A Study of Dynamic Response for a Vertical Tube Heat Transfer

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垂直管熱傳達에 있어서의 動應答

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The dynamic responses of the temperatures of the outlet water and the wall for a vertical tube heat exchanger, were studied as a function of the sinusoidal change in the inlet water temperature.

The experimental response data for the outlet water temperature were compared with the calculated data from the derived equation. The comparison was shown to be in good agreement between the experiment and the theory in the present range of the experiment.

The inlet water-to-wall temperature transfer function was derived, and the theoretical frequency response characteristics were compared with the experimental data. The experimental data were lower than those predicted from the theory.

Introduction

Heat exchangers are widely used in process industries and any other engineering fields, and the mechanism of heat transfer in the exchangers has been investigated to obtain better design and performance. Traditionally, the heat exchanger has been designed on the steady state basis without much consideration of the dynamic behavior, however, with the increase of the automatic control of the process, special interests have been laid on the dynamic behavior of the heat exchanger which is so vitally important in determining performance under automatic control. The dynamic charcteristics of the process components and overall plant may be indicated by frequency response data or by transient response data. Among these two methods, the frequency response method of analysis is the most commonly used method for studying the dynamic behavior of the process components and the overall dynamic behavior of several pieces of equipment can readily be synthesized if the frequency response characteristics are available.

The first step required to study the dynamic characteristics of the heat exchanger is a theoretical analysis of the process in question from the standpoint of unsteady state behavior. Generally, a complete description of the performance of heat exchanger contain the equations for conservation of mass, energy, momentum, and equations of state and equations for the rate processes occuring between phases. Since the parameters in the equations are functions of time and space, the equations in most cases become nonlinear. Unless the sets of equations can be uncoupled and enduced quasilinear or linear, little hopes exists for a quantitatvie evaluation of the dynamic characteristics of heat exchanger.

In the lirerature, many investigators could be found who studied the dynamic characteristics of various heat exchanger by linearizing equations by using simplifying assumptions[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. Cohen and Johnson[1] have studied the dynamics of a double-pipe heat exchanger in which steam condensed in the outer annulus and water flowing through the inner pipe. They have found the transfer function for the response of the inner fluid to changes in both its temperature

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and the steam temperature. Their emphasis has been in the evaluation of the frequency response and have disclosed a significant characteristic resonance in the amplitude ratio of the temperature of one of the fluids at a certain frequency. Lees and Hougen(5) in a study of pulse testing a model heat exchanger have demonstrated the applicability of pulse testing methods to the investigation of the dynamics of heat exchangers.

The purpose of the present investigation was to study the dynamic response for a vertical tube heat transfer process by establishing the theoretical transfer functions where the coolant flows upward through a round stainless steel tube and the heat was generated in the tube wall. The test section was a round stainless steel tube, 8.7 mm inside diameter x 0.4 mm wall and the heated height was 195.5cm. The heating power was put into the tube wall by passing an constant A.C. current and the water was forced to flow upward through the test section steadily. The temperature of the water entering the test section was and the inlet-water-to-wall-temperature transfer function were obtained, and the frequency response characteristics were compared whith the experimental results. The present experiments were carried out in the range of the insignificant theoretical changes of the amplitude ratio and phase angle, and the experimental results for the response of the outlet water temperature were in fairly good agreement with the theoretical results, and for the response of the wall temperature, the experimental results were lower than those predicted from theory.

Theory

(A) Basic Differential Equations

The physical system analyzed is shown in Fig. 1. This consists of a circular tube through which a coolant flows steadily upward and in the solid walls of which energy is generated. In the steady state, all of this energy appears as a flow of heat at the interface between coolant and the solid causing the coolant to increase in enthalpy (and temperature) as it flows through the tube. During the transient, however, both the tube wall and the coolant experience local temperature excursions. In analyzing the system the following assumptions are imposed.

(a). The liquid water temperature and the velocity are constant across the flow cross.

(b). The tube wall temperature dose not depend on radius.

(c). Axial heat conduction in the tube wall is negligible.

(d). The heat transfer coefficient is constant with length and time.

(e). The liquid water is incompressible and all the fluid properties are constant.

(f). Heat generation within the tube wall is constant with length but is time dependent.

(g). The temperature of the inlet-water is time dependent.

