Optimization of Some Radiators With Fins and With Evolute Reflectors

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The maximum heat rejection per unit weight of radiator with evolute type reflectors is compared with those of some radiators with optimum rectangular fins by means of examples. The mutual irradiations among the surfaces of neighboring radiator parts are considered in computing the view factors.

The heat rejection system for space power plants is a major weight item. For this reason a number of papers in recent years treated the utilization of the some kind of fins or reflectors. Conducting fins that act as extended heat transfer surface are spaced between the tubes. If the cylindrical radiator geometry is selected, a radiator with evolute reflectors may be interesting. Some radiator configurations are shown in Fig. 1.

An approximate solution of maximizing heat rejection per unit weight with tube and fin geometry is given in Callinan and Berggren[1] by assuming that the fin view factor equals one. It is also proposed in his work, that as a result of future analytical work, a generalized correctior factor could be generated, which could readjust the fin heat rejection rate, as calculated by the approximate method, to show closer agreement with the exact solution. Instead of finding a correction factor the fin and tube view factors are derived analytically only as a function of 2B/D in Appendices A and B. Using these view factors the heat rejection per unit weight is maximized with optimum rectangular fins.

Reynolds[6] presented the optimization of the fin geometry in consideration of the incident irradiation and the associated structure weight. According to his paper the optimum fin height and thickness are expressed in terms of the heat $(Q/L)_f$, which must be rejected from each fin per unit of span. It means, one cannot determine the optimum fin height and thickness, until the radiator geometry including fin view factor and efficiency is known.

A method and examples of maximizing heat rejection per unit weight per unit radiator length are given and results for some possible radiator configurations are compared.

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The following assumtions are made:

1. Material properties k, e are independent of the temperature.
2. The fin thicknees is constant.
3. The fin base temperature is constant at T_o
4. Incident solar radiation is excluded.
5. The increase of the associated structure weight according to the increase of the fin height is not considered.
6. The radiator is infinitely long in tube axial direction.
7. The surface heat transfer is entirely by radiation.

(1) A tube and Two Fins with Double-active Surfaces

Considering a section which includes half a tube and a fin in Fig. 1 (a), the heat rejected from the fin per unit length is

$$(Q/L)_f = b\eta \cdot F_f e\sigma T_o^4 B \tag{1, 1}$$

The fin view factor F_f is calculated in Appendix A and illustrated in Fig. 4.

$$F_f = \sqrt{1+D/2B} - D/2B \text{ arc cos}\left(\frac{D/2B}{2+D/2B}\right) \tag{1, 2}$$

If the irradiation from the tube section is neglected, $(Q/L)_f$ may be expressed as in Hwang-Bo[4].

$$(Q/L)_f = \frac{2}{1.76} F_f \cdot e\sigma T_o^4 \cdot B \tag{1, 3}$$

The heat rejected from the tube section is

$$(Q/L)_t = \left(\frac{\pi D}{2} - d\right) e\sigma T_o^4 F_t \tag{1, 4}$$

The tube view factor F_t is calculated in Appendix B and illustrated in Fig. 5.

$$F_t = \frac{1}{2} + \frac{2}{\pi}\left(1 + \frac{2B}{2} - \sqrt{\left(\frac{2B}{D}\right)^2 + \frac{2B}{D}} - \frac{1}{2} \text{arc sin} \frac{0.5}{2B/D+0.5}\right) \tag{1, 5}$$

The heat rejection per unit weight of radiator may be described as a function of the optimum fin height.

$$(Q/L \cdot G)_f = \frac{\left(\frac{\pi D}{2} - d\right) e\sigma T_o^4 F_t + 2\eta B_{opt} \cdot e\sigma T_o^4 F_f}{\frac{\pi}{2}\left[\frac{D^2}{4} - \frac{(D-2s)^2}{4}\right]\rho_1 + B_{opt} \cdot d\rho_2} \tag{1, 6}$$

The optimum rectangular fin thickness d is given in Hwang-Bo[4] in Fig. 3.

$$d_{opt} = \frac{5}{2} \cdot \frac{1.46 \cdot (Q/L)_f^2}{kT_o \cdot e\sigma T_o^4 F_f \cdot b} (QL)_f^2 \tag{1, 7}$$

Ex. 1.

