

# Microstrainer 設計를 爲한 改良濾過 理論

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## Modified Filtration Theory for Microstrainer Design

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### Abstract

The use of microstrainers in the water and wastewater engineering field is expected to increase in the future. Currently employed design methods for microstrainers are based on Boucher's straining law which assumes a constant rate of flow. The assumption of a constant pressure head is more appropriate than that of a constant flow rate. Governing equations describing the hydraulics of filtration through microstrainers based on these two assumptions are analyzed and compared.

### INTRODUCTION

The merits of microstrainers for removing suspended solids from water are well recognized. Microstrainers for water clarification have been in operation in many countries since 1945, when the process was introduced in England. The process also seems to have a great potential for upgrading the quality of secondary effluent. It is expected that the use of microstrainers in water and wastewater treatment will increase in the future.

Currently employed methods<sup>1)3)9)</sup> of designing microstrainers assume that the hydraulic characteristics of the filtration process may be described by Boucher's straining law. This law assumes that the rate of filtration is constant<sup>2)7)</sup>. The purpose of this paper is to demonstrate that filtration in microstrainers actually takes place under constant-pressure conditions for which Boucher's straining law is not valid. Equations describing the relationship between head loss, rate of filtration, speed of drum rotation, and submerged

area of screen based on constant-pressure conditions have been derived which will hopefully replace the currently used equations based on constant rate conditions.

### HYDRAULICS OF FILTRATION

#### Constant Rate Filtration

Boucher<sup>2)</sup> described his experimental investigations on the hydraulics of filtration in a thesis completed in 1944. These studies were undertaken to solve certain engineering problems in the design of large-scale microstrainers for water and wastewater treatment. The experimental data indicated that under constant-rate conditions of filter operation, the rate of increase of hydraulic resistance with respect to the volume of water filtered is proportional to the hydraulic resistance:

$$\frac{dH}{dV} = IH \quad (1)$$

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where  $H$  is hydraulic resistance or head loss after the passage of a volume  $V$  of raw water. The constant  $I$  was designated as a filterability index by Boucher.

Hermans and Bredée<sup>6)</sup> studied the clogging of filter fabric and suggested two possible mechanisms: (1) complete blocking, in which single particles somewhat larger than the holes in the filter medium plug up individual holes and (2) standard blocking, in which particles smaller than the holes are attached to the fibers along or within holes, or to other particles previously retained. Two additional mechanisms of filtration are explained as follows.

The filtration of suspended solids from a liquid medium proceeds by the formation of a layer of solids, commonly called 'cake', at the surface of the filter cloth. As filtration progresses, the liquid flows through the cake layer; this condition is called cake filtration. Filtration occurring between standard blocking and cake filtration where cake is under formation is called intermediate blocking.

Hermans and Bredée<sup>6)</sup> and Gonsalves<sup>4)</sup> derived filtration laws on the assumption that separate physical mechanisms govern pore plugging for complete blocking, standard blocking, intermediate blocking, and cake filtration. The work of Hermans and Bredée and of Gonsalves has shown that various filtration laws may be derived from a general differential equation for constant-rate filtration, which is:

$$\frac{dH}{dV} = I(H)^n \quad (2)$$

where the value of the constant 'n' defines the filtration mechanism involved, and the value of  $I$  for a given filtration mechanism is an empirically derived parameter which depends upon the characteristics of the filter medium used and the suspension being filtered.

According to Grace<sup>5)</sup> values of 'n' for various filtration mechanisms are:

Filtration Mechanism	n
Complete Blocking	2
Standard Blocking	1.5

Intermediate Blocking	1
Cake Filtration	0

Equation (1) proposed by Boucher is seen to be a special case of the general differential equation (2) for the intermediate blocking mechanism with  $n=1$ . Solution of equation (2) for  $n=0$  yields the well-known cake filtration equation which applies to constant-rate filtration.