(h). The flow channel has constant area.

(i). The outside surface of the tube is adiabatic.

(j). The mechanical energy (kinetic and potential) of the fluid is negligible compared with the thermal energy.

With these assumptions the application of the first law of thermodynamics and the law of conservation of mass for an incompressible fluid to the system in Fig. 1(b) produces the following two differential equations, one for the fluid, the other for the tube wall.

Fluid

$$(1) \qquad \frac{\partial T_o}{\partial t} = \alpha \frac{\partial^2 T_0}{\partial x^2} - u \frac{\partial T_o}{\partial x} + \frac{1}{\tau}(T_{wo} - T_o)$$

Wall

$$(2) \qquad \frac{\partial T_{wo}}{\partial t} = Q(t) - \frac{1}{\tau_w}(T_{wo} - T_o)$$

where

$$(3) \qquad \tau = \frac{c \, \rho \, r_i}{2h_i} \quad (\text{min})$$

$$(4) \qquad \tau_w = \frac{\pi (r_o^2 - r_i^2) C_w \rho_w}{2\pi r_i h_i} \quad (\text{min})$$

$$(5) \qquad Q(t) = \frac{q'(t)}{\pi (r_o^2 - r_i^2) C_w \rho_w} = Q_{xo} + q(t) \left(\frac{°F}{\text{min}} \right)$$

in which Q_{xo} is the function of the initial uniform heat generation rate and $q(t)$ is the function of the time-variant heat generation rate, having a zero value at zero time. The other symbols are defined in the Nomenclature.

The heat exchanger is considered to be in steady state operation at $t=0$, then both functions $T_o(x,t)$ and $T_{wo}(xt)$ of equations (1) and (2) are split into two components, one being a steady state part $T_s(x,o)$ and $T_{ws}(x,o)$ the other of the transient contribution $T(x,t)$ and $T_w(x,t)$. For purpose of transient analysis, it is necessary only to carry out detailed mathematical analysis of the latter functions. Hence the governing differential equations become

Fluid

(6) $\quad \dfrac{\partial T}{\partial t} = a\dfrac{\partial^2 T}{\partial x^2} - u\dfrac{\partial T}{\partial x} + \dfrac{1}{\tau}(T_w - T)$

Wall

(7) $\quad \dfrac{\partial T_w}{\partial t} = q(t) - \dfrac{1}{\tau_w}(T_w - T)$

If the variation in temperatures for both the fluid and wall from the steady state is zero at $t=0$, the initial conditions become

(8) $\quad T(x,0)=0$

(9) $\quad T_w(x,0)=0$

(B) The Inlet-to-outlet Water Temperature Transfer Function When the Axial Heat Conduction in Fluid Is Negligible.

If the axial heat conduction in the fluid is assumed negligible, equation (6) becomes

(10) $\quad \dfrac{\partial T}{\partial t} = -u\dfrac{\partial T}{\partial x} + \dfrac{1}{\tau}(T_w - T)$

Equation (7) and (10) are operated on by the Laplace Transform technique, using the initial conditions (8) and (9) to give the following transformed equation.

(11) $\quad \overline{T}(x,s) = \dfrac{f_2(s)}{f_1(s)}\left[1 - e^{-\frac{f_1(s)}{u}x}\right]\,q(s)$
$$+ e^{-\frac{f_1(s)}{u}x}\,\overline{T}(o,s)$$

where

(12) $\quad f_1(s) = s + \dfrac{\tau_w s}{\tau(\tau_w s + 1)}$

(13) $\quad f_2(s) = \dfrac{\tau_w}{\tau(\tau_w s + 1)}$

where the barred symbols indicate Laplace transforms and a denote the Laplace transform of the time according to usual practice.

Equation (11) represents the total response of the outlet-water temperature at position x to the multiple disturbances of heat generation and the inlet-water temperature. If the inlet-water temperature is held constant, equation (11) reduces to

(14) $\quad G_1(s) = \dfrac{\overline{T}(x,s)}{\overline{q}(s)} = \dfrac{f_2(s)}{f_1(s)}\left[1 - e^{-\frac{f_1(s)}{u}x}\right]$

This is the transfer function which determines the relationship between the temperature of the fluid anywhere in the tube $0 < x < L$ and the heat generation.

