

ESTIMATION OF VISCOSITY

I. Polar or Nonpolar Gases and Nonpolar Gas Mixtures at Low Pressures (0.2 - 5 Atm.)

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Two estimation methods are recommended for calculating the viscosity of polar or nonpolar gases and nonpolar gas mixtures at low pressures (0.2-5 atm.): (1) the theoretical approach using the Chapman-Enskog expression, and (2) the empirical approach using the principle of corresponding states. These methods allow one to estimate the low pressure viscosity of gases or gas mixtures at any temperature if either the value of Lennard-Jones or Stockmayer potential parameters or the critical properties for pure gases or components of gas mixtures are known.

Errors less than 2 to 3 percent are to be expected over a temperature range from near the freezing point to reduced temperatures around 10 or higher using the Chapman-Enskog expression. Using the empirical correlations by the principle of corresponding states, the errors are expected to be from 2 to 4 percent.

Various methods for estimating the viscosity of gases and gas mixtures at low pressures in the absence of experimental data have been suggested by many investigators and the results up to 1964 were summarized by Reid and Sherwood⁽¹⁾.

The most reliable methods were selected to estimate the viscosity by either theoretical method or corresponding states method.

Theoretical Method

Pure Gas, Polar or Nonpolar

The general expression for viscosity from the results of Chapman-Enskog solution of Maxwell-Boltzmann equation⁽³⁾ is

$$\eta = \frac{266.93 \sqrt{MT}}{\sigma^2 \Omega_V} \times 10^{-7} \quad (1)$$

where M is the molecular weight, σ is the collision diameter of a molecule, T is the absolute temperature. Ω_V is a dimensionless collision integral and is a function of the reduced temperature, $T^* = \frac{kT}{\epsilon}$.

The parameters of an assumed intermolecular potential function are chosen to provide agreement between experimental and calculated values of the viscosity by Eq. (1). While this equation applies rigorously only to spherical molecules having no internal degrees of freedom, it has been possible to obtain useful information about other types of simple molecules by assuming that a central force law also describes their interactions. The potential function parameters thus obtained represent an average of the actual forces over all orientations of the colliding pair of molecules. Excellent discussions on the intermolecular potential functions are found elsewhere^(3,7).

The values of the collision integrals for an assumed intermolecular force model are needed to calculate the

viscosity. The Lennard-Jones potential for nonpolar gases and the Stockmayer potential for polar gases have been the most successfully used intermolecular potentials for which the collision integrals are available. The Lennard-Jones potential has been treated a number of times; the best results available have been summarized by Hirschfelder, Curtiss, and Bird⁽⁹⁾. Since then, more accurate results were obtained with better high-speed computers. The most accurate results presently available are those of Monchick and Mason⁽⁸⁾ up to $T^*=20$ to 30; at higher temperatures the results of Itean, Glueck, and Svehla⁽⁴⁾ are more accurate. Monchick and Mason⁽⁸⁾ have determined the collision integrals using the Stockmayer potential (Lennard-Jones potential plus the dipole-dipole interaction term).

The functional correlations between Ω_V and T^* , from the results given by Monchick and Mason, can be obtained as

$$\frac{1}{\Omega_V} = A + B \ln T^* + C (\ln T^*)^2 \quad (2)$$

where $\Omega_V = \langle Q^{(2,2)*} \rangle$ as given by Monchick and Mason, the constants A , B , and C are determined by the non-linear least-square fit method.

Multicomponent Nonpolar Gas Mixtures

Several methods, both rigorous and empirical, have been developed for predicting the viscosities of multicomponent gas mixtures^(3,8). Most of these methods are either unwieldy, require prolonged calculations, or necessitate a knowledge of the pure component viscosity at given temperature. Such data are often not available.

Strunk and coworkers^(14,15) suggested a method for predicting the viscosities of multicomponent nonpolar gas mixtures. This method is based on a simplified model which assumes that the nonpolar molecules are rigid spheres and that the Lennard-Jones potential holds for all nonpolar mixtures and components over all temperature ranges. The results of this proposition can be expressed by

$$\eta_m = \frac{266.93 \sqrt{TM_m}}{\sigma_m^3 \Omega_{Vm}} \times 10^{-7} \quad (3)$$

where

$$\eta_m = \sum_{i=1}^n x_i \sigma_i \quad (4)$$

$$M_m = \sum_{i=1}^n x_i M_i \quad (5)$$

$$\frac{\epsilon_m}{k} = \sum_{i=1}^n x_i \left(\frac{\epsilon_i}{k} \right) \sigma_i^3 / \sigma_m^3 \quad (6)$$

From the analysis of the experimental data, Strunk and coworkers⁽¹⁵⁾ suggested some modification of Eq. (3) for mixtures containing three or more components as follows:

$$\eta_m = \frac{276.27 \sqrt{TM_m}}{V_m^2 \Omega_{Vm}} \times 10^{-7} \quad (7)$$

The Principle of Corresponding States Method

Since one needs to know the values of the intermolecular potential parameters to calculate the viscosity of gases or gas mixtures by Eqs. (1), (3), or (7), the values calculated with any of these equations are of questionable accuracy for gases or gas mixtures for which such data are not available. Even though the relationship between the critical properties and the potential parameters of the gas has been given elsewhere^(2,13) the uncertainty is too big to use the calculated values of the potential parameters in the viscosity calculation as desired. Therefore, it would be nice to relate directly the viscosity to the critical constants of gas, since these constants are more readily available than the experimental viscosity data.

The kinetic theory of a gas at low pressures indicates that the viscosity is a function of temperature and molecular weight and the viscosity is expected to depend on the critical properties. Hence, one can write

$$\eta = f(\tau, T_c, M, P_c, v_c) \quad (8)$$

Applying the Raleigh method of dimensional analysis to these variables, one gets

$$\eta = \alpha T^a T_c^b M^c P_c^d v_c^e R^f \quad (9)$$

The dimensions involved are mass, length, time, and temperature. Dimensional analysis produces the following result:

$$\eta \left(\frac{T_c^{1/6}}{M^{1/2} P_c^{2/3}} \right) = \beta Z_c^m T_r^n \quad (10)$$

where $Z_c = P_c v_c / R T_c$. For convenience the group $T_c^{1/6} / M^{1/2} P_c^{2/3}$ will be referred to in this paper as ζ . This group is a characteristic constant for each substance^(2,5,6).

The following equations were obtained by Stiel and Thodos^(11,12) by fitting the experimental data to Eq. (10) and they seem to be the best available.

Nonpolar Gases

$$\eta \zeta = 34.0 T_r^{0.94} \times 10^{-7} \text{ for } T_r \leq 1.5 \quad (11)$$

$$\eta \zeta = 17.78(4.58 T_r - 1.67)^{5/8} \times 10^{-7} \text{ for } T_r > 1.5 \quad (12)$$

Polar Gases

Hydrogen bonding types

$$\eta \zeta = (7.55 T_r - 0.55) Z_c^{-5/4} \times 10^{-7} \text{ for } T_r < 2.0 \quad (13)$$

Non-hydrogen bonding types

$$\eta \zeta = (1.90 T_r - 0.29)^{0.9} Z_c^{-2/3} \times 10^{-6} \text{ for } T_r < 2.5 \quad (14)$$

These simple equations are applicable to all nonpolar and polar gases except hydrogen and helium. Stiel and Thodos⁽¹¹⁾ present special techniques to calculate the viscosities for these gases (see Appendix 1). Also, although acetic acid is known to exhibit hydrogen bonding, the predicted behavior is found to be anomalous, probably cyclic dimers are formed and it best be considered a polar gas without hydrogen bonds.

