特別寄稿
SPECIAL CONTRIBUTION

Turbulence in Thermally Stratified Shear Flows*

T. Mizushina

Kyoto University, Japan

INTRODUCTION

Heat transfer in thermally stratified shear flow is of great practical interest in the problems of disposal of waste water, because stratification often has a governing influence on the dispersion of the waste heat. The flow behavior and heat transfer of the stratified flow are affected by buoyance force arising from the stratification and it is important to understand this influence to predict the flow characteristics and consequently the dispersion of the discharged waste heat.

Attempts at predicting the buoyancy effect on turbulence have ranged from similarity theories based on dimensional arguments to turbulence closure assumptions in the transport equations for the turbulent shear stresses and heat fluxes. All hypotheses involve empirical functions or coefficients that need to be experimentally verified.

Meteorologists have provided most of the experimental results, because one of the important practical cases of stratified shear flow is the atmospheric boundary layer.

However, the standards of accuracy desired or attained in meteorological experiments are significantly poorer than those in laboratory experiments. This is due mainly to the difficulty in carrying out atmospheric measurements under steady state conditions. Laboratory measurements in stable stratified flows have been made by some investigators; Ellison and Turner in an inclined rectangular tube flow, Webster in a wind tunnel equilibrium air flow, Arya and Plate in a wind tunnel boundary layer flow and Schiller and Sayre in an open channel flow. Under unstable stratified conditions, experimental results, particularly laboratory measurements, have been extremely limited. Some experiments were conducted in the atmospheric surface layer by Businger et al. and others. All the experimental results of both stable and unstable flows are not conclusive, because of a wide scattering among the individual experiments and because of the limited experimental condition of rather weak stability or instability.

The objective of this paper is to present experimental results of the eddy diffusivities of momentum and heat in both stable and unstable stratified shear flows in order to offer the conclusive bases on the closure problem of the buoyancy effect on turbulence. An open channel flow has the great advantage that steady two-dimensional strongly stratified flows can be obtained by limiting the temperature field to the region near the free surface where the velocity gradient is small.

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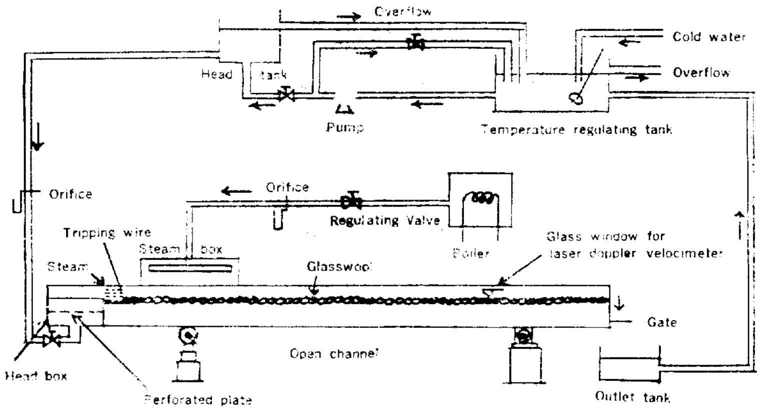

Figure 1. Flow System of Experimental Appratus

EXPERIMENT

First of all, I will explain the experimental procedure. *Figure* 1 illustrates the experimental apparatus.

The experiments were carried out in the 6.1m flume with a cross section of 30cm wide and 6cm deep. The side walls and the bottom were made of smooth stainless steel plates lined with 3cm thickness glasswool to maintain adiabatic conditions. The measurement stations were located at the center of the flume and at distance 1.5m downstream for securing the developed stratified flows. At the measuring station the side walls were made of optical glass plates for use with a laser Doppler velocimeter. To create the turbulent boundary layer, seven tripping wires were mounted at short distance from the inlet of the flume. The slope of the flume could be varied in combination with the variation in the flow conditions. Water was recirculated through the flume by a pump and the temperature of the water was controlled within ±0.05°C in a temperature regulating tank. Water from the head tank passed through a calibrated orifice and a flow control valve into a head box which was connected to the flume. Two perforated plates with 5mm diameter holes were installed in the head box and three screens were placed at the entrance of the flume to obtain a developed subcritical flow in the flume.