Similarly, if the heat generation is constant with time, that is $q(t)=0$, and the inlet-water temperature is allowed to vary, equation (11) becomes

(15) $\quad G_2(s) = \dfrac{\overline{T}(x,s)}{\overline{T}(o,s)} = e^{-\frac{f_1(s)}{u}x}$

This is the transfer function between the outlet-water temperature and the inlet-water temperature. The transfer function $G_2(s)$ also has the form at $x=L$

(16) $\quad G_2(s) = \exp\left\{-t_d s\left(1 - \dfrac{\tau_w}{\tau(\tau_w s + 1)}\right)\right\}$

where t_d is the "dead time" L/u related to the flow in the tube over the length L. The frequency response is obtained simply by replacing s in the transfer function by jw, where w is the angular frequency in radians per minute and $j=\sqrt{-1}$. The amplitude and phase of the resulting complex number corresponds to the amplitude ratio and the phase angle of the frequency response.

The complex number corresponding to the transfer function $G_2(s)$ becomes

(17) $\quad G_2(j\omega) = e^{A}(\cos B - j\sin B)$

where

(18) $\quad A = -\dfrac{(\tau_w^2 t_d/\tau)\omega^2}{1 + \omega^2 \tau_w^2}$

(19) $\quad B = \dfrac{t_d \tau_w^2 \omega^3 + t_d(1 - \tau_w/\tau)\omega}{1 + \omega^2 \tau_w^2}$

Therefore, the amplitude ratio and the phase angle become simply

(20) $\quad$ Amplitude ratio $= \exp(A)$

(21) $\quad$ Phase lag angle $= -B$

(C) The Inlet Water-to-wall Temperature Transfer Function When the Heat Generation Is Constant With Time

As in (B) the heat conduction in the fluid is assumed negligible. If the heat generation is constant with time, equation (7) becomes

(22) $\quad \dfrac{\partial T_w}{\partial t} = -\dfrac{1}{\tau_w}(T_w - T)$

Laplace transform of the equation (22) with respect to time is given by

(23) $\quad s\overline{T}_w(x,s) = -\dfrac{1}{\tau_w}[\overline{T}_w(x,s) - \overline{T}(x,s)]$

Rearranging

(24) $\quad \dfrac{\overline{T}_w(x,s)}{\overline{T}(x,s)} = \dfrac{1}{s\tau_w + 1}$

This is the transfer function between the wall temperature and the fluid temperature in the tube. The transfer function between the inlet-water temperature and the wall temperature at x is obtained as follows

(25) $\quad G_3(s) = \dfrac{\overline{T}_w(x,s)}{\overline{T}(x,s)} \cdot \dfrac{\overline{T}(x,s)}{\overline{T}(o,s)} = \dfrac{\overline{T}_w(x,s)}{\overline{T}(o,s)}$
$$= \dfrac{1}{s\tau_w + 1}\,e^{-\frac{f_1(s)}{u}x}$$

Substituting jw into s in equation (25), the amplitude ratio and the phase angle are expressed as follow:

(26) $\quad$ Amplitude ratio $= \dfrac{e^A}{\sqrt{1 + \omega^2 \tau_w^2}}$

(27)   Phase lag angle $= \tan^{-1}\left(-\dfrac{\omega\tau_w \cos B - \sin B}{\cos B + \omega\tau_w \sin B} \right)$

where $t_x = x/u$.

Experimental

The experimental apparatus used to check the theory is shown schematically in Fig. 2. It consists mainly of a supply thank, circulating pump, preheater, test section, holdup tank, an ion-exchanger system and the measuruments.

The test section was a round stainless steel tube, 0.8mm inside diameter x 0.4mm wall and the heated height of 195.5 cm. The test section was heated uniformly along its axis by passing as A.C. current through it, and was isolated from the piping by the Lucite flanges at both ends of it to prevent the passage of current through the piping of the system. (see Fig. 3) The outside surface of the test section was insulated with glass wool to prevent the possible heat loss to the surrounding. The preheater used was a Chromalox type (AR- 2519. 240v, 9Kw, 1PH). The on-off control of the preheater varies the temperature of the water flowing through it sinusoidally. The frequencies generated in the experiment were: 0.5, 0.66, 1.0, 1.2, 1.5, 2.0, and 3.0 cycles per minute. The liquid water used was distilled water and was continuously purified by forcing it through the vertical ion-exchanger during experiment. The flow rate of water was measured by means of a calibrated sharp-edged orifice and controlled by adjusting the valve opening. The selected flow rate