Dean and Stiel⁽¹⁾ suggested that Eqs. (11) and (12) might be applicable to low pressure gas mixtures, if in evaluating ζ , suitable pseudocritical constants were used. After a careful study of low pressure data, it was recommended that Eqs. (11) and (12) be modified slightly to give

$$\eta_m \zeta_m = 34.0 T_r^{0.94} \times 10^{-7} \text{ for } T_r < 1.5 \quad (15)$$

$$\eta_m \zeta_m = 166.8(0.1338 T_r - 0.0932)^{5/8} \times 10^{-7} \text{ for } T_r \geq 1.5 \quad (16)$$

where

$$\zeta_m = T_{cm}^{1/6} / \left(\sum_{i=1}^n x_i M_i \right)^{1/2} P_{cm}^{2/3} \quad (17)$$

T_{cm} and P_{cm} are pseudocritical values and are to be determined from the modified Prausnitz and Gunn rule:

$$T_{cm} = \sum_{i=1}^n x_i M_{ci} \quad (18)$$

$$Z_{cm} = \sum_{i=1}^n x_i Z_{ci} \quad (19)$$

$$v_{cm} = \sum_{i=1}^n x_i v_{ci} \quad (20)$$

$$P_{cm} = \frac{R Z_{cm} T_{cm}}{v_{cm}} \quad (21)$$

Discussion

Table I illustrates the methods recommended and their accuracy. These methods allow one to calculate the viscosity using either the given potential parameters or critical properties of gases. The intermolecular potential parameters and the basic constants are given in Table II.

The collision integrals for the Stockmayer potential were expressed as a function of the reduced temperature, T^* , and the constants in Eq. (2) are presented in Table III. The relative errors of the calculated value of collision integrals are also given. These errors are less than 0.2 percent in most cases and less than or about one percent for $T^*=0.1-1.0$.

The methods presented in this paper are the best available in simplicity and are capable of predicting the viscosities of polar or nonpolar gases and nonpolar gas mixtures at low pressures with high accuracy.

NOMENCLATURE

k = Boltzmann constant	M = Molecular weight
P = Pressure, atm.	P_c = Critical pressure, atm.
R = Gas constant	T = Absolute temperature, °K
T_c = Critical temperature, °K	T_r = Reduced temperature ($=T/T_c$)
T^* = Reduced temperature ($=kT/\epsilon$)	v_c = Critical molar volume, cc/g mole
x = Mole fraction	Z_c = Critical compressibility factor

Greeks

α, β = Constants	σ = Polar characteristic constant
ϵ = Minimum potential energy	η = Viscosity, poise
Ω_V = Collisional integrals for viscosity, $\Omega^{(2,2)*}$	σ = Collision diameter

Subscripts

i = Component
m = Mixture

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Appendix

Viscosity of Hydrogen and Helium

Due to the quantum effects on the transport properties of hydrogen and helium, the viscosity behavior of

these gases do not follow the normal behavior of other gases. Stiel and Thodos⁽¹¹⁾ suggested the following equations to estimate the viscosities of hydrogen and helium at low pressure:

Hydrogen

$$\eta = 6.43 T^{0.94} \times 10^{-7}; \quad T \leq 50^\circ\text{K}$$

$$\eta = 90.71(0.1375 T - 1.67)^{5/8} \times 10^{-7}; \quad T > 50^\circ\text{K}$$

Helium

$$\eta = 46.78 T^{0.656} \times 10^{-7}; \quad T > 15^\circ\text{K}$$

TABLE I. THE METHODS USED FOR VISCOSITY ESTIMATION

Methods	Applicability	Required Data	Accuracy	Remarks
1. Theoretical				
a. Chapman-Enskog	polar or nonpolar gases	potential parameters	<1-2%	
b. Strunk, et al.	binary nonpolar gas mixtures	potential parameters	<5-6%	Tested for 201 nonpolar gas mixtures, 80°K < T < 550°K
c. Strunk, et al.	multicomponent non-polar gas mixtures	potential parameters	<3%	Tested for 136 nonpolar gas mixtures 278°K < T < 1287°K
2. Corresponding States				
a. Stiel-Thodos	polar gases, hydrogen bonding or nonhydrogen bonding	critical properties	<2-4%	Ave. 1.47% for 11 hydrogen bonding polar gases and ave. 2.95% for 42 nonhydrogen bonding polar gases.
b. Dean-Stiel	nonpolar gas or gas mixtures	critical properties	<1-3%	Ave. 1.44% for 984 experimental points.

TABLE III. CONSTANTS A, B, and C IN

$$1/\eta_V = A + B \ln T^* + C(\ln T^*)^2$$

δ	T^*	A	B	C	$[1 - \eta_V(1/\eta_{V-c})] \times 100$
0.0	0.1-1.0	0.61820	0.28715	0.054558	1.18%
	1.0-3.5	0.62217	0.36213	-0.047731	0.06%
	3.5-30.0	0.75869	0.20111	-0.001665	0.13%
0.25	0.1-1.0	0.61550	0.27894	0.049624	1.06%
	1.0-3.5	0.61861	0.35523	-0.042641	0.05%
	3.5-30.0	0.74862	0.20718	-0.002607	0.13%
0.50	0.1-1.0	0.60017	0.25770	0.042347	0.91%
	1.0-3.5	0.60184	0.34742	-0.033010	0.07%
	3.5-30.0	0.71959	0.22463	-0.005309	0.14%
0.75	0.1-1.0	0.57296	0.26067	0.038124	0.70%
	1.0-3.5	0.57288	0.34112	-0.023838	0.09%
	3.5-30.0	0.67364	0.25199	-0.009508	0.16%
1.0	0.1-1.0	0.53338	0.25768	0.03892	0.68%
	1.0-3.5	0.53723	0.33330	-0.074354	0.14%
	3.5-30.0	0.61399	0.28706	-0.014826	0.18%
1.5	0.1-1.0	0.46565	0.24275	0.039547	0.59%
	1.0-3.5	0.46484	0.30902	-0.020619	0.16%
	3.5-30.0	0.47376	0.36600	-0.026339	0.19%
2.0	0.1-1.0	0.40541	0.21980	0.037314	0.72%
	1.0-3.5	0.42710	0.23487	0.063307	1.00%
	3.5-30.0	0.32850	0.44159	-0.036500	0.19%
2.5	0.1-1.0	0.35967	0.20024	0.035114	0.85%
	1.0-3.5	0.36232	0.23636	0.064220	0.69%
	3.5-30.0	0.20179	0.49805	-0.042663	0.13%