With the following assumed values

Two fins $N=2$

Double-active surface $b=2$

Tube diameter $D=14$[mm]

Tube wall thickness $s=2$[mm]

Thermal conductivity $k=0.2$[W/cm$\cdot C$]

Fin base temperature $T_o=800$[°C]

Emissivity of the tube surface $e_t=0.9$

Emissivity of the fin surface $e_f=0.85$

Density of the tube $\rho_1=7.9$ [g/cm³]

Density of the fin $\rho_2=4.3$ [g/cm³]

The maximum heat rejection is found at the optimum fin height $B_{opt}=13.7$ [mm] from Fig. 2.

$\therefore$ $(Q/L\cdot G)_{max}=5.9$ [W/cm·g]

The optimum fin thickness is from Fig. 3

$d_{opt}=1.14$ [mm]

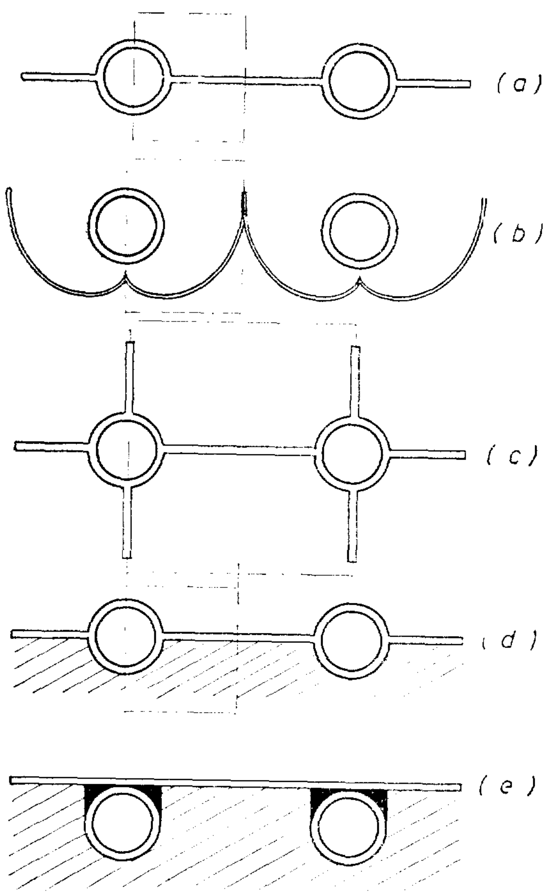

Fig. 1. Schematic Radiator Configurations with Fins or Reflectors.