Stable and unstable stratified flows were obtained with the following procedures. For stable stratified flows, saturated steam at 100°C was condensed on the free surface of the flow in the fully developed turbulent flow region using a steam box as shown in the *Figure*. Unstable stratified flows were obtained by providing the water heated up to 60~70°C in the temperature regulating tank into the flume and by evaporating the water from the free surface into the atmosphere.

Figure 2 shows a schematic diagram of stable and unstable stratified flows.

In stable stratified flow, temperature and velocity gradients are positive in the vertical y-direction. For a positive temperature gradient, a turbulent eddy moving upward, for instance, is normally be colder than the

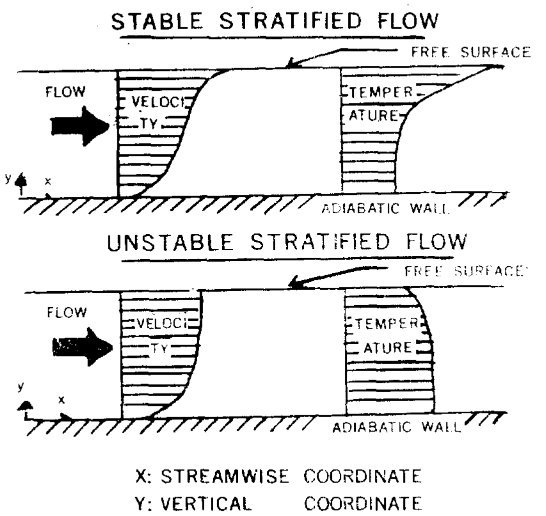

Figure 2. Schematic Diagram of Stratified Flow

surrounding fluid. Then, buoyancy forces will tend to decelerate the eddy moving upward. Similarly, an eddy moving downward will be decelerated. Therefore, the flow with a positive temperature gradient is called "Stable stratified flow." In unstable stratified flow, a temperature gradient is negative in the vertical direction. For a negative temperature gradient, it can be seen that the buoyancy effect will be opposite to that in stable stratified flow; that is, the buoyancy forces will tend to accelerate the eddy motions.

In *Figure* 3, I will show a block diagram of the measuring system used in this study. A

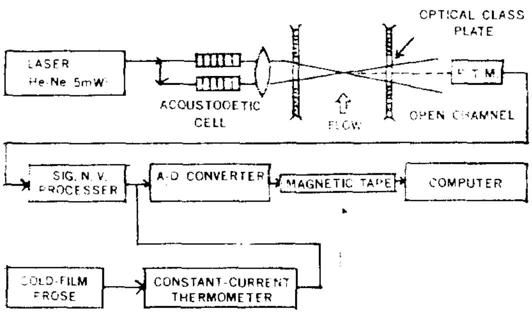

Figure 3. Block Diagram of Measuring System.

laser Doppler velocimeter with an adapter for frequency shift was used for measuring the local time-averaged and fluctuating velocities. The local time-averaged and fluctuating temperatures were measured with thermocouples and a constant current thermometer connected to a cold-film probe. The voltage output from these instruments was directly transmitted to a data acquisition system which could store the digital voltage output on a digital magnetic tape. In order to compute various statistical turbulent quantities, the digital signals were processed with the computer system at the Data Processing Center of Kyoto University. The sampling interval and the sample size were 0.01 second and about 25,000, respectively.

When we apply a laser Doppler velocimetry to non-isothermal flow, laser beams encounter a succession of hot and cold pockets formed by turbulence eddies with various sizes and various temperatures, which modify phase and direction of the laser beams, consequently the measured velocity should be corrected. In this study, the correction for the velocity was accomplished by inserting a small acrylic plate at the point of beam intersection, because the particles in the acrylic plate or on the surface of it worked as a stationary acattering obstacle.

The table in *Figure* 4 gives all the essential parameters in the present experiments.

