucture of hexagonal cells has been observed. The attempt to analyze convective charge transport was made by Hopfinger and Gosse (38) and Laroix, et al. (52).

The effects of buoyancy forces on ionic mass transfer existing in the vicinity of working electrodes were investigated experimentally by Wagner (94), Ibl and Müller (44), Fench and Tobias (25), and Tobias and Hickman (91). These mass transfer characteristics have been known to be similar to those in heat transfer of Bénard convection.

Ackrowledgements

This work was performed under the National Science Foundation Grant during the author's graduate study at the Clarkson College of Technology, Potsdam, N.Y., U.S.A. The author wishes to thank professor E.J. Davis for his suggestions and encouragement.

References

1. M. Akiyama, G.J. Hwang and K.C. Cheng, *J. Heat Transfer*, **93** (1971), 335.
2. P. Atten, and J.P. Gosse, *J.Chem. Phys.*, **51** (1969), 2804.
3. D. Avsec, and M. Luntz, *Compt. Rend.*, **204** (1937), 420.
4. H. Bénard, *Rev. Gen. Sci. Pures Appal.*, **11** (1900), 1261.
5. J.C. Berg, A. Acrivos, and M. Boudart, *Adv. Chem. Eng.*, **6** (1966), 61.
6. A.E. Bergles, and R.R. Simonds, *Int.J. Heat Mass Transfer*, 14 (1971), 1989.
7. L.M. Blair, and J.A. Quinn, *J. Fluid Mech.*, **36** (1969), 385.
8. M.J. Block, *Nature*, **178** (1956), 650.
9. F.H. Busse, *J. Math. Phys.*, **46** (1967), 140.
10. F.H. Busse, and J.A. Whitehead, *J. Fluid Mech.*, **417** (1971), 305.
11. D.R. Caldwell, *J. Fluid Mech.*, **42** (19 70), 161.
12. K. Chandra, *Proc. Roy. Soc.*, **A164** (1938), 231.
13. S. Chandrasekhar, "Hydrodynamic and Hydromagnetic Stability", Clarendon Press, Oxford (1916).
14. S.W. Churchill, and R. Usagi, *AIChE. J.*, **18** (1972), 1121.
15. R.M. Clever, and F.H. Busse, *J. Fluid Mech.*, **65** (1974), 625.
16. P.J. Cressman, *J. Appl. Phys.*, **34** (19 63), 2327.
17. I.G. Currie, J. *Fluid Mech.*, **20** (1967), 337.
18. C. Dauzére, *Compt. Rend.*, **156** (1913), 1228.