for the experiment were: 6.77, 9.13, 14.42, and 18 lb/min. The temperature of the water and the tube wall were measured by using calibrated iron-constantan thermocouples (20 gage). The inlet-and outlet-water temperatures were measured by bare thermocouples in the stream and the wall temperatures were measured at 5 different points along the axis by using thermocouples silver-soldered directly to the outside of the test tection. The Fischer Recordall was used to take the readings of these temperatures.
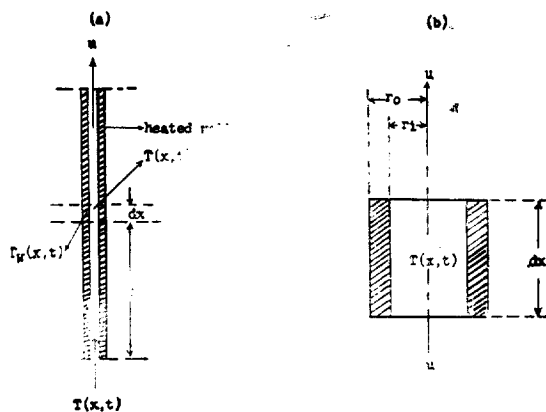

Fig. 1. Simulated coolant channel.

The heating power to the test section was measured by reading the test section voltage and current. The average heating power in the experiment was 3.94Kw and the heat flux based on the inner surface of the tube was 2.34×10^4 Btu/(sq. ft) /(hr).

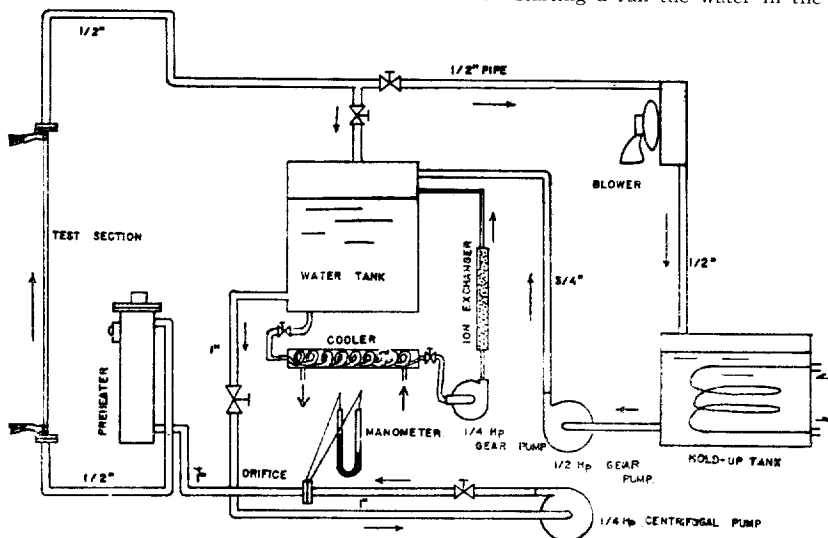In starting a run the water in the tank at the room

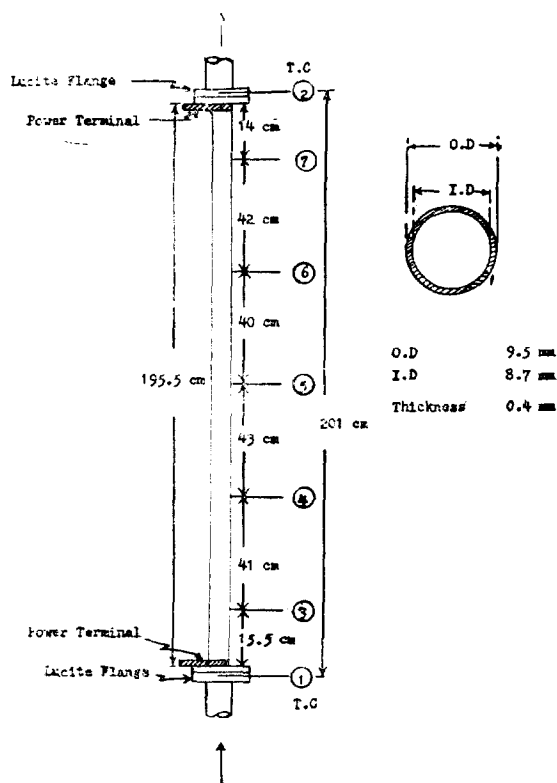Fig. 2. Schematic diagram of the experimental apparatus.