A bulk Richardson number, $\overline{Ri}$, in the table can be defined in terms of density differences across the flow-layer and the expansion coefficient of the fluid,

$$\overline{Ri}=\beta g\frac{(T_{suf}-T_{bot})R}{U_{ave}^2}$$

and is equal to the reciprocal of densimetric Froude number. The value of $\overline{Ri}$ is equal to zero for neutral flows, and is large positive

Run No.	Key	$\overline{Ri}$[-]		Re[-]	δ[m]	R[m]	U_{ave}[m/s]	u^*[m/s]
I		0.0	(neutral)	8600	0.040	0.031	0.071	0.0046
II		2.31×10^{-2}	(stable)	9800	0.040	0.031	0.074	0.0047
III		6.39×10^{-2}	(stable)	17000	0.039	0.031	0.104	0.0061
IV		1.53×10^{-2}	(stable)	10500	0.039	0.031	0.069	0.0043
V		2.13×10^{-2}	(stable)	12600	0.033	0.031	0.074	0.0045
VI		-1.23×10^{-2}	(unstable)	41700	0.040	0.032	0.152	0.0030
VII		-2.03×10^{-2}	(unstable)	18300	0.039	0.031	0.080	0.0046
VIII		-3.53×10^{-2}	(unstable)	21200	0.040	0.032	0.079	0.0045
IX		-4.90×10^{-2}	(unstable)	30000	0.040	0.031	0.079	0.0043

Figure 4. Parameters in Experiments

for strongly stable stratified flows, and is large negative for strongly unstable stratified flows. The present nine runs can be divided into 3 stability groups, depending on the value of a bulk Richardson number $\overline{Ri}$. The first is the neutral group of Run Number I. The second is the stable group of Run Numbers II, III, IV, V and the third is the unstable group of Run Numbers VI, VII, VIII, IX. Reynolds numbers, Re, based on hydraulic radius R ranged from 8,600 to 41,700 and the Froude numbers were less than 0.24. The flow depth δ was maintained at approximately 4cm and the cross-sectional mean velocities, U_{ave}, ranged from 6.9cm/sec to 15.2cm/sec.

EXPERIMENTAL RESULTS

Next, I will discuss the experimental results. In order to predict the buoyancy effect in the measured turbulence quantities, we introduce a well-known local gradient Richardson number. The local gradient Richardson number indicates the stability of the flow and can be defined in terms of density and velocity gradients and the expansion coefficient of the fluid.

$$Ri=\beta g\frac{\partial T/\partial y}{(\partial U/\partial y)^2}$$

For stable conditions, the local gradient Richardson number is positive, and for unstable conditions, it is negative.

The values of turbulent quantities measured in the range where the local Richardson number could be more reliably estimated were used for deriving empirical relationships between turbulent quantities and the local gradient Richardson number Ri.

CORRELATION OF TURBULENCE INTENSITIES, REYNOLDS STRESS AND TURBULENT HEAT FLUXES WITH RICHARDSON NUMBER

In *Figure* 5, turbulence quantities are plotted against the local gradient Richardson number for stable stratified flows.

It can be seen that the turbulence quantities is well correlated with Richardson number Ri. In the figures, u denotes a streamwise velocity fluctuation, v is a vertical velocity fluctuation and θ is a temperature fluctuation. Overbar and prime designate a time-averaged value and root mean square value, respectively. For instance, $\overline{v^2}$ is the mean square value of vertical velocity fluctuation and v' is the root mean square value of vertical velocity fluctuation.