TABLE I POTENTIAL PARAMETERS DETERMINED FROM VISCOSITY DATA AND BASIC CONSTANTS

Gas	Basic Constants										Ref.	
	μ (debye)	σ^{**}	$\sigma(\text{Å})$	$\epsilon/k(^{\circ}\text{K})$	Ref.	M	$T_v(^{\circ}\text{K})$	$P_v(\text{atm})$	$v_c(\frac{\text{cc}}{\text{g mole}})$	Z_c		ζ
Monatomic												
Helium			2.576	10.22	(3)	4.003	5.206	2.26	57.76	0.306	0.381	(9,11)
Neon			2.858	27.5	(3)	20.183	44.5	26.86	41.70	0.307	0.0466	(9,11)
Argon			3.465	116.	(3)	39.944	151.2	48.00	75.32	0.291	0.0276	(9,11)
Krypton			3.61	190.	(3)	83.80	209.4	54.18	76.17	0.292	0.0184	(9,11)
Xenon			4.055	229.	(3)	131.3	289.8	57.64	113.68	0.290	0.0151	(9,11)
Diamotic												
Hydrogen			2.915	38.	(3)	2.016	33.3	12.8	65.03	0.305	0.230	(9,11)
Nitrogen			3.749	79.5	(3)	28.02	126.2	33.5	90.10	0.291	0.0407	(9,11)
Oxygen			3.541	88.	(3)	32.00	154.8	50.1	74.42	0.293	0.0301	(9,11)
Carbon monoxide			3.706	88.	(3)	28.01	133.0	34.5	90.06	0.294	0.0402	(9,11)
Nitric oxide			3.599	91.	(3)	30.01	180.0	64.	57.71	0.251	0.0271	(9,11)
Fluorine			3.653	112.	(3)	38.00	144.0	55.			0.0396	(9,11)
Chlorine			4.400	257.	(3)	70.91	417.2	76.1	124.0	0.276	0.0180	(11)
Bromine			4.268	520.	(3)	149.8	584.	102.	188.44	0.306	0.0115	(9,11)
Iodine			4.982	550.	(3)	253.8	785.	116.			0.00905	(11)
Simple Polyatomic												
Carbon dioxide			4.00	190.	(3)	44.01	304.2	72.85	95.67	0.275	0.0224	(9,11)
Nitrous oxide			3.816	237.	(3)	44.02	309.66	71.7	97.82			(9)
Sulfur dioxide	1.63	0.42	4.04	347.	(8)	64.06	430.7	77.7	123.2	0.269	0.0189	(9,11)
Carbon oxysulfide			4.13	335.	(3)	60.07	378.2	61.0				(9)
Carbon disulfide			4.438	488.	(3)	76.13	552.	78.0	172.5		0.0180	(11)
Carbon tetrachloride			5.581	327.	(3)	153.8	556.4	45.0	275.3		0.0183	(9,11)
*Water	1.85	1.00	2.52	775.	(8)	18.016	647.4	218.4	55.78	0.231	0.0192	(9,12)
*Ammonia	1.47	0.7	3.15	350.	(8)	17.03	405.5	111.5	72.4	0.242	0.0284	(9,12)
Inorganic Vapors												
*Hydrogen chloride	1.08	0.34	3.36	328.	(8)	36.47	324.5	81.6	86.83	0.266	0.0231	(9,12)
Hydrogen bromide	0.80	0.14	3.41	417.	(8)	80.92	363.2	84.0				(9)
Hydrogen iodide	0.42	0.029	4.13	313.	(8)	254.8	424.2	82.0				(9)
Arsenic hydride			4.06	281.	(3)	77.93						
Mercuric iodide			5.625	698.	(3)	327.53						
Mercuric bromide			5.414	530.	(3)	280.53						
Hydrogen cyanide			27.02	456.7	(3)	27.02	456.7	53.2	135.1	0.197	0.378	(9,12)
Stannic tetrabromide			6.666	435.	(3)	438.36						
Stannic tetrachloride			4.540	1550.	(3)	260.54	591.9	37.0	351.13			(11,12)
Mercury			2.898	851.	(3)	200.61	>1823.	>200.	40-50			(12)
Boron trifluoride			4.22	178.	(3)	67.82	261.9 ^a	49.2 ^a				
Silicon tetrafluoride			4.95	148.	(3)	104.09	287.26 ^a	37.5 ^a				
Nitrosyl chloride			3.57	668.	(3)	65.47	440.7 ^a	90. ^a				

TABLE I (continued)—2

Gas	Basic Constants											
	μ (debye)	δ^{**}	σ (Å)	$\epsilon/k^{\circ}K$	Ref.	M	T_c (°K)	P_c (atm)	v_c ($\frac{cc}{g\text{mole}}$)	Z_c	ξ	Ref.
Hydrocarbons												
Methane	3.808		3.808	140.	(2)	16.04	191.1	45.8	99.0	0.289	0.0464	(11)
Ethane	4.384		4.384	236.	(2)	30.07	305.5	48.2	148.1	0.285	0.0357	(11)
Propane	5.240		5.240	206.	(2)	44.09	370.0	42.0	200.4	0.277	0.0334	(11)
n-Butane	5.869		5.869	208.	(2)	58.12	425.2	37.5	254.9	0.274	0.0321	(11)
i-Butane	5.341		5.341	313.	(3)	58.12	408.5	36.0		0.282	0.0327	(11)
n-Pentane	6.099		6.099	269.	(2)	72.15	469.8	33.3	311.0	0.269	0.0317	(11)
n-Hexane	5.916		5.916	324.	(2)	86.17	507.9	29.9	368.2	0.264	0.0315	(11)
n-Heptane	7.407		7.407	333.	(2)	100.2	540.2	27.0	426.4	0.260	0.0316	(11)
n-Octane	8.302		8.302	266.	(2)	114.2	569.4	24.6	490.1	0.258	0.0318	(11)
n-Nonane	8.407		8.407	266.	(2)	128.2	595.4	22.6	540.	0.250	0.0320	(11)
2-Methylbutane	5.819		5.819	217.	(2)	72.15	461.0	32.9		0.268	0.0318	(11)
2,2-Dimethyl propane						72.15	433.5	31.6		0.269	0.0290	(11)
2,2-Dimethyl butane						86.17	489.4	30.7		0.274	0.0308	(11)
2,2-Dimethyl pentane						100.2	520.3	27.4	420.	0.268	0.0312	(11)
2,2,4-Trimethyl butane	7.621		7.621	178.	(2)	100.2	531.5	29.75	394.	0.260	0.0296	(11)
2,2,4,4-Trimethyl pentane						114.2	544.1	25.4		0.262	0.0321	(11)
Ethylene	4.066		4.066	230.	(2)	28.05	282.4	50.0	123.6	0.267	0.0356	(11)
Propylene	4.670		4.670	303.	(2)	42.08	365.1	45.4	180.6	0.280	0.0324	(11)
Butene-1	5.198		5.198	319.	(2)	56.10	419.6	39.7	239.7	0.276	0.0314	(11)
Butene-2	5.508		5.508	259.	(2)	56.10	430.	41.0	235.7	0.274	0.0308	(11)
i-Butene	4.776		4.776	425.	(2)	56.10	417.9	39.5	238.7	0.257	0.0315	(11)
i-Amylene	5.829		5.829	283.	(2)	70.13	464.8	33.9	293.0	0.260	0.0317	(11)
Pentene-2	6.476		6.476	204.	(2)	84.16	475.0	36.0	289.7	0.268	0.0306	(11)
Hexene-1	4.114		4.114	212.	(2)	26.04	503.7	31.2	262	0.262	0.0310	(11)
Ethylene	4.742		4.742	261.	(2)	40.06	309.2	61.6	112.7	0.274	0.0326	(9, 11)
Propyne						40.06	401.2	52.8	266	0.266	0.0305	(11)
Cyclopropane						42.08	406.7	57.2	287	0.287	0.0283	(11)
Cyclopentane						70.13	412.1	44.6	260.	0.276	0.0269	(11)
Methyl cyclopentane						84.16	532.8	37.4	307.	0.272	0.0278	(11)
Cyclohexane	6.143		6.143	313.	(2)	84.16	553.2	40.0	319.	0.272	0.0263	(11)
Cyclooctane	5.628		5.628	335.	(2)	78.11	620.2	32.9	261	0.261	0.0269	(11)
Benzene	5.932		5.932	377.	(2)	92.13	562.2	48.6	260.4	0.274	0.0244	(11)
Toluene	7.706		7.706	234.	(2)	120.19	594.0	41.6	317.7	0.271	0.0251	(11)
1, 3, 5-Trimethylbenzene						120.19	641.2	33.	449.	0.282	0.0260	(11)
Alcohols												
*Methanol	1.70	0.5	3.69	417.	(8)	32.04	513.2	78.5	117.7	0.220	0.0272	(12)
*Ethanol	1.69	0.3	4.31	431.	(8)	46.07	516.	63.0	248	0.248	0.0264	(12)
*n-Propanol	1.69	0.2	4.71	495.	(8)	60.09	537.	49.95	219.8	0.251	0.0266	(9, 12)
*i-Propanol	1.69	0.2	4.64	418.	(8)	60.09	508.8	53.0		0.278	0.0258	(12)
Amines												
n-Butyl amine						73.14	527.			0.270	0.0282	(12)
i-Butyl amine						73.14	518.			0.278	0.0276	(12)
Diethyl amine						73.14	496		297.	0.267	0.0299	(12)