Fig. 3. Drawing of the test section.

temperature was forced to circulate the system at the desired rate and then the power was put into the test section, adjusting the current and voltage at the desired values by the transformer. The constant heating power was maintained by the auto-trnsformer.

After the temperature of the water at the exit of the test section reached a constant value, the preheater was operated by putting on and off the current regularly on the selected time intervals. After the sine waves of the water temperature were formed at the entrance of the test section, the outlet-water temperature and the wall temperature were started to be measured. Experimental data were taken after the steady sine waves of the outlet water and the wall temperature were formed.

Experimental Results and Discussion

The frequency response characteristics of the bulk water temperature plotted on Bode diagram are shown in Fig. 4 and Fig 5. The solid lines represent the theoretical curves calculated from equations (20) and

(21) and the circles represent the experimental data. The theoretically calculated amplitude ratio approaches a limiting value greater than zero for increasing frequency, while the lag angle increases infinitely with increasing frequency. The physical properties of water were assumed constant and were taken at the average temperature of the inlet and outlet water temperature, and the heat transfer coefficient h_i were calculated by the use of the equation correlated by Colburn (14):
$h_i = 0.023 \ (Re)^{0.8} \ (k) \ (Pr)^{0 \ 3}/D$. The theoretical amplitude ratios are nearly equal to 1 over the frequency range $0-20$ rad/min. and thereafter decreases gradually. The experimental data taken within the frequency range, $0-3$ cyles/min. fall fairly well on the theoretical curves. The fact that the amplitude ratio decreases very slowly at low frequency can be explained by the time constants used. In the experiment, the dead time constant, t_d, were all in the order of 10^{-2} min, which means the water entered the test section flows out too fast that the variation in the water temperature cannot be developed in the tube at low frequencies and that the axial mixing and the heat conduction in the water cannot occur significantly. The wall time constants, τ_w, were in the order of 10^{-3} min, which means that the energy retained by the tube wall is negligible in comparison to that transferred to the coolant water. The frequency response of the wall temperature was investigated at the point of 7 where $x=141.5$cm for the flow rate of water 6.77 lb/min. In Fig. 6 and 7, the frequency response characteristics of the wall temperature are shown. The dotted lines in the Figures represent the response of the bulk water temperature calculated from equation (20) and (21) at $x=141.5$cm for the flow rate of water 6.77 lb/min. As shown in Figure, the experimental data for the wall temperature lie below the theoretical curves both in the amplitude ratio and the phase angle. Part of this discrepancy is probably due to the fact that the experimental data include the response of the thermocouple and the dynamic behavior of the outer insulating material, while the theoretical curves indicate the response of the heat exchanger alone exclusive of the outer insulating material.