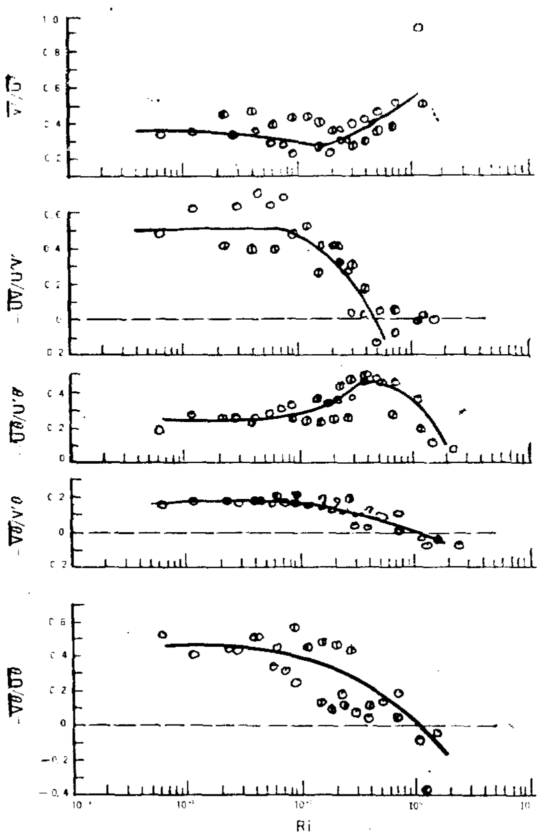

Figure 5. Turbulence Quantities vs. *Ri*. (Stable)

The ratio of the mean square value of mwise vvelocity fluctuation to that of strea-Atical elocity fluctuation, $\overline{v^2}/\overline{u^2}$, decreases

slightly as the stratification shifts from neutral to weakly stable conditions and then increases as Ri becomes larger. The behavior shows the interesting fact that the vertical turbulence is promoted under strongly stable conditions. This may be due to the buoyancy term in the transport equation for $\overline{v^2}$.

The correlation coefficient of the Reynolds stress, $-\overline{uv}/u'v'$, decreases with increasing Ri and approaches zero in the range of $Ri\approx$ 1.0. This means that the vertical turbulent momentum transfer decreases and is suppressed in the extreamly stable range.

The correlation coefficient of the streamwise turbulent heat flux, $\overline{u\theta}/u'\theta'$, increases with increasing Ri and then decreases under strongly stable conditions.

The correlation coefficient of the vertical turbulent heat flux, $-\overline{v\theta}/v'\theta'$, decreases with increasing stability and, in the extreamly stable range of $Ri\approx1$, becomes zero. This shows that the vertical turbulent heat transfer is stopped at $Ri\approx1$.

The ratio of the vertical turbulent heat

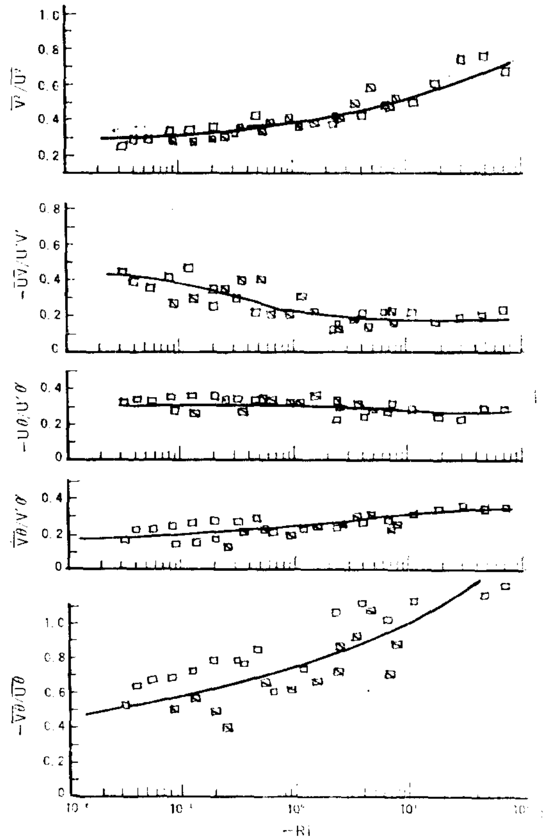

Figure 6. Turbulence Quantities vs. *Ri*. (Unstable)

flux to the streamwise heat flux, $-\overline{v\theta}/\overline{u\theta}$, decreases rapidly with increasing stability, in the range of $Ri \approx 1$. This indicates that the vertical turbulent heat transfer is more strongly suppressed than the streamwise heat transfer.