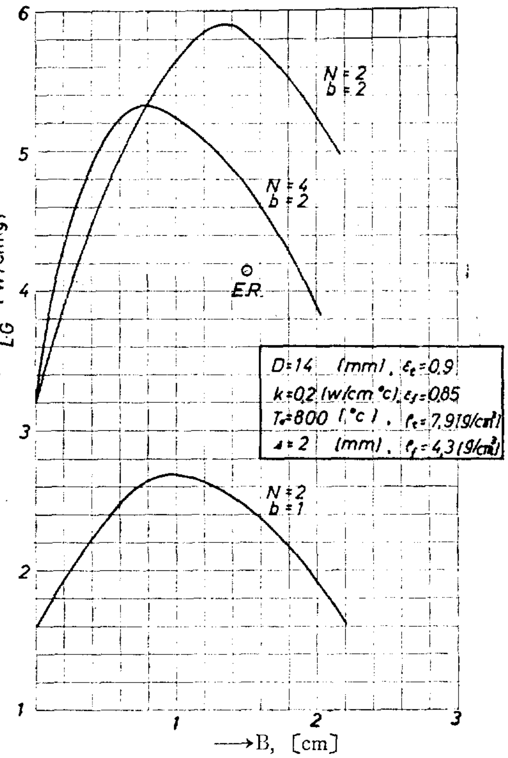

Fig. 2. Heat Rejection per Unit Weight of Radiator with Optimum Rectangular Fins

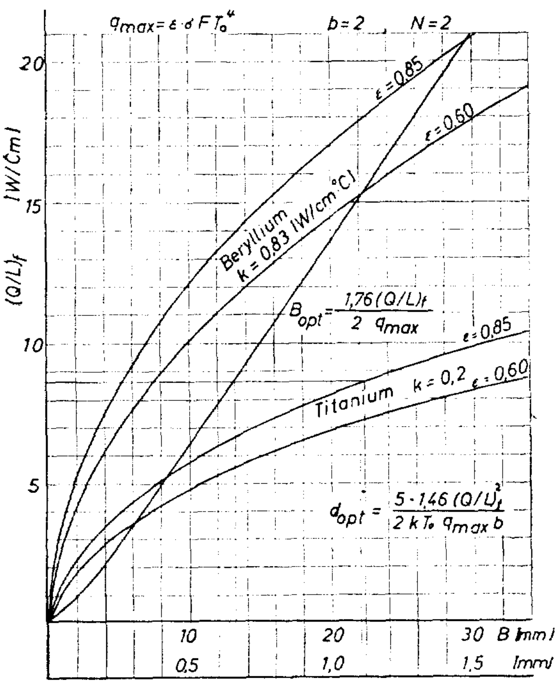

Fig. 3. Rejected Heat per Unit Span of Fin vs Optimum Fin Height and Thickness

(2) A Tube and Four Fins with Double-active Surfaes

Considering a section which includes two half tubes and four fins in Fig. 1(c), the heat rejected from the section is

$$(Q/L)_2=4B\eta_2F_{23}(e_f\sigma T_o^4+\gamma_2H_2)+4B\eta_1F_{13}(e_f\sigma T_o^4+\gamma_1H_1)+(\pi D-4d)(e_t\sigma T_o^4+\gamma_tH_t)F_{t3} \quad (1,8)$$

Where H_1 is the irradiation leaving the adjacent radiator surface and may be given approximately as follows.

Neglecting the twice or more reflected radiation among the surfaces A_1, A_2, and A_t,

$$H_t\simeq 8e_f\cdot\sigma T_o^4B(\eta_1F_{1t}+\eta_2F_{2t})+(\pi D-4d)e_t\cdot\sigma T_o^4F_{tt} \quad (1,9,1)$$

$$H_2\simeq 4e_f\cdot\sigma T_o^4B(2\eta_1F_{12}+\eta_2F_{22})+(\pi D-4d)e_t\sigma T_o^4F_{t2} \quad (1,9,2,)$$

$$H_1\simeq 4e_f\cdot\sigma T_o^4B\eta_2F_{21}+(\pi D-d)e_t\cdot\sigma T_o^4F_{t1} \quad (1,9,3)$$

The heat rejection per unit weight of radiator may be expressed as a function of the optimum fin height. Both fin efficiencies are slightly different, since the irradiation from the adjacent radiator surfaces are not equal. A dimensionless parameter α, defined as $H/be\sigma T_o^4F$, is introduced to compute the fin efficiencies. With the view factors given in Appendix C and Fig. 5 (a) in Reynolds $\eta_1=0.61$ and $\eta_2=0.59$ are found.

$$(Q/L\cdot G)_2=\frac{(Q/L)_2}{\pi\left[\frac{D^2}{4}-\frac{(D-2s)^2}{4}\right]\rho_1+4B\cdot d\rho_2} \quad (1,10)$$

With the same values assumed in Example 1, the optimum fin height is, from Fig. 2, $B_{opt}=8$ [mm], and the optimum fin thickness is, from Fig. 3, $d_{opt}=0.45$ [mm] with

$$(Q/L\cdot G)_{max}=5,3\ [W/cm\cdot g]$$

(3) A Tube and Two Fins with One Active surface

The heat rejection per unit weight can be written from equation (1,6)

$$(Q/L\cdot G)_3=\frac{\left(\frac{\pi D}{4}-d\right)e_t\sigma T_o^4F_t+\eta\cdot B_{opt}e_f\sigma T_o^4}{\frac{\pi}{2}\left[\frac{D^2}{4}-\frac{(D-2s)^2}{4}\right]\rho_1+2B_{opt}\cdot d\cdot\rho_2} \quad (1,11)$$

With the same conditions as in Example 1, the heat rejection is presented as a function of B_{opt} in Fig. 2.