TABLE II (Continued)-3

Gas	μ (debye)	δ	$\sigma(\text{\AA})$	$\epsilon/k(^{\circ}\text{K})$	Ref.	M	$T_c(^{\circ}\text{K})$	$P_c(\text{atm})$	Basic Constants			ξ	Ref.
									$v_c\left(\frac{\text{cc}}{\text{g/mole}}\right)$	Z_c	Z_c		
Esters													
Ethyl formate						74.08	508.5	46.65	229.35	0.257		0.0252	(9, 12)
Propyl formate						88.10	538.1	40.1	285.11	0.259		0.0259	(9, 12)
i-Butyl formate						102.13	540.			0.272		0.0267	(12)
Methyl acetate	1.72	0.2	5.04	418.	(8)	74.08	506.9	46.3	227.94	0.254		0.0254	(9, 12)
Ethyl acetate	1.78	0.16	5.24	499.	(8)	88.10	523.	37.8	286.04	0.252		0.0258	(9, 12)
Methyl propionate						88.10	519.7	39.3	282.37	0.270		0.0264	(9, 12)
Ethyl propionate						102.13	536.5	33.0	344.45	0.770		0.0271	
Ethers													
Dimethyl ether	1.30	0.19	4.21	432.	(8)	46.07	400.1	52.0	169.75	0.287		0.0283	(12)
Diethyl ether	1.15	0.08	5.49	362.	(8)	74.08	467.	35.6 ^a		0.261		0.0299	(12)
Methyl ethyl ether						60.07	438.			0.267		0.0288	(12)
Methyl propyl ether						74.08	475.			0.269		0.0290	(12)
Methyl isopropyl ether						74.08	469.			0.275		0.0285	(12)
Ethyl propyl ether						88.10	499.			0.270		0.0291	(12)
Ethyl isopropyl ether						88.10	491.			0.277		0.0287	(12)
Dipropyl ether						102.13	529.			0.296		0.0274	(12)
i-Propyl propyl ether						102.13	522.			0.271		0.0291	(12)
Di-isopropyl ether						102.13	506.			0.274		0.0285	(12)
Diphenyl ether						168.23	764.			0.247		0.0235	(12)
Freons and Related Halides													
11 C Cl ₃ F						137.38	471.2	43.2	247.98	0.277		0.0193	(12)
12 C Cl ₂ F ₂						120.92	384.7	39.6	217.87	0.273		0.0211	(12)
21 CH Cl ₂ F						102.92	451.7	51.0	197.16	0.271		0.0198	(12)
22 CH Cl F ₂						86.48	359.5	48.5	164.72	0.263		0.0216	(12)
113 C ₂ Cl ₂ F ₂						132.93	487.3	33.7	230.78	0.274		0.0196	(12)
CF ₂ H ₂ Cl						68.48	426.7			0.257		0.0228	(12)
CF ₃ Cl						104.47	543.5			0.157(?)		0.0243	(12)
CF ₂ H ₂ Br						112.94	476.7			0.316(?)		0.0158	(12)
CF ₂ HBr						130.93	415.9			0.337(?)		0.0159	(12)
CF ₃ Br						148.93	340.8			0.368(?)		0.0159	(12)
Halides													
Chloroform	1.013	0.07	5.31	355.	(8)	119.39	536.6		231.38	0.293		0.0182	(9, 12)
Methylene chloride	1.57	0.2	4.52	483.	(8)	84.94	510.			0.277		0.0200	(12)
Methyl chloride	1.87	0.5	3.94	414.	(8)	50.49	416.3	65.8	136.46	0.276		0.0235	(9, 12)
Methyl bromide	1.80	0.4	4.25	382.	(8)	94.95	464.0			0.360		0.0149	(12)
Ethyl chloride	2.03	0.4	4.45	423.	(8)	64.52	460.4	52.0	195.52	0.293		0.0182	(9, 12)
Cyclics													
Thiophene						84.13	580.	48.0		0.288		0.0208	(9, 12)
Methyl thiophene						611.				0.278		0.0221	(12)
Pyridene						79.10	622.	60.0		0.259		0.0238	(9, 12)

*Hydrogen Bonding Type ** σ values are zero except given otherwise
 a From the Matheson Gas Data Book, The Matheson Company, Inc.

II. Polar or Nonpolar Gases and Nonpolar Gas Mixtures at High Pressures (up to $\rho^R=2.5$)

The methods for estimating the viscosity of polar or nonpolar gases and nonpolar gas mixtures at high pressures (up to $\rho^R=2.5$) using the residual viscosity correlations have been reviewed.

The residual viscosity group, $(\eta-\eta_0)\xi$, has been expressed as a function of reduced density, ρ^R . Errors less than 5 percent are anticipated over a wide range of density (up to $\rho^R=2.5$) for nonpolar gases and gas mixtures. For polar gases, errors are expected to be less than 10 to 15 percent.

Many excellent reviews of the methods proposed to estimate or correlate dense gas viscosities have shown that there appears to be no accurate theory upon which to base the estimation techniques^(8,10,16,22).

For pressures up to 5 atm., the estimation of viscosity is to be made as presented in the previous paper⁽¹³⁾. At higher pressures, however, the viscosity increases with pressure or density. In the description of a gas at higher densities, the simplifications afforded by (1) the neglect of the collisional transfer contribution to the momentum transfer and (2) the simple analysis of the dynamics of the gas in terms of binary collisions are not available.