Figure 6 shows the correlations of the turbulence quantities with the local Richardson number in unstable stratified flows.

In unstable etratified flow, the ratio, $\overline{v^2}/\overline{u^2}$, increases with increasing instability. This shows that the velocity fluctuation in the vertical direction is more strongly promoted by the destabilizing buoyancy forces, in comparison with that in the streamwise direction.

The correlation coefficient of the Reynolds stress, $-\overline{uv}/u'v'$, decreases with increasing instability.

The correlation coefficient of the streamwise turbulent heat flux, $-\overline{u\theta}/u'\theta'$, is almost constant under unstable conditions.

The correlation coefficient of the vertical turbulent heat flux, $\overline{v\theta}/v'\theta'$, increases gradually as the stratification shifts to unstable conditions.

The ratio of the vertical turbuent heat flux to the streamwise turbulent heat flux increases with increasing instability. This means that the vertical turbulent heat transfer is more strongly prmooted by the destabilizing buoyancy effect than the streamwise heat transfer.

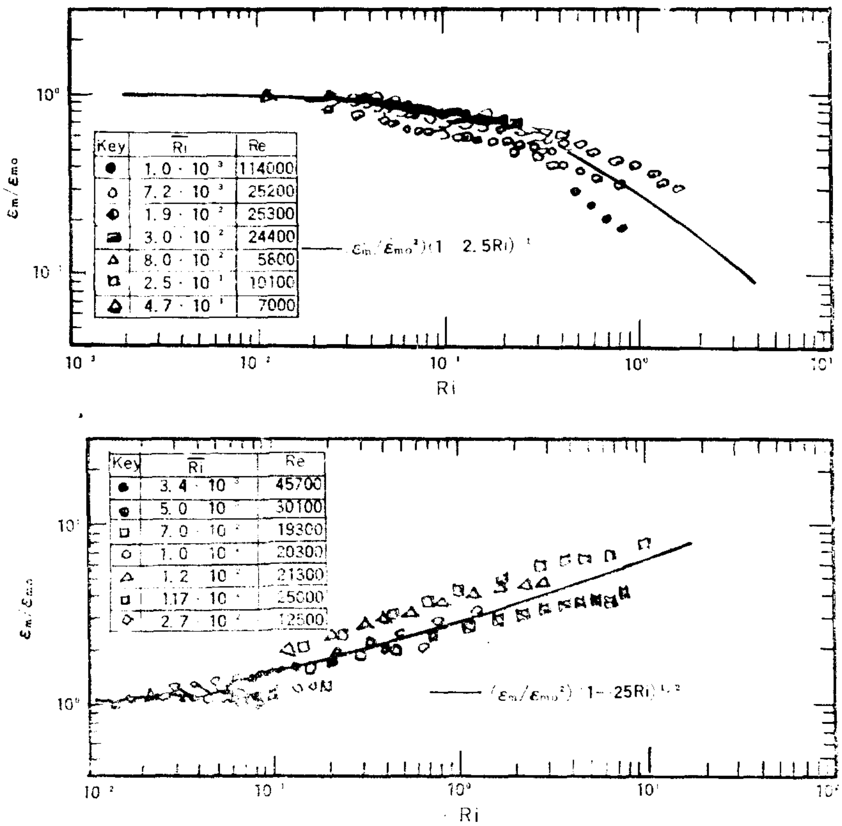

Figure 7. Variation of $\varepsilon_m/\varepsilon_{m0}$ with Ri

CORRELATION OF EDDY DIFFUSIVITIES WITH RICHARDSON NUMBER

In *Figure* 7, the ratio of the eddy diffusivity of momentum under stratified conditions to that under neutral conditions at the same position in the flow, $\varepsilon_m/\varepsilon_{m0}$, is plotted against gradient Richardson number Ri for stable and unstable stratified flows.

The upper figure shows $\varepsilon_m/\varepsilon_{m0}$ for stable stratified flows and the lower figure shows that for unstable stratified flows. Although there is much scattering in the range $Ri>0.3$, the ratio $\varepsilon_m/\varepsilon_{m0}$ in stable stratified flows can be correlated fairly well with the gradient Richardson number and its values decrease with increasing Ri.