The present investigation was carried out in the range of the insignificant change of the amplitude ratio and the phase angle, however, from equations (20)
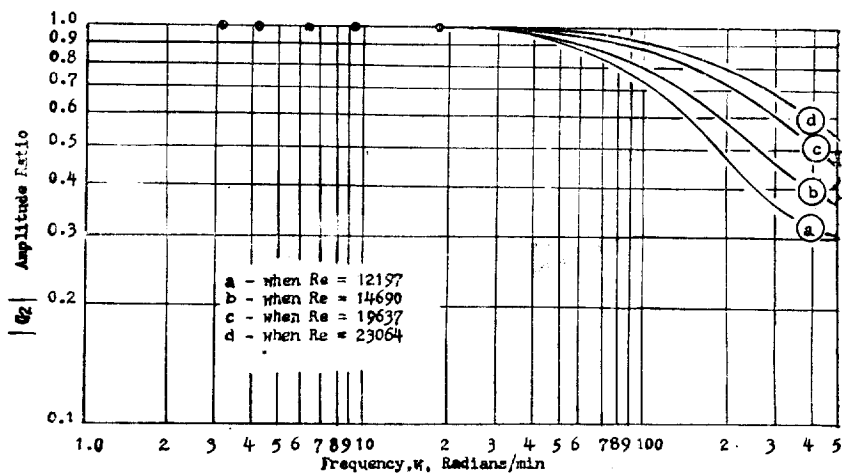
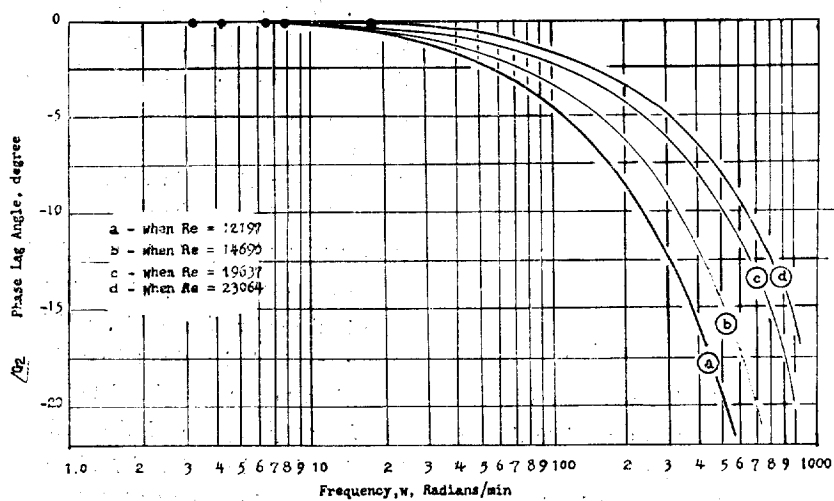Fig. 4. Amplifude-ratio respon of the bulk water temperature.

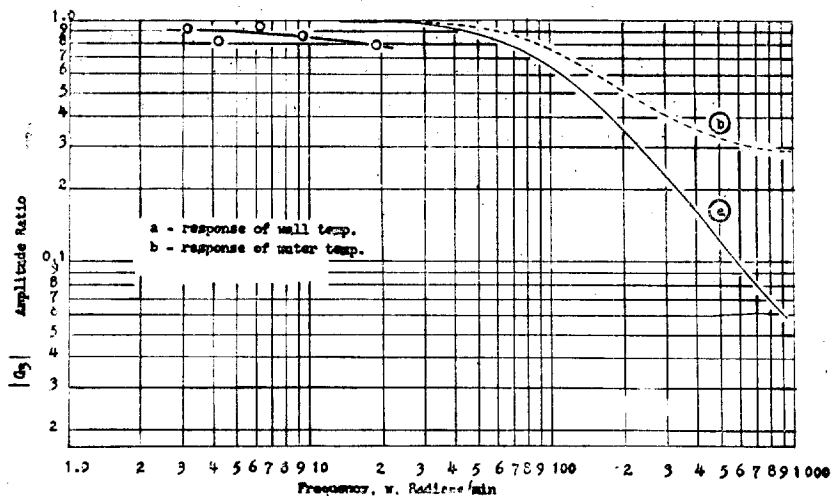Fig. 5. Phase-angle response of the bulk water temperature.

Fig. 6. Amplitude-ratio response of the wall temperature.
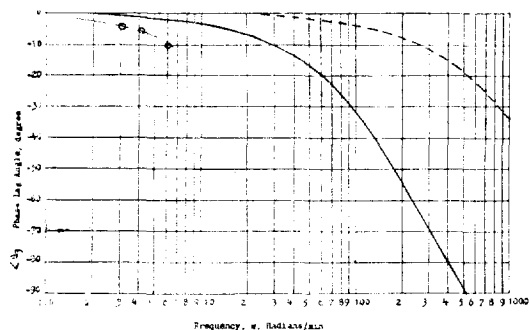

Fig. 7. Phase-angle response of the wall temperature.

(21), (26), and (27), the significant changes of the amplitude ratio and the phase angle would be expected if the time constants, t_d, τ, and τ_w became larger, and in other to make the time constants larger, it is required to have the thicker tube wall, lower velocity of water as can be seen from the equations (3) and (4) of the definitions of the time constants.

Summary

Dynamic characteristics of a vertical tube heat exchanger having constant uniform heat generation in the wall was investigated by the frequency response method.