$(Q/L\cdot G)_{max}=2.7[W/cm\cdot g]$

at $B_{opt}=9.5$ [mm]

$d_{opt}=1$ [mm]

(4) A tube with Evolute Reflectors

Reflector efficiency is defined as the ratio of the heat rejected by the tube with reflectors to that which would be rejected by the tube without reflectors with no incident irradiation.

For very long radiator in axial direction the reflector efficency is given in Hwang-Bo[3].

$$\eta_{ref}=0.196\gamma^2+0.402\gamma+0.402 \qquad (1,12)$$

With the following values assumed in Example 1,

$e_t=0.9$ $\quad T_o=800$ [°C] $\quad D=14$ [mm]

$\rho_1=7.9$ [g/cm³] $\quad s=2$ [mm]

and reflector reflectivity

$\gamma=0.94$ $\quad d_{ref}=0.18$ [mm]

$\eta_{ref}=0.95$ $\quad \rho_3=8$ [g/cm³]

The rejected heat from half a tube with a evolute reflector per unit weight per unit radiator length is

$$(Q/L\cdot G)=\frac{\eta_{ref}(\pi/2)D\ e_t\sigma T_o^4}{\frac{\pi}{2}\left[\frac{D^2}{4}-\frac{(D-2s)^2}{4}\right]\rho_1+\frac{D}{4}\pi^2 d_{ref}\rho_3}$$

$$=\frac{0.95(2.2)(6.88)}{2.975+0.495}=4.14 \quad [W/cm.g]$$

Conclusion

The addition of two more fins per tube, centered in a plane at right angles to the other fin and of the same fin height, does not improve the radiation capacity per unit mass. The use of shorter fin and the variation of reflectivity of fin material may improve the radiation capacity, but not much better than that of the optimized two fin configuration with double-active sides.

The puncture of meteorites decrease the reflectivity of the reflector. But the radiation capacity of the radiator with evolute reflectors is still better than that of the optimized two fin system with one active side, even if a tenth of reflecting surface were destroyed.

Appendix A. Fin View Factor F_f

The fin view factor is equal to that fraction of the radiation. leaving the surface of fin A_1 in all directions, which is intercepted within the bounds of the imaginary surface A_2, bounded by the tangents of the opening angle $(\Phi+\Phi')$ in Fig. 4.

For a very long tube with fins the fiin view factor may be given as follows[5].

$$F_f=\frac{1}{2B}\int_o^{2B}\left(\frac{\sin\Phi+\sin\Phi'}{2}\right)dx \qquad (A,\ 1)$$

Where

$$\sin\Phi=\frac{\sqrt{x^2+xD}}{x+D/2}$$

$$\sin\Phi'=\frac{\sqrt{\left(\frac{D}{2}+2B-x\right)^2-\left(\frac{D}{2}\right)^2}}{\frac{D}{2}+2B-x}$$

The integration of equation (A, 1) results

$$F_f=\sqrt{1+(D/2B)}-(D/4B)\ \text{arc}\ \cos\left(\frac{D/2B}{2+D/2B}\right) \qquad (A, 2)$$

Appendix B. Tube Geometric Factor F_t

The tube view factor is equal to that fraction of the radiation, leaving the tube surface A_t in all direction, which is intercepted within the bounds of the imaginary surface A_2 in Fig. 5. For the area of infinite extent in one direction, generated by a straight line moving always parallel to itself: all cross sections normal to the infinite dimension are identical,the view factor may be given as follows[2].

$$F_t=\frac{A_3+A_2-(A_4+A_5)}{2\cdot A_3} \qquad (A, 3)$$

$$F_t=\frac{\frac{\pi D}{4}+(2B+D)-\left[\sqrt{\left(2B+\frac{D}{2}\right)^2-\left(\frac{D}{2}\right)^2}+\left(\frac{D}{2}\right)\theta\right]}{2\left(\frac{\pi D}{4}\right)} \qquad (A, 4)$$

Where

$$\theta=\text{arc}\ \sin\frac{D/2}{2B+D/2}$$

$$F_t=0.5+\frac{2}{\pi}\left(1+\frac{2B}{D}-\sqrt{\left(\frac{2B}{D}\right)^2+\left(\frac{2B}{D}\right)}-\frac{1}{2}\text{arc}\ \sin\frac{0.5}{\frac{2B}{D}+0.5}\right) \qquad (A,\ 5)$$