The ratio $\varepsilon_m/\varepsilon_{m0}$ under unstable conditions also can be well correlated with Ri. The values of $\varepsilon_m/\varepsilon_{m0}$ increase with increasing instability and at $Ri=-10$ become 5~6 times as large as those under neutral conditions.

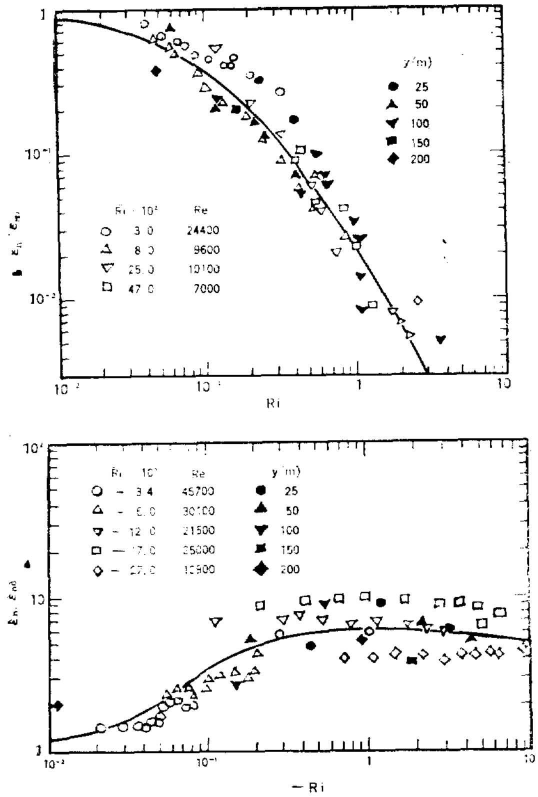

Figure 8. Variation of $\varepsilon_h/\varepsilon_{h0}$ with *Ri*

Similarly, *Figure* 8 shows the ratio, $\varepsilon_h/\varepsilon_{h0}$, of the eddy diffusivity of heat under stratified conditions to that under neutral conditions.

In stable stratified flows, $\varepsilon_h/\varepsilon_{h0}$ decreases more rapidly than $\varepsilon_m/\varepsilon_{m0}$. In the strongly stable range near $Ri=1$, $\varepsilon_h/\varepsilon_{h0}$ becomes less than 0.01. Under unstable conditions, $\varepsilon_h/\varepsilon_{h0}$ increases with increasing instability and then approaches a constant value of 6. Thus, we can see one thousand-fold variation of the eddy diffusivity of heat depending on Richardson number.

In the same diagram the values measured by one of my co-workers Dr. Ueda recently in the atmospheric boundary layer are plotted with the filled symbols. The measurements were conducted at 25~200m height above the ground. In both stable and unstable cases, the results in an open channel are in good agreement with those in the atmospheric boundary layer flow.

In *Figure* 9, the author's results of the

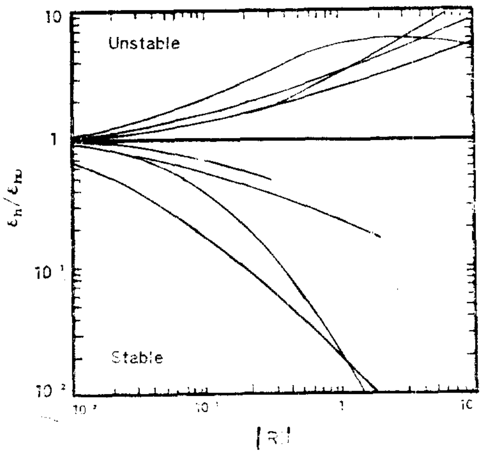

Figure 9. Comparison $\varepsilon_h/\varepsilon_{h0}$ with other Investigations in the Atmospheric Boundary Layer

ratio, $\varepsilon_h/\varepsilon_{h0}$, are compared with other investigations in the atmospheric boundary layer. All of the measurements by other investigators, except Depardorff's correlation curve were conducted in the atmospheric surface layer of several meters above the ground surface. The results in the atmospheric surface layer indicates a much smaller dependence of the eddy diffusivity of heat on thermal stratification. The difference of $\varepsilon_h/\varepsilon_{h0}$ between author's results and others is larger in the stable case. This may be due mainly to the difficulty in obtaining the steadiness of the flow and flow-direction, and the constancy of the heat flux in the atmospheric surface layer.