Enskog^(3,7) was the first to treat the dense rigid sphere fluid and several workers^(10,18,19, and others) have extended his original work. The Enskog theory explains the main trend of the density dependence of the transport properties, but it does not appear to provide an accurate basis for dense gas viscosity. Difficulties due to a singularity in the density expansions of the transport coefficients have caused some pause for additional evaluation of the kinetic theory procedures for dense gases^(8,20).

Empirical relationships for the interpolating transport properties have traditionally been based on the principle of corresponding states⁽¹²⁾. The situation is not as good for the transport properties at high densities because there are insufficient data to develop the correlation and the principle of corresponding states may not be as generally applicable to non-equilibrium properties.

The residual viscosity (the viscosity at a given pressure and temperature minus the low pressure viscosity at the same temperature) of a gas or a mixture has been correlated as a monotonic function of the density^(1,2,21,23). Though there is no theoretical justification for this type of correlation, it has been extensively used in the dense gas viscosity estimation.

Thodos co-workers^(1,14,21,23) applied the residual viscosity concept to pure monatomic and diatomic gases. The applications of this concept to the hydrocarbons and their mixtures are those of Ellington and co-workers^(6,15) and of Giddings and Kobayashi⁽⁴⁾. Flynn, et al⁽⁹⁾, Hyun⁽¹²⁾, Reynes and Thodos⁽¹⁷⁾ contributed very accurate viscosity data to confirm that the residual viscosity concept is essentially correct. Despite all these efforts to correlate the dense gas viscosity with density, there is no single equation presently available for accurate prediction of viscosity for all dense fluids. An excellent summary of the correlations of different types was given by Reid and Sherwood⁽¹⁶⁾.

Nonpolar Gases

Jossi, Stiel, and Thodos⁽¹⁴⁾ developed the analytical relations between the group $(\eta-\eta_0)\xi$ and ρ^R for pure nonpolar substances. Dean and Stiel⁽⁴⁾ modified these relationships with a correlation suggested by Eakin⁽⁵⁾ to develop the residual viscosity correlation for low-molecular weight substance. Furthermore, they showed that the same formula could be applicable to the dense gas mixture viscosity estimation, if in evaluating ξ and the low viscosity, η_0 , the proper pseudocritical constants were used.

The Dean-Stiel relationship for estimating the viscosity of nonpolar gases and gas mixtures at high pressures,

given below in Eq. (1), is the most reliable at present.

$$(\eta - \eta_0)\xi = 10.8 \times 10^{-7} [\exp. (1.439\rho_R) - \exp. (-1.11\rho_R^{1.858})] \quad (1)$$

where η is the viscosity at a given temperature and pressure, η_0 is the low pressure viscosity at the same temperature, $\xi (T_{Cm}^{1/6}/M_m^{1/2}P_{Cm}^{2/3})$ is the characteristic constant, and ρ_R is the reduced density. The reduced density and the molecular weight of gas mixture can be obtained as follows:

$$\rho_R = v_{Cm}/v_m \quad (2)$$

$$M_m = \sum x_i M_i \quad (3)$$

where v_{Cm} , v_m are the critical molar volume and molar volume, respectively, and M_m , M_i are the molecular weight of mixture and i th component, respectively. x_i is the mole fraction of i th component gas.

After a careful study and review, Dean and Stiel⁽⁴⁾ suggested the modified Prausnitz and Gunn rule to be used for obtaining the pseudocritical values:

$$T_{Cm} = \sum x_i T_{Ci}, \quad Z_{Cm} = \sum x_i Z_{Ci}, \quad v_{cm} = \sum x_i v_{Ci} \quad (4)$$

$$P_{Cm} = R Z_{Cm} T_{Cm} / v_{Cm} \quad (5)$$

where T_C , P_C , Z_C are the critical temperature, critical pressure, and critical compressibility, respectively, R is the gas constant, and the subscripts m and i represent the mixture and i th component,

Polar Gases

Stiel and Thodos⁽²³⁾ suggested the following correlations to estimate the viscosity of pure polar gases at high densities:

$$(\eta - \eta_0)\xi = 16.56\rho_R^{1.111} \times 10^{-7} \text{ for } \rho_R \leq 0.10 \quad (6)$$

$$(\eta - \eta_0)\xi = 0.607 (9.0450\rho_R + 0.63)^{1.739} \times 10^{-7} \text{ for } 0.10 < \rho_R < 0.90 \quad (7)$$

$$\log \{-\log [(\eta - \eta_0)\xi] - 2\} = 0.6439 - 0.1005 \rho_R - \Delta \text{ for } 0.9 < \rho_R < 2.6 \quad (8)$$

where $\Delta = 0$ for $0.9 < \rho_R < 2.2$ and $\Delta = 4.75 (\rho_R^3 - 10.65)^2$ for $2.2 < \rho_R < 2.6$

Discussion

The residual viscosity correlations are recommended for estimating the high pressure viscosity of gases and gas mixtures. Table I shows the methods recommended in this paper and the each method.

Using Eq. (1), errors are expected to be less than 5 percent for mixtures of nonpolar low-molecular weight gases. A pure gas can be considered as one component mixture. For high-molecular weight nonpolar gas mixtures, and for mixtures containing one or more polar components, no satisfactory method has not been devised. Eq. (1) could still probably be used but with the realization that larger errors may be expected. Dean and Stiel⁽⁴⁾ found that for several mixtures the experimental viscosity data for certain ranges of temperature or density did not follow the consistent behavior showed in other viscosity data. These anomalies are listed as follows:

Mixture	Range
CH ₄ -C ₃ H ₈	$\rho_R \geq 1.2$
C ₂ H ₆ -C ₃ H ₈	$\rho_R \leq 1.0$
C ₂ H ₆ -C ₂ H ₄	$T = 233^\circ\text{K}$
Natural Gas	$T = 273^\circ\text{K}$
Natural Gas	$\rho_R \geq 1.6$
Air	$\rho_R \geq 1.25, T = 423^\circ\text{K}, 473^\circ\text{K}$

It is difficult to assess the accuracy of the correlations for polar gases [Eq. (6) to (8)] as they have not been extensively tested. Probably errors less than 10 to 15 percent are to be expected.

The viscosity data for hydrogen and water were found to be inconsistent with those of the other substances treated by Jossi, Stiel, and Thodos⁽¹⁴⁾. The reason for this abnormal behavior is uncertain, but may be due

to the quantum effects for hydrogen and the excessive hydrogen bonding effects for water. The analytical relationships for these gases given by Jossi, Stiel, and Thodos⁽¹⁴⁾ are presented in the Appendix.

TABLE I
THE METHODS USED FOR DENSE GAS VISCOSITY ESTIMATION

<u>Methods</u>	<u>Applicability</u>	<u>Required Data</u>	<u>Accuracy</u>	<u>Remarks</u>
Dean-Stiel	Low molecular weight, nonpolar gases and gas mixtures	Critical properties, ρ_R, M, Z	<5%	Jossi, <i>et al.</i> plus recent data on Ar, Ne, N ₂ , C ₃ H ₈ , n-C ₄ H ₁₀ . At 1,396 data pts. for 9 mixture, the overall aver. deviation is 3.73%.
*Jossi, et al	Low molecular weight, non-polar gases	Critical properties, ρ_R, M, Z	10-15%	Based on the available data up to 1962.
Stiel-Thodos	Polar gases	Critical properties, ρ_R, M, Z	10-15%	Based on the high pressure viscosities for the gaseous and liquid phase for 14 polar substances.