To apply Boucher's straining law to microstrainers, the following variables will be used:

$q$ =local rate of filtration per unit area of screen,  $ft^3/sec/ft^2$

$S$ =speed of drum rotation,  $ft^2/sec$

$Q$ =overall rate of filtration through the entire submerged area of screen,  $ft^3/sec$

$A$ =submerged area of screen,  $ft^2$

$V$ =cumulative volume of filtrate per unit screen area at time  $t$ ,  $ft^3/ft^2$

The cumulative volume  $V$  is set equal to the total filtrate passed through a unit area ( $1 ft^2$ ) of the screen as this unit area travels through the water during each drum revolution. Since the time of travel of the drum during each revolution is equal to  $A/S$  and the local filtration rate is constant under the constant-rate assumption, the following relation would be obtained:

$$\begin{aligned} V &= q A/S \\ &= Q/S \end{aligned} \quad (3)$$

Integration of equation (1) yields:

$$H = H_0 \exp (I V) \quad (4)$$

where  $H_0$ =initial head loss through a clean screen. Substitution of equation (3) into equation (4) yields:

$$H = H_0 \exp (IQ/S)$$

$$= \frac{C_f Q}{A} \exp(IQ/S) \quad (5)$$

where  $C_f$  = resistance factor of a clean screen. Equation (5) is the well-known Boucher's straining law which is currently used for designing microstrainers.

### Filtration Mode Applicable to Microstraining

In the microstraining process, a fabric of metallic or plastic construction is fastened to a drum which may be circular, belt type, or star shaped. The influent enters the open end of the drum and passes through the screen, leaving the solids embedded on the interior periphery of the screen. As the drum revolves, the solids are carried to the top of the unit and washed into a hopper for disposal by means of a jet. During each revolution of the drum, solids are deposited onto the submerged screen. As the solid mat builds up, it too provides additional filtration, although at a decreasing rate.

A simplified cross-sectional view of a circular drum-type microstrainer is shown in Figure 1. Let ABC be the completely submerged portion of the screen. The section A'A and C'C are filled by the influent from inside the drum, and the section A'DC' is not contacted by either the influent or the effluent. The surface area of the screen bounded by the sections A'A and C'C is considerably smaller than the submerged section ABC; therefore, for all practical purposes, the effect of the sections A'A and C'C can be ignored with the result that the screen at A can be considered clean. The raw water enters the drum at the open end with total piezometric head equal to  $h_1$ . At equilibrium under steady-state conditions, let  $h_2$  be the piezometric head of the water in the tank, giving a head loss through the screen equal to  $h_1 - h_2 = H$ . As the raw water passes through the screen, the pores of the screen become plugged with the suspended solids in the water with maximum plugging occurring at C. Obviously the rates of flow at A, B,

and C in the submerged part of the screen are not the same; the rate of flow at A is greater than that at B, and similarly the rate of flow at B is greater than that at C. Thus, the assumption of constant rate of flow through the screen does not hold. On the other hand, the pressure head causing the flow all along the periphery ABC is equal to  $H$ .

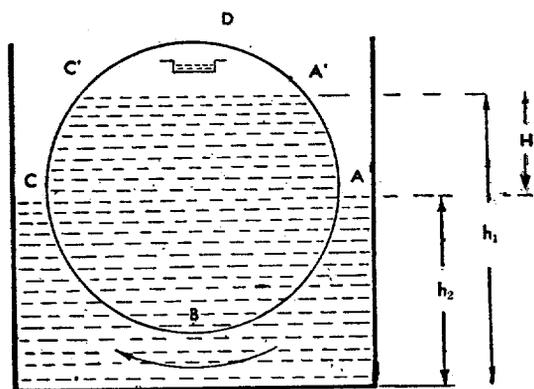


FIG. 1. SIMPLIFIED CROSS-SECTIONAL VIEW OF A CIRCULAR DRUM-TYPE MICROSTRAINER.

Thus, although the overall rate of water flow through the entire submerged area is constant, the local flow rate along the perimeter of the screen varies as the thickness of solids varies. Furthermore, the difference of the liquid levels between the inside and the outside of the drum provides a constant hydrostatic head across all parts of the submerged area of the screen; therefore, the filtration mode which applies to microscreening is that of constant-pressure filtration.

### Derivation of Straining Law for Constant-Pressure Filtration

The differential equation applicable to constant pressure filtration is:

$$\frac{d^2 t}{dV^2} = I_p \left( \frac{dt}{dV} \right)^n \quad (6)$$

where  $t$  = time of filtration, sec.