On the other hand, Deardorff tried to simulate the motions of the atmosphere under stratified conditions numerically, and he concluded that his correlation curve in the diagram gave the best simulation. His correlation curve is in good agreement with speaker's one.

In *Figure* 10, the ratio of the eddy diffusivity of heat to that of momentum under stable and unstable conditions is plotted against Richardson number.

Although there is much scattering, the reciprocal turbulent Prandtl number $\varepsilon_h/\varepsilon_m$ can be correlated fairly well with the gradient Richardson number.

In stable stratified flows, $\varepsilon_h/\varepsilon_m$ decreases with increasing Ri and falls rapidly at $Ri \approx 1$.

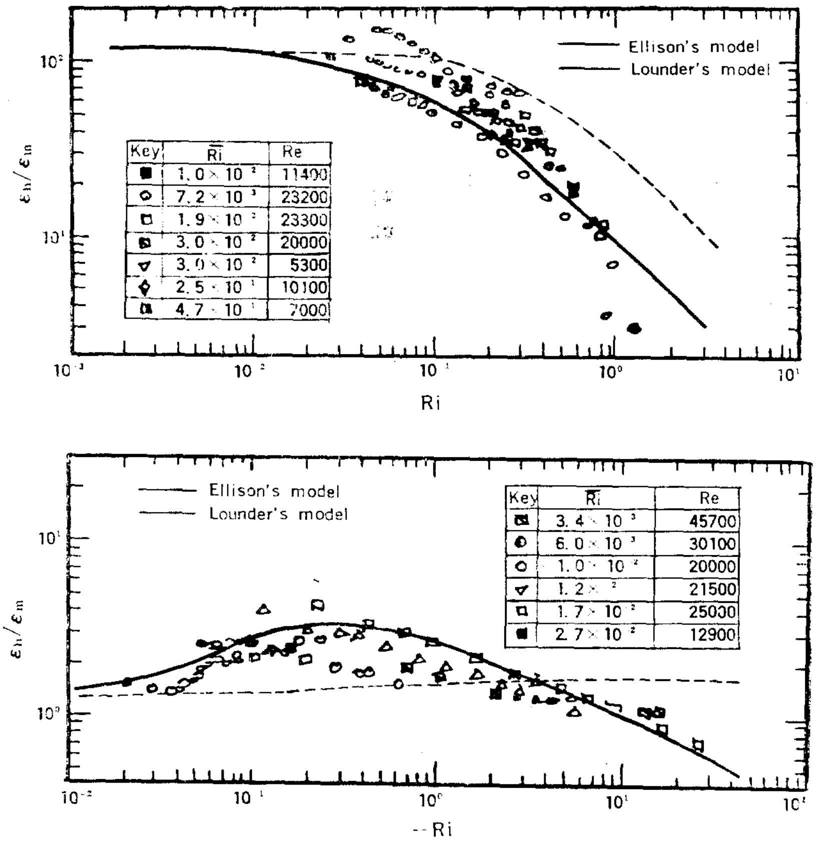

Figure 10. Variation of $\varepsilon_h/\varepsilon_m$ with *Ri*

In unstable flows, $\varepsilon_h/\varepsilon_m$ increases as the stratification shifts from neutral to weakly unstable conditions and then falls gradually as $-Ri$ increases. It has a maximum value of about 3 at $Ri \approx -0.2$, and is down to a value less than unity at $-Ri > 10$.