* Included for comparison

NOMENCLATURE

M	Molecular weight
n	Number of components
P_C	Critical pressure, atm.
R	Gas constant, 82.0567 cc-atm./g-mole, °K
T_C	Critical temperature, °K
v	Molar volume, cc/g-mole
v_C	Critical molar volume, cc/g-mole
x	Mole fraction
Z	Compressibility factor
Z_C	Critical compressibility factor
Greek Symbols	
η	Viscosity, poise
η_0	Low pressure viscosity, poise
ρ_R	Reduced density
Subscripts	
i	Component
m	Mixture

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APPENDIX

The following analytical relationships are presented for estimating the viscosity values of hydrogen and water in the dense gas region by Jossi, Stiel, and Thodos⁽¹⁴⁾

Hydrogen

$$[100(\eta-\eta_0)\xi+10^{-4}]^{1/4}=0.10616-0.042426\rho_R+0.17553\rho_R^2-0.12295\rho_R^3+0.028149\rho_R^4$$

Water

$$[100(\eta-\eta_0)\xi+10^{-4}]^{1/4}=0.10721+0.040646\rho_R+0.0026282\rho_R^2-0.0054430\rho_R^3+0.0017979\rho_R^4$$

III. Liquids

For the temperature range from freezing point to normal boiling point and the pressures below 1000-4000 atm., the modified Andrade equation

$$\eta_{lp}=A(v_l v_p)^{-1/6}(k_l/k_p)^{1/2} \exp(C/v_p T)$$

was chosen for estimating liquid viscosities. The constants A and C for 116 substances have been tabulated. At high temperatures (above a T_r of 0.8), the correlations for the high pressure gas viscosity were recommended. The errors are anticipated to be less than 1 to 2 percent for many inorganic and organic liquids, fused salts and mineral oils, and liquid metals. The errors are greater for water, tertiary alcohols, and other highly polar compounds.

Being different from the viscosities of dilute gases, the viscosities of liquids are much larger numerically and decrease with temperature in an exponential manner. These differences are mainly due to different mechanisms of momentum transfer in the dilute gas and liquid phases. While the momentum transfer in a dilute gas is caused by individual collisions between randomly moving molecules, in a liquid the intermolecular force

fields predominate the mechanism of momentum transfer. The molecules of liquids are so closely packed that the average molecular separation distance is small and comparable to the range of intermolecular forces. The viscosity behavior of a dense gas is intermediate between the behavior of a dilute gas and a liquid, and both mechanisms are important. Many attempts were made to explain the liquid viscosity behavior. Brush⁽⁸⁾ reviewed the theories of liquid viscosity up to 1961, and discussed many phases of problems involved in this rather difficult task of theory development of liquid viscosity. Several references^(1,7,11,13,17,18) are available which summarize most of the key concepts. Reid and Sherwood⁽²¹⁾ gave additional references in their monograph.

Despite all the efforts to understand the liquid viscosity, no theory comes up with a simple form to allow liquid viscosities to be calculated *a priori*. For most practical purposes, the empirical (and often inaccurate) estimation techniques must be used. Of course, these methods do not conflict with theory; they just allow some of the unknown or incalculable theoretical constants to be determined from experimental data or approximated from structure or other physical properties. Over the past century hundreds of formulas have been proposed and tested for one or more liquids by various experimentors^(9,20). Since the initial emphasis of this study is to provide the best available estimation methods which can be used in the computer program, the most simple and reliable viscosity equations recommended by several reviewers^(4,10,21) will only be considered in the following section.

η_l - T Correlation

When you have two or more viscosity values and you want to estimate the viscosity values certain temperatures, the "Andrade" equation^(2,3) is the best simple temperature-liquid viscosity correlation;

$$\eta_l = A' \exp(B/T) \quad (1)$$

Where A' and B are positive. A' is the same order of magnitude as the gas viscosity (η_0) and B is approximately represented as the heat of fusion divided by the gas constant^(14,15). Since Eq. (1) was first suggested, literally hundreds of other viscosity-temperature relations have been proposed^(8,9,20). Eq. (1) was also modified to include the specific volume term⁽¹⁾:

$$\eta_l v^{1/3} = A \exp(C/vT) \quad (2)$$

Where A, C are constants and v is the specific volume.

Andrade⁽¹⁾ claimed that Eq. (2) gave a slightly better fit for large range of viscosity than Eq. (1). Srinivasan and Prasad⁽²³⁾ found that both equations gave a comparable result for several liquids and concluded that additional complication is unnecessary in Eq. (2). However, the constant C in Eq. (2) appears in the Andrade's correlation of high-pressure liquid viscosity⁽¹⁾. Eq. (2) is chosen as a basis in estimating liquid viscosity.

Extensive testing of subsequent forms suggested (see Appendix), showed that for a two-constant relation, Eq. (2) is reasonably accurate compared to others and usually much more convenient to use. Of course, Eq. (2) may not be suitable for complex compounds, or for very low or very high temperatures, but no other simple relation has been proved reliable under such conditions. Eq. (2) is best suited in the temperature range from freezing point to normal boiling point. In the high temperature range (above a T_r of about 0.8), the correlation used for high-pressure gas viscosity estimation⁽¹³⁾ is probably a best fit.

PRESSURE EFFECT

Liquid viscosity increases with pressure for all liquids except water below 30°C (and some aqueous solutions): the increase is almost linear with pressure up to about 1,000–4,000 atm. (the exact value depends on the liquid), whereas at high pressures the increase becomes exponential. However, the pressure effect is rarely significant when the pressure is less than 10–40 atm. (the exact value depends on the liquid). Pressure

effect on the liquid viscosity becomes great for liquids of complex molecular structure and higher compressibility. For example, at 11,700 atm., η_{1p}/η_{11} for mercury is only 1.32; for methanol, 10; and for eugenol (4-allyl-methoxy phenol), it is 10^{11} . Andrade⁽³⁾ derived the following relationship for the highpressure liquid viscosity:

$$\frac{\eta_{1p}}{\eta_{11}} = \left(\frac{v_1}{v_p}\right)^{1/6} \left(\frac{k_1'}{k_p'}\right)^{1/2} \exp \left[\frac{C(v_1 - a_p)}{Tv_1 v_p} \right] \quad \text{Eq. (3)}$$

where suffix p denotes at high pressure and 1 denotes at one atmosphere. k' is the adiabatic compressibility. The constant C is the same as in the exponential term of Eq. (2). Since very few values of k' are available and for the liquids the ratio of the isothermal compressibilities (k) will not differ much from the ratio of the adiabatic compressibilities. Hence, if we use k in place of k' and combine Eq. (2) and Eq. (3), we will get

$$\eta_{1p} = A(v_1 v_p)^{-1/6} (k_1/k_p)^{1/2} \exp(C/v_p T) \quad \text{Eq. (4)}$$

when $p=1$, Eq. (4) reduces to Eq. (2).