19. R. P., Davies-Jones, *J. Fluid Mech.*, **49** (1971), 193.
20. J. W. Deardorff, *J. Atmos. Sci.*, **21** (1964), 419
21. J. W. Deardorff, *Phys. Fluids*, **8** (1965), 1027.
22. J. W. Deardorff, and G. E. Willis, *J. Fliud Mech.*, **28** (1967), 675.
23. J. W. Elder, *J. Fluid Mech.*, **32** (1968), 69.
24. R. Farhadieh, and R. S. Tankin, *J. Fluid Mech.*, (1974), 739.
25. E. J. Fench, and C. W. Tobias, *Electrochemica Acta*, **2** (1960), 311.
26. T. D. Foster, *Phys. Fluids*, **8** (1965), 1249.
27. T. D. Foster, *J. Fluid Mech.*, **37** (1969) 81.
28. J. E. Fromm, *Phys. Fluids*, **8** (1965), 1757.
29. K. S. Gage, and W. H. Reid, *J. Fluid Mech.*, **33** (1968), 21.
30. A. P. Gallagher, and A. McD. Mercer, *Proc. Roy. Soc.*, **A286** (1965), 117.
31. S. Globe, and D. Dropkin, *J. Heat Transfer*, **81** (1959), 156.
32. H. Görtler, *Ingenieur-Archiv*, **28** (1959), 71.
33. A. Graham, *Phil. Trans. Roy. Soc.*, **A232** (1933), 285.
34. P. M. Gresho, and R. L., Sani, *Int. J. Heat Mass Transfer*, **14** (1971), 207.
35. P. G. Grodzka, and T. C., Bannister, *Science*, **187** (1975), 165.
36. J. R. Herring, *J. Atmos. Sci.*, **20** (1963), 325.
37. K. G. T. Hollands, G. D. Raithby, and L. Konicek, *Int. J. Heat Mass Transfer*, **18** (1975), 879.
38. E. J. Hopfinger, and J. P. Gosse, *Phys. Fluids*, **14** (1971), 1671.
39. L. N. Howard, "Convection at High Rayleigh Number", Proc. 11th Int. Congress Applied Mechanics, Munich (ed., H. Görtler), p. 1109, Springerverlag, Berlin (1966).
40. L. N. Howard, *J. Fluid Mech.*, **17** (1963), 405
41. S. C. Hung, and E. J. Davis, *AIChE J.*, **20** (1974). 194.
42. G. J. Hwang. and, K. C. Cheng, *J. Heat Transfer*, **95** (1973), 72.
43. G. J. Hwang, and K. C. Cheng, "A Boundary Vorticity Method for Finite Amplitude Convection in Plane Poiseuille Flow", Developments in Mechanics, *Proc.* 12th Midwestern Mechanics Conference, Vol. 6, p. 207 (1971).
44. N., Ibl, and R. H. Müller, *J. Electrochem. Soc.*, **105** (1958), 346.
45. P. Ingersoll, *Phys. Fluids*, **9** (1966), 682.
46. E. L. Koshmieder, *Bull. Amer. Phys. Soc.*, **16** (1971), 1309.
47. E. L. Koshmieder, and S. G. Pallas, *Int. J. Heat Mass Transfer*, **17** (1974), 991.
48. R. Krishnamurti, *J. Fluid Mech.*, **42** (1970), 295.
49. R. Krishnamurti, *J. Fluid Mech.*, **42** (1970), 309.
50. R. Krishnamurti, *J. Fluid Mech.*, **60** (1973), 285.
51. H. L. Kuo, *Pheys. Fluids*, **6** (1963), 195.
52. J. C. Laroix, P. Atten, and E. J. Hopfinger, *J. Fluid Mech.*, **69** (1975), 539.
53. W. Lick, *J. Fluid Mech.*, **21** (1965), 565.
54. C. C. Lin, "The Theory of Hydrodynamic Stability", Cambridge Univ. Press (1955).
55. A. R. Low, *Proc. Roy. Soc.*, **A125** (1929), 180.
56. E. J. Mahler, and R. S. Schecter, *Chem. Eng. Sci.*, **25** (1970), 955.
57. W. V. R. Malkus, *Proc. Roy. Soc.*, **A225** (1954), 196.
58. W. V. R. Malkus, and G. Veronis, *J.*

Fluid Mech., **4** (1958), 225.