In the diagrams, the solid and dashed lines show the values calculated by Launder's multi-equation turbulece model and by the Ellison's model. The calculated values by the Ellison's model are in good agreement with the results both under stable and unstable conditions. However, it is not possible to make very definite statements on the net buoyancy effect on shear stresses, heat fluxes and consequently the turbulent Prandtl number by the Ellison's model, because he used only three equations of turbulent energy, mean square of the temperature fluctuation and turbulent heat flux. Nevertheless, the agreement with the present experimental results is impressive.

In *Figure* 11, the present results of the ratio $\varepsilon_h/\varepsilon_m$, depicted by the solid curves, are compared with observed data in the atmospheric surface layer along with data obtained in laboratory experiments by the other investigators. ε_s and ε_w in this diagram represent the eddy diffusivities of salt and water vapor. This figure shows that as a whole, heat transfer is more affected by buoyancy than momentum transfer. In the stable case, the present results are in good agreement with the ratio of eddy diffusivity of mass (salt) to that of momentum obtained by Ellison and Turner in an inclined rectangular tube. In contrast to this, the field experiments in the atmospheric surface boundary layer have indicated a much smaller dependence of the ratio $\varepsilon_h/\varepsilon_m$ on thermal stratification as shown in the figure. In the surface layer, the values at $Ri = 0.2$ are twice as large as the value of the present result $\varepsilon_h/\varepsilon_m = 0.47$ and the difference is accentuated with increasing Ri. However, the results from field observations should be treated with some reserve in respect to the steadiness of the flow and flow-direction, constancy of the momentum and heat

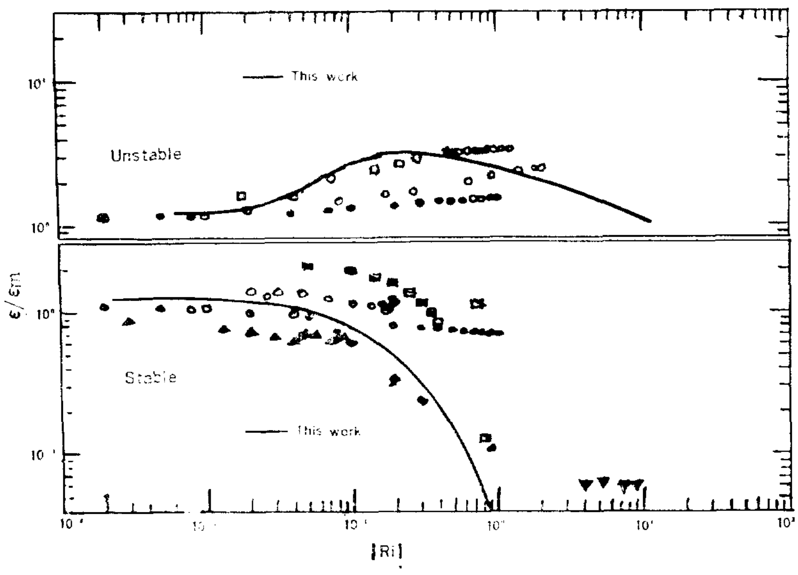

Figure 11. Comparison of the Present Results with those of Other Investigators

fluxes, and radiation effect.

In the unstable case, the present results cover a much wider range of unstable conditions by more than one order of magnitude of $-Ri$ than has been reported up to date. The results of the field observations show the increase of $\varepsilon_h/\varepsilon_m$ with increasing $-Ri$ in the range of $-Ri$ which is normally encountered in the atmospheric boundary layer. This is in accordance with the present results, especially the results of Charnock et al. showing good agreement. In the strongly unstable condition, it is surprising that the ratio $\varepsilon_h/\varepsilon_m$ decreases with $-Ri$, having the value of $\varepsilon_h/\varepsilon_m=1$ at $Ri\approx-10$. This has not been found in any previous works which have only covered the range $-Ri<2$.

CONCLUSIONS

In conclusion, the present work has produced much new information concerning the variation with stability and instability of the various quantities associated with turbulence and turbulent transport processes. However, I should say that we are standing still far from the goal and I would like to urge many young scientists join us for such an interesting problems.