WHEN YOU HAVE NO DATA

When you have no data, it is usually impossible to calculate η with a high degree of accuracy. Many attempts^(4,6,14,15,16,19,21,22,24) were made to correlate the viscosity to molecular structure and a thorough discussion was written by Bondi⁽⁵⁾. All attempts to estimate A (or A') or C (or B) from group contributions have met with indifferent success⁽²¹⁾. Kierstead and Thurkevitch⁽¹⁶⁾ studied this problem for simple case of pure hydrocarbons, but could not recommend any quantitative basis for predicting A' and B from structure alone. Their conclusions were summarized by Reid and Sherwood⁽²¹⁾ as follows:

"In homologous series, $1/A'$ appears to be a linear function of the number of carbon atoms, but B increases slowly as the number of carbons in the skeleton increases. A' decreases but B increases with branching, but the effect of the branching varies, i. e., each branch appears to have more effect than the preceding. Unsaturation increases A' and decreases B for similar compounds; cyclic compounds have much larger values of B and smaller values of A' than the corresponding aliphatic compounds."

Several empirical methods suggested to estimate the viscosity of liquids seem to be unreliable. Detailed discussion on these methods, the reader is referred to review articles^(10,18,21) or original sources^(6,19,21,22,24)

DISCUSSION

We have been very fortunate to have the abundant source of liquid viscosity data. For many chemical compounds, Eq. (2) has been tested extensively by several investigators^(3,25) are listed in Table I. The C values given by Wright⁽²⁵⁾ were found to be in error by the factor of 2.303 and were corrected by the author. also, listed in Table I are the density parameters needed to calculate the specific volume $v = \frac{L}{\rho_t}$, where $\rho_t = \rho_0 + \alpha t + \beta t^2 + \delta t^3$ and t is the temperature, °C. For those for which the density parameters are not given, density values can be obtained from International Critical Tables⁽²⁶⁾ The values of isothermal compressibility data can be obtained from several references^(26,27).

Eq. (4) was used in the computer program to estimate the liquid viscosity at temperatures between freezing point and normal boiling point and pressures up to 1,000–4,000 atm. For high temperatures (above a T_r of 0.8), the correlations for high pressure gas viscosity estimation are recommended to be used to calculate the liquid viscosity.

The correlations using molecular structure to predict the viscosity were not discussed in this report, because our initial emphasis was on selecting the best available estimation techniques.

Using Eq. (4) at pressures below 10-40 atm., the errors are anticipated to be less than 1 to 2% for many inorganic and organic liquids, for fused salts and mineral oils, and for liquid metals. The errors are greater for water, tertiary alcohols, and other highly polar compounds. Eq. (4) was tested for only four organic liquid (diethyl ether, acetone, ethyl bromide and ethyl iodide) by Andrade⁽³⁾. The agreement was

good up to 2,000 atm., The differences between observed and calculated values were less than 10%. This was considered to be satisfactory.

NOMENCLATURE

A, A', B, C	Constants
k	Isóthermal compressibility
k'	Adiabatic compressibility
T	Absolute temperature, °K
t	Temperature, °C
v	Specific volume, cc/g
Greek Symbols	
α, β, δ	Density parameters
ρ	Density
Subscripts	
o	At 0°C
1	At 1 atm. pressure
l	Of the liquid
p	At some higher pressure
t	At $t^\circ\text{C}$

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APPENDIX

The following formulas have been proposed and tested for one or more liquids by various experimentors^(8,20):

$$\eta = \frac{A}{(1+BT+CT^2)}$$

$$\eta = \frac{C}{(A+T)^n}$$

$$\eta = \frac{C}{(A+T)^n}$$

$$\eta = \frac{AT^{1/2}[(T-B)^2+C]}{(D+T)^2+E}$$

$$\eta = CA \arctan A(T+B)$$

$$\eta = A(1+Be^{-CT})^n$$

$$\eta = C/T^3$$

$$\eta = A-BT-[(BT-A)^2]^{1/2}$$

$$\eta = \frac{(A-BT)^n}{(A+DT)^m}$$

$$\eta = T(Ae^{B/T} + Ce^{D/T})$$

$$\log \eta = \frac{A}{T} + B \log T - CT + D$$

$$\eta = ATe^{B/T} + CT$$

$$\log \eta = B + AT^{-n}$$

$$\log \eta = -Ae^{D/T}[-Ei(-D/T)] - B + Ce^{D/T}$$

(where $Ei(x)$ is the exponential integral)

$$\log \eta = AT^{-2} + BT^{-1} + C$$

$$\log \eta = A + BT^{-3}$$

$$\eta = \frac{C}{A+T} - B$$

$$\eta = \frac{A(T_c - T)}{T - B}$$

$$\eta = \frac{A(T_c - T)}{T - B}$$

$$\eta = C \left(\frac{T+A}{T+B} \right)^n$$

$$\eta = Ce^{-BT}$$

$$\eta = CT^{-n}e^{-A/T}$$

$$\eta = \frac{A}{T-B}$$

$$\eta = A^{(T-B)/(T-C)}e$$

$$\eta = Ae^{B/T} + Ce^{D/T}$$

$$\eta = (A+B/T)^3$$

A number of other equations involve quantities such as pressure, volume, surface tension, density, velocity of sound, refractive index, vapor concentration, vapor pressure, latent heat of evaporation, latent heat of fusion, molecular weight, and chemical composition.

TABLE 1

**VALUES OF PARAMETERS A, C IN EQUATION (4)
AND DENSITY PARAMETERS FOR $\rho_t = \rho_0 + \alpha t + \beta t^2 + \delta t^3$**

Liquid	$A \times 10^6$	C	ρ_0	$\alpha \times 10^3$	$\beta \times 10^6$	$\delta \times 10^9$	Density Temp. Range	Reference
Acetaldehyde	480	610						(3)
Acetic acid	428	927	1.0724	-1.1229	+0.0058	-2.0	9 to 100°C	(3, 26)
Acetic anhydride	457(447)	809(867)	1.1054	-1.1983	-0.4261	0.0		(3, 25)
Acetylene bromide	652	315						(3)