Appendix C. View Factors between Two Sur faces of $A_1, A_2, A_3,$ and A_t

The view factor F_{13} is equal to that fraction of the radiation leaving the surface A_1 in all direction, which is intercepted within the bounds of the surface A_3 in Fig. 6. The method in Appendix B is still valid for all other view factors and the results are given in terms of $2B/D$ in Fig. 6.

$$F_{13}=\sqrt{\frac{5}{4}+\frac{3}{2}\left(\frac{D}{2B}\right)+\frac{1}{2}\left(\frac{D}{2B}\right)^2}-\sqrt{\frac{1}{2}\left(\frac{D}{2B}\right)+\frac{1}{4}}$$
$$-\frac{1}{2}\left(\frac{D}{2B}\right)\text{arc sin}\frac{D/2B}{1+D/2B} \qquad (A,6)$$

$$F_{t2}=\frac{2}{\pi}\left[1+\frac{2B}{D}+\sqrt{5\left(\frac{B}{D}\right)^3+\frac{3B}{D}+\frac{1}{2}}\right.$$
$$-\sqrt{\left(\frac{2B}{D}\right)^2+\frac{2B}{D}}-\frac{1}{2}\text{arc sin}\frac{1}{\frac{4B}{D}+1}$$
$$\left.-\sqrt{5\left(\frac{B}{D}\right)^2+\frac{4B}{D}+1}\right] \quad (2B/D>0.3)$$

oder $F_{t2}=\frac{2}{\pi}\left[\frac{1}{2}\text{arc cos}\frac{1}{\frac{2B}{D}+1}+\frac{B}{D}-\sqrt{\left(\frac{B}{D}\right)^2+\frac{B}{D}}\right]$

(A, 7)

$$F_{t3}=\frac{2}{\pi}\left[\sqrt{\frac{5}{4}\left(\frac{2B}{D}\right)^2+\frac{4B}{D}+1}+\sqrt{\left(\frac{B}{D}\right)^2+\frac{B}{D}}\right.$$
$$+\frac{1}{2}\text{arc sin}\left(\frac{1}{2B/D+1}\right)-\sqrt{\frac{5}{4}\left(\frac{2B}{D}\right)^2+\frac{3B}{D}+\frac{1}{2}}$$
$$\left.-\frac{B}{D}\right] \qquad (2B/D>0.29) \qquad (A,8)$$

$$F_{23}=2.5+\frac{D}{B}-\sqrt{4.25+\frac{4D}{B}+\left(\frac{D}{B}\right)^2} \qquad (A,9)$$

$$F_{12}=\frac{1}{4}\left[\sqrt{4+\frac{2D}{B}}+\frac{D}{2B}\text{arc sin}\frac{D/B}{4+D/B}+\sqrt{1+\frac{D}{B}}\right.$$
$$+\frac{D}{2B}\text{arc sin}\frac{D/B}{2+D/B}-\frac{\pi D}{4B}$$
$$\left.-\sqrt{5+\frac{3D}{B}+\frac{1}{2}\left(\frac{D}{B}\right)^2}\right] \qquad (A,\ 10)$$

$$F_{22}=\sqrt{5+\frac{4D}{B}+\left(\frac{D}{B}\right)^2}-2-\frac{D}{B} \qquad (A,11)$$

$$F_{tt}=\frac{2}{\pi}\left[2\sqrt{\left(\frac{2B}{D}\right)^2+\frac{2B}{D}}+\frac{1}{2}\text{arc sin}\frac{0.5}{2B/D+0.5}\right.$$
$$\left.-2\left(\frac{2B}{D}\right)-1\right] \qquad (A,12)$$

$$F_{t1}=\frac{1}{2}+\frac{2}{\pi}\left[\frac{2B}{D}-\sqrt{\left(\frac{2B}{D}\right)^2+\frac{2B}{D}}\right.$$
$$\left.-\frac{1}{2}\text{arc sin}\frac{0.5}{2B/D+0.5}\right] \qquad (A,13)$$