The experimental ratio and lag angle for the inlet-to-outlet-water temperature are nearly 1 and 0 respectively in the low range of frequency (0−3 cycles/min.), which are identical to those values from the theory.

The inlet-water-to-the wall-temperature transfer function was obtained by combining the water-to-the wall temperature transfer function and inlet-to-out-water temperature transfer function. The experimental frequency response data obtained were lower in the amplitude ratio and in the phase angle than those predicted from theory.

The time constants of the wall and the water and the dead time constant of the heat exchanger were shown to have a significant role in the frequency response of the heat exchanger, and in order to observe the range where significant changes of the amplitude ratio and the phase angle of the temperatures of the water and the wall occur, the thicker tube wall, the lower velocity of water are required.

Nomenclature

A = defined by equation (18), dimensionless

B = defined by equation (20), dimensionless

c = specific heat of water, Btu/(lb) (°F)

c_w = specific heat of wall, Btu/(lb) (°F)

$f_1(s)$ = defined by equation (12)

$f_2(s)$ = defined by equation (13)

h_i = heat transfer coefficient between wall and fluid, Btu/(ft²) (hr) (°F)

$G_1(s)$ = transfer function between the temperature of water and heat generation, defined by equation (14)

$G_2(s)$ = transfer function between the inlet-and outlet-water temperature, defined by equation (15)

$G_3(s)$ = transfer function between the inlet-water-and the wall-temperature, by equation (25)

$|G|$ = amplitude ratio, dimensionless

$\angle G$ = phase lag angle, degree

k = conductivity of the water, Btu/(ft) (hr) °F

L = length of the heated tube, ft

$q^\cdot(t)$ = heat generation in the tube wall, Btu/(min) (ft of tube wall)

$Q(t)$ = defined by equation (5), °F/min

r_o = outer radius of the tube, ft

r_i = inner radius of the tube, ft

s = Laplace transform of time

t = time, min

t_d = dead time, L/u, min

$T_o(x, t)$ = water temperature in tube, °F

$T_{wo}(x, t)$ = wall temperature, °F

$T_s(x, 0)$ = water temperature in tube at steady state, °F

$T_{ws}(x, 0)$ = wall temperature at steady state, °F

$T(x, t)$ = variation in water temperature from steady state, °F

$T_w(x, t)$ = variation in wall temperature from steady state, °F

u = velocity of water, ft/min

x = axial distance of the tube, ft

ρ = density of water, lb/ft³

ρ_w = density of wall, lb/ft³

τ = time constant, defined by equation (3), min

τ_w = time constant, defined by equation (3), min

τ_w = time constant, defined by equation (4), min

ω = angular frequency of sinusoidal oscillation, rad/min

Bibliography

1. W. C. Cohen and E. F. Johnson, *Ind. Eng. Chem.* **48**, 1031-1034 (1956)

2. J. P. Hsu and N. Gilbert, *A. I. Ch. E. Journal*, 593-598, (1962)

3. J. M. Mozley, *Ind. Eng. Chem.*, **48**, 1035-1051 (1956)

4. M. Masubuchi, *Trans. ASME*, Series D, **81**, 51 (1960)

5. S. Lees and J. O. Hougen, *Ind. Eng. Chem*, **48**, 1064-1068 (1956)

6. J. A. Clark, V. S. Arpaci, and K. M. Treadwell, *Trans. ASME.* **80**, 612-624, (1958)

7. V. S. Arpaci, and J. A. Clark, *Trans. ASME.* **80**, 625-634, (1958)

8. V. S. Arpaci, and J. A. Clark, *Journal of heat transfer, Trans. ASME.* Series C, **81**, 253-266 (1959)

9. W. J. Yang, J. A. Clark, and V. S. Arpaci, *Journal of heat transfer, Trans. ASME.* Series C, **83**, 321-338 (1961)

10. L. M. Zoss, N. W. Gollin, and R. I. Edelman, *Ind. Eng. Chem.*, **48**, 1069-1073 (1956)

11. D. P. Campbell, "Process Dynamics", Wliey, New York, (1958)

12. N. H. Ceaglske, "Automatic Process Control for Chemical Engineers", Wiley, New York (1961)

13. R. R. Rohde, "The Methods and Apparatus used in the Experimental Determination of Water Film Coefficients", Argonne National Laboratory, (1955)