Allyl alcohol	133	1609						(3)
Allyl bromide	508	444						(3)
Allyl chloride	478(465)	611(624)	0.9637	-1.3101	0.0	0.0		(3, 25)
Allyl iodide	523	389						(3)
Amyl alcohol(active)	29.8	2688						(3)
Benzene	331	1007	0.90005	-1.0636	-0.036	-2.213	11 to 72°C	(3, 26)
Benzonitrile	481	963	1.02279	-0.8758	-0.14	0.0	0 to 60°C	(25, 26)
Bromine, liquid	708	213						(3)
n-butyl alcohol	78.3	2174	0.82390	-0.699	-0.32	0.0	0 to 47°C	(3, 26)
n-butyric acid	419	1107	0.9780	-0.9831	-0.080	0.0	25 to 81°C	(3, 26)
Butyric anhydride	395	1119						(3)
Carbon disulfide	729	356						(3)
Carbon tetrachloride	397	560						(3)
n-capronitrile	398	889	0.8215	-0.7843	-0.4632	0.0		(25)
Chlorine, liquid	1153	197						(3)
Chloroform	607	412	1.52643	-1.8563	-0.5309	-8.81	-53 to 50°C	(3, 26)
Cyclohexene	408	1027	0.8299	-0.9750	0.0	0.0		(25)
n-decene	424	1216	0.7567	-0.7733	-0.1458	0.0		(25)
Diallyl	416	854						(3)
Diallyl ether	388	917	0.8291	-1.0436	-0.4405	0.0		(25, 26)
Diethyl carbonate	428	887	0.9972	-1.0713	-0.8437	0.0		(25)
Diethyl ether	444	728	0.73629	-1.1138	-1.237	0.0	0 to 70°C	(3, 26)
Diethyl ketone	476	848	0.8337	-0.940	-0.962	0.0	0 to 53°C	(3, 26)
Diisopropyl ketone	435	1002	0.9972	-1.0713	-0.8437	0.0		(25)
Dimethyl carbonate	395	751	1.0958	-1.2583	-0.9166	0.0		(25)
Dimethyl ethyl carbinol	9.74	3111						(3)
Dimethyl ketone	491	720						(3)
Dipropyl ether	414	950						(3)
Ethyl acetate	440	767	0.92454	-1.168	-1.95	+20.0	0 to 40°C	(3, 26)
Ethyl alcohol	228	1491	0.80625	-0.8461	+0.160	-8.5	0 to 80°C	(3, 26)
Ethyl benzene	458	922						(3)
Ethyl bromide	546	378	1.50138	-2.0644	+0.2673	0.0	-119 to 34°C	(3, 26)
Ethylene bromide	479	446	2.2223	-2.090	-0.20	0.0	0 to 30°C	(3, 26)
Ethylene chloride	444	663	1.28428	-1.4217	-0.933	+2.29	0 to 74°C	(3, 26)
Ethyl formate	492	675						(3)
Ethylidene chloride	498	557						(3)
Ethyl iodide	568	319	1.98049	-2.217	-1.55	+3.0	0 to 63°C	(3, 26)
2-Ethyl-1-hexene	459	979	0.7432	-0.8204	-0.5833	0.0		(25)
Ethyl isobutyl ether	412	925						(3)
Ethyl propionate	433	838	0.9124	-1.1223	+0.203	-7.31	0 to 100°C	(3, 26)
Ethyl propyl ether	425	843						(3)
Ethyl sulfide	508	779						(3)
Formic acid	225	1036	1.2441	-1.221	+0.126	0.0	0 to 40°C	(3, 26)
Heptane	453	929	0.70048	-0.8476	+0.1880	-5.23	0 to 100°C	(3, 26)
Hexane	455	929						(3)
1-Hexene	379	928	0.6049	-0.9650	0.0	0.0		(25)
1-Hexyne	375	938	0.7347	-0.9800	0.0	0.0		(25)
Iodine, liq.	1535	220						(3)
Isoamyl alcohol	51.5	2640						(3)
Isobutyl alcohol	31.3	2658	0.8169	-0.951	-0.28	-0.8	0 to 50°C	(3, 26)
Isobutyl bromide	480	605						(3)
Isobutyl chloride	443	797						(3)
Isobutylene bromide	479	446						(3)
Isobutyl iodide	490	499						(3)
Isobutyric acid	455	1048	0.96820	-0.9849	+1.036	0.0	-45 to 18°C	(3, 26)
Isocapronitrile	378	907						(3)
Isoheptane	449	974						(3)
Isohexane	454	900						(3)
Isopentane	436	856						(3)
Isoprene	449	731						(3)
Isopropyl alcohol	35.2	2466	0.8014	-0.809	-0.27	0.0	0 to 25°C	(3, 26)
Isopropyl bromide	491	492						(3)

Isopropyl chloride	459	683						(3)
Isopropyl iodide	530	410						(3)
Mercury	2467	21.0						(3)
Methyl acetate	463	668						(3)
Methyl alcohol	269	1171	0.80999	-0.9253	-0.41	0.0	0 to 60°C	(3, 26)
Methyl n-butyrate	425	862	0.92006	-1.0704	-7.14	+1.19	0 to 100°C	(3, 29)
Methyl-n-amyl ketone	416	1094	0.8324	-0.8470	-0.20	0.0		(25)
Methylene chloride	577	422						(3)
Methyl ethyl ketone	453	834	0.82551	-1.022	-0.46	0.0	0 to 50°C	(3, 26)
Methyl formate	524	571	1.00319	-1.4174	-0.776	-8.62	0 to 100°C	(3, 26)
Methyl iodide	540	247						(3)
Methyl isobutyl ether	434	825						(3)
Methyl isobutyrate	437	827						(3)
Methyl-n-nonyl ketone	330	1483	0.8405	-0.7252	-0.351	0.0		(25)
2-Methyl-1-pentene	328	986	0.7083	-0.9975	0.0	0.0		(25)
4-Methyl-2-pentene	316	995	0.6857	-0.9467	0.0	0.0		(25)
Methyl propionate	498	719	0.93871	-1.1891	+4.19	-0.0106	0 to 100°C	(3, 26)
Methyl-n-propyl ether	446	746						(3)
Methyl-n-propyl ketone	472	884	0.8261	-0.923	-0.766	0.0	0 to 54°C	(3, 26)
Methyl sulfide	530	610						(3)
n-Octane	437	1098	0.71848	-0.8239	+0.4459	0.0	0 to 130°C	(3, 26)
1-Octene	452	1041	0.7321	-0.8023	-0.6333	0.0		(25)
2-Octene	447	1027	0.7365	-0.8412	-0.1917	0.0		(25)
n-Pentane	434	855	0.64539	-0.9398	-0.6243	-7.53	-53 to 100°C	(3, 26)
2-Pentene	218	1094	0.6695	-0.9625	0.0	0.0		(25)
Phenyl bromide	571	563						(3)
Phenyl chloride	548	703						(3)
Phenyl fluoride	514	695						(3)
Phenyl iodide	547	516						(3)
Pinacoione (tert butyl methyl ketone)	413	1064	0.8262	-0.9538	0.0	0.0		(25)
Propionic acid	513	904						(3)
Propionic anhydride	418(440)	952(932)	1.0343	-1.1182	-0.1125	0.0		(3, 25)
Propionitrile	426	585	0.8015	-0.9675	-0.7501	0.0		(25)
Propylacetate	406	891						(3)
n-Propyl alcohol	104.5	1986						(3)
Propyl bromide	529	473						(3)
Propyl chloride	502	655						(3)
Propylene bromide	473	505						(3)
Propyl formate	451	799						(3)
Propyl iodide	543	406	1.7844	-1.845	-1.25	0.0	0 to 52°C	(3, 26)
Tetrachloroethylene	680	436	1.6475	-1.62	0.0	0.0	0 to 90°C	(3, 26)
Thiophene	440	739	1.08717	-1.124	-0.93	0.0	0 to 50°C	(3, 26)
Toluene	439	912	0.88412	-0.92248	-0.0152	-4.223	0 to 99°C	(3, 26)
Trimethylcarbinol	3.95	3574						(3)
Trimethyl ethylene	470	721						(3)
1-vinyl-2-cyclohexene	447	988	0.8465	-0.8317	-0.4722	0.0		(25)
o-xylene	417	1007						(3)
m-xylene	465	893	0.88151	0.8515	-1.09	-1.73	0 to 100°C	(3, 26)
p-xylene	436	931						(3)
Water	58.8	1534						(3)