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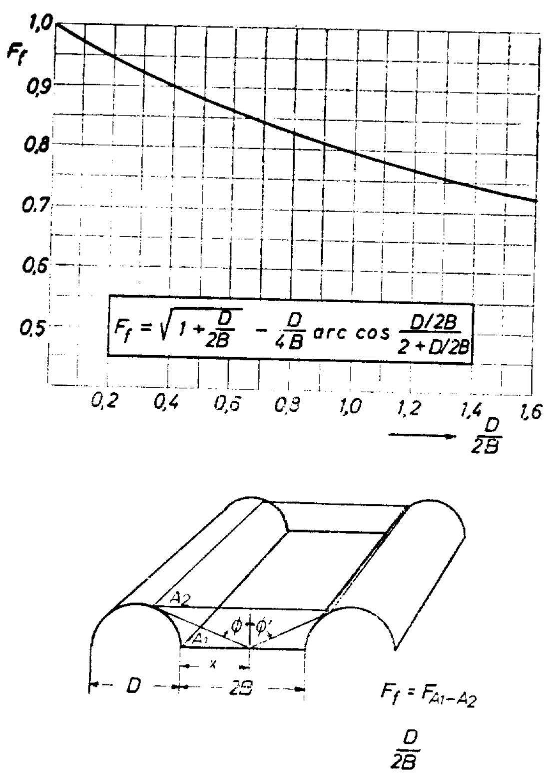

Fig. 4. Fin View Factor vs $\frac{D}{2B}$

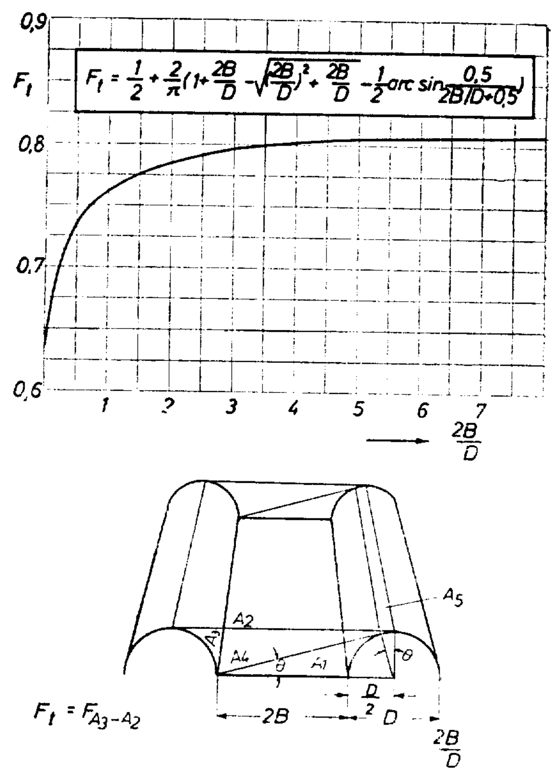

Fig. 5. Tube Geometric Factor vs $\frac{2B}{D}$

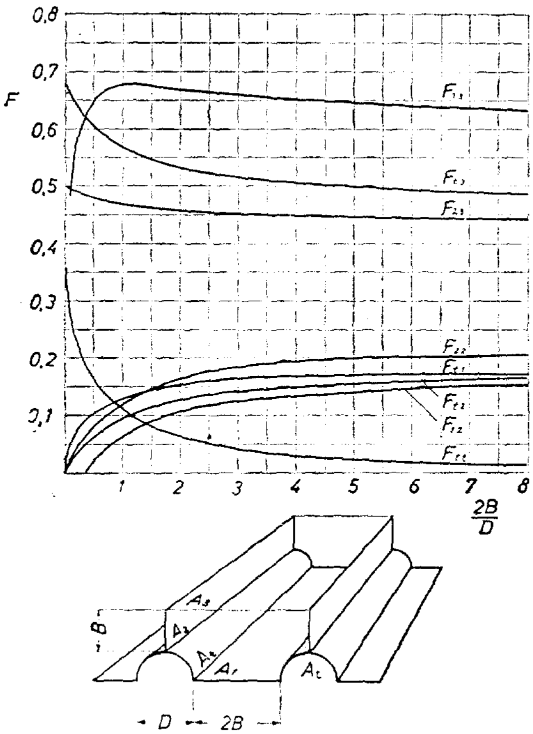

Fig. 6. Fin and Tube View Factors vs $\frac{D}{2B}$

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