59. J. R. Melcher, *Phys. Fluids*, **9** (1966), 1548.
60. J. R. Melcher, *J. Fluid Mech.*, **40** (1970) 641.
61. D. R. Moore, and N. O. Weiss, *J. Fluid Mech.*, **58** (1973), 289.
62. S. M. Morcos, and A. E. Bergles, *J. Heat Transfer*, **97** (1975), 212.
63. Y. Mori, and Y. Uchida, *Int. J. Heat Mass Transfer*, **9** (1966), 803.
64. B. R. Morton, *J. Mech. Appl. Math.*, **10** (1957), 433.
65. D. A. Nield, *J. Fluid Mech.*, **19**, (1964), 341.
66. R. C. Nielsen, and R. H. Sabersky, *Int. J. Heat Mass Transfer*, **16** (1973), 2407.
67. Y. Ogura, and Yagihashi, *J. Meterological Soc., Japan*, **47** (1969), 205.
68. S. Ostrach, and Y. Kamotani, *J. Heat Transfer*, **97** (1975), 220.
69. G. A. Ostroumov, *Sov. Phys. JETP*, **14** (1962), 317.
70. E. Palm, T. Ellingsen, and B., Gjevik, *J. Flid Mech.*, **30** (1967), 651.
71. J. R. A. Pearson, *J. Fluid* Mech., **4** (1958), 489.
72. A. R. Pellew, and R. V. Southwell, *Proc. Roy. Soc.*, **A176** (1940), 312.
73. R. E. Plevan, and J. A. Quinn, *AIChE. J.*, **12** (1966), 894.
74. W. H. Flows, *Phys. Fluids*, **11** (1968), 1593.
75. Rayleigh, Lord, *Phil. Mag.*, **32** (1916), 529.
76. W. H. Reid, and D. L. Harris, *Phys. Fluids*, **1** (1958), 102.
77. H. T. Rossby. *J. Fluid Mech.*, **36**(1969), 309.
78. A. Schlüter, D. Lortz, and E. Busse, *J. Fluid Mech.*, **28** (1965), 223.
79. R. J. Schmidt, and S. W. Milverton, *Proc. Roy. Soc.*, **A152** (1935), 586.
80. J. M. Schneider, and P. K. Watson, *Phys. Fluids*, **13** (1970), 1955.
81. L. E. Scriven, and C. V. Sterning, *J. Fluid Mech.*, **19** (1964), 321.
82. L. Segel, "Nonlinear Hydrodynamic Stability Thery and its Applications to Thermal Convection and Curved Flows", Non-equilibrium Thermodynamics, Variational Techniques and Stability (R. Donnelly, R. Herman and I. Progogine, eds.), p. 165, Univ. of Chicago Press (1966).
83. P. L. Silveston, *Forsch. Geb. Ing Wes.*, **24 29** (1958), 59.
84. K. A. Smith, *J. Fluid Mech.*, **24**(1966), 401.
85. R. K. Soberman, *Phys. Fluids*, **2** (1959), 131.
86. E. F. C. Somerscales, and T. S. Dougherty, *J. Fluid Mech.*, **2** (1970), 755.
87. F. M. Sparrow, R. J. Goldstein, and V. I., Jonsson, *J. Fluid Mech.*, **18** (1964), 513.
88. F. M. Sparrow, and R. B. Husar, *J. Fluid Mech.*, **37** (1969), 251.
89. O. M. Stuetzer, *J. Appl. Phys.*, **31** (1960), 136.
90. H. Tippelskirch, *Beitr. Phys. Atmos.*, **29** (1956), 219.
91. C. W. Tobias, and R. G. Hickman, *Z. Phys. Chem.*, **229** (1965), 145-166.
92. A. A. Townsend, *J. Fluid Mech.*, **5** (1959), 209.
93. W. Velte, *Z. angew. Math. Phys.*, **13** (1962), 591.
94. C. Wagner, *J. Electrochem. Soc.*, **95** (1949), 161.
95. G. E. Willis, and J. W. Deardoff, *Phys. Fluids.* **10** (1967), 1861.
96. G. E. Willis, and J. W. Deardorff, and R. C. J. Sommerville, *J. Fluid Mech.*, **54** (1972), 351.

97. R. W. Zeren, and W. C., Reynolds, *J. Fluid Mech.*, **53** (1972), 305.

著者略歷

崔昌均博士는 서울工大 化工科에서 B. S. (1967), 陸軍兵器將校(R. O. T. C.), 韓肥現場勤務를 거쳐 美國 University of Iowa의 化工科에서 Gas permeability through membranes關聯으로 M. S. (1972), Clarkson College of Technology의 化工科에서 Thermal convection in liquid film flow에 關한 論文으로 Ph. D. (1976)를 받았으며, 現在 韓國科學技術硏究所 化學工程硏究室의 先任硏究員으로 在職中이다.