$I_p$  = a constant equivalent to  $I$  in equation (2). The constant  $n$  defines, as in equation (2), the various filtration mechanisms involved. Following the deriva-

tion of Boucher's law based on the intermediate blocking mechanism ( $n=1$ ), solution of equation (6) yields the following equation for constant-pressure filtration<sup>9</sup>:

$$\frac{1}{q} = \frac{1}{q_0} + I_p t \tag{7}$$

where  $q_0=q$  at  $t=0$ , i. e., the local rate of filtration per unit screen area at the start of filtration, ft<sup>3</sup>/sec/ft<sup>2</sup>. In constant-pressure filtration, the local rate of filtration  $q$  is not assumed to be constant along the periphery of the submerged screen. The total outflow  $Q$  from the microstrainer would, therefore, be obtained by considering a screen element of unit area with a corresponding filtration rate  $q$ , and by integrating  $q$  with respect to the time taken by the screen element in moving from section A to section C as follows:

$$Q = S \int_0^{A/S} q \, dt \tag{8}$$

Substitution of  $q$  from equation (7) into equation (8) and integration of the resulting equation yields:

$$Q = \frac{S}{I_p} \ln \left( 1 + \frac{I_p A q_0}{S} \right) \tag{9}$$

The variable  $q_0$  can be expressed in terms of the head loss  $H$  and the resistance factor  $C_f$  of a clean screen as follows:

$$q_0 = H/C_f \tag{10}$$

Substitution of  $q_0$  from equation (10) into equation (9) and rearranging the resulting equation yields the

following:

$$H = \frac{C_f S}{I_p A} [\exp(I_p Q/S) - 1] \tag{11}$$

Equation (11) is a new straining law proposed for constant-pressure filtration studies of microstrainers. Parameters appearing in equation (9) are the same as those contained in Boucher's law except for  $I_p$ , which might be named a filterability index for constant-pressure filtration. It can be shown that as the drum speed  $S$  is increased to a very large value, both equations (5) and (11) reduce to  $H=C_f Q/A$ , which applies to filtration through a clean screen. In this trivial case, no distinction exists between constant-rate and constant-pressure filtration.

### DISCUSSIONS AND CONCLUSIONS

It should be pointed out that the straining law expressed by equation (11) is based upon the intermediate blocking mechanism which is one of the several mechanisms mentioned earlier. Additional experimental studies would be needed to establish which mechanism adequately describes the microscreening process. Depending upon the pore size of the screen, the particle size of the suspended solids, and the deposition rate of the solids on the screen, more than one mechanism might be involved along the different sections of the submerged screen. To establish the filtration mechanism(s) involved in a given situation, a laboratory experiment could be performed to obtain a filtration curve in terms of the cumulative volume of filtrate and the filtration time. The filtration experiment would be conducted under a constant-pressure head comparable to the actual head to be employed on the microstrainer and the total filtration time comparable to the time of each revolution of the drum. The filtration curve, in its entirety or in segments thereof, would be fitted to one or more of the four filtration equations obtained by solving the differential equation (6) with the values of the con-

stant  $n$  set equal to 0, 1, 1.5, and 2. The curve-fitting procedure would also establish the value of the filterability index  $I_p$ .

This study has attempted to show that a new straining law derived on the basis of constant-pressure filtration would be more realistic than the existing law based on constant-rate filtration for describing the operation of commercial microstrainers. It is hoped that the new straining law proposed in this study would provide an improved and more reliable method of designing microstrainers.

### APPENDIX I-NOTATION

- $C_f$ =Initial resistance of clean microscreening, dimensions  $[T]$   
 $H$ =Head loss through microscreening, dimensions  $[L]$   
 $A$ =Effective submerged area of microscreening, dimensions  $[L^2]$   
 $I$ =Filterability index (constant flow rate), dimensions  $[L^{-1}]$   
 $I_p$ =Filterability index (constant pressure head), dimensions  $[L^{-1}]$   
 $n$ =A constant having values of 2, 3/2, 1, and 0 for complete blocking, standard blocking, intermediate blocking, and cake filtration respectively (dimensionless)  
 $Q$ =Rate of outflow, dimensions  $[L^3/T]$   
 $q$ =Filtration rate per unit fabric area at time  $t$ , dimensions  $[L/T]$   
 $q_0$ =Filtration rate per unit fabric area at initial time, dimensions  $[L/T]$   
 $S$ =Speed of drum rotation, dimensions  $[L^2/T]$   
 $t$ =Time of filtration, dimensions  $[T]$   
 $t_0$ =Time taken by the microscreen in moving from

section A to section C of Figure 1, dimensions  $[T]$   
 $V$ =Cumulative volume of filtrate per unit fabric area at time  $t$ , dimensions  $[L]$

### APPENDIX II-REFERENCES

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