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REDUCING THE EFFECTS OF FAILURE PROPAGATION IN PERIODIC PROCESSES INVOLVING INTERMEDIATE STORAGE WITH MULTIPLE INPUT/MULTIPLE OUTPUT STREAMS

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Abstract—Batch processes are susceptible to long term production failures that adjacent intermediate storage cannot absorb totally and therefore force the shut-down of adjacent units. Because batch processes have many timing constraints, a careless storage operation leads to the propagation of the failures along the production line which results in a great additional loss of productivity. This article develops operational algorithms for the basic MIMO storage system in the presence of failures of the processing units, up or down-stream of the storage facility. Specially, we consider a class of long term failures which affect all or a subset of the multiple input/output streams. Algorithms are carefully designed to satisfy physical constraints and fully utilize the storage capacity, while minimizing the duration of forced shut-downs. An example study simulating failures demonstrates the effectiveness of our approach.

Key words: *Failure, MIMO, Storage, Operation, Periodic*

INTRODUCTION

Batch processes are susceptible to production failures. These failures consume production time as well as constituting material loss. Because batch process production timing is closely related between adjacent units, failures can propagate through the production line in a explosive pattern which results in great loss of productivity. Badly managed production failure time can become a great of cost in today's quality and consumer oriented market.

Failure time management can be achieved in past through effective intermediate storage operation in most a real plant. One of the main roles of intermediate storage in production facilities is to mitigate the effects of production failures. While short duration, infrequent failures may be absorbed by modest amounts of storage capacity, long duration failures can not be absorbed totally, thus forcing the shut-down of adjacent units. The systematic analysis of the dynamics of failure prone systems with intermediate storage has been carried out by Yi (1992) for the basic single input/single output (SISO) storage system as well as serial trains. Operating algorithms were developed which minimize the effects of the propagation of the failures under various physical constraints and time limitations. The present study extends this work to address the Multiple Input/Multiple Output (MIMO) storage case. As in the SISO case, the storage is assumed not to have enough capacity or initial hold-up to absorb all of the failures. Thus, the operator must decide the sequence and lengths of shut-downs of incident streams (thus units) so that the storage facility is neither depleted nor caused to overflow. The approach used in MIMO case is the same as the SISO storage case except for the introduction of the concept of distribution of failure and shut-down times. Since the transient behavior of hold-up is too complicated to analyze when arbitrary sequences of failures and shut-downs are considered, we impose some reasonable assumptions on the allowable failure patterns. Moreover, since the exact analytical solution of the minimum/maximum hold-up for a general MIMO storage system is not available, we make use of conservatively approximating model. On that basis, a straightforward algorithmic procedure is developed and tested with appropriate examples. To handle more complex phenomena, optimality is somewhat relaxed in order to achieve operational simplicity. Operating algorithms are designed to be explicit, compact, suitable for real-time application.

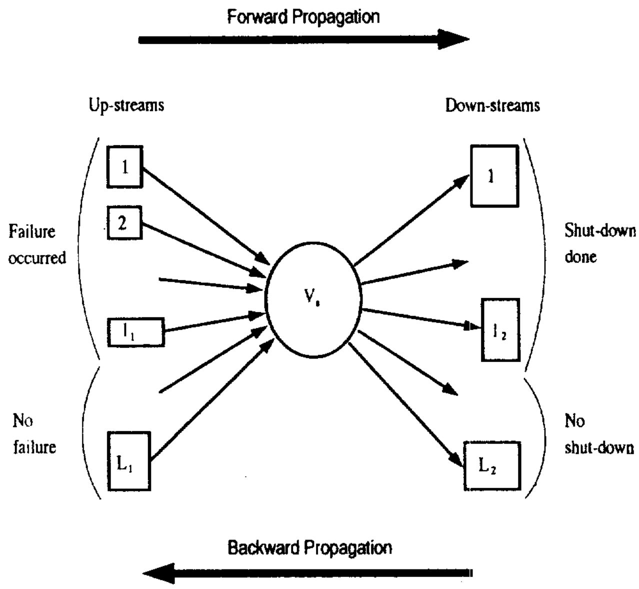

Fig. 1. Schematic diagram of MIMO storage with long term failures.

PROBLEM DESCRIPTION

Fig. 1 represents the schematic diagram of the MIMO storage system which is composed of L_1 up-stream and L_2 down-stream

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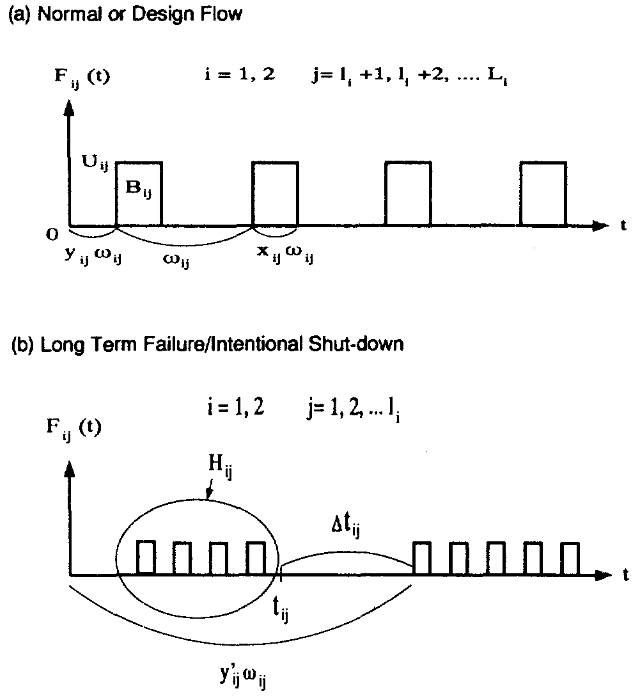

Fig. 2. Parametrization of batch flow and long term failure in MIMO storage.

flows which originate from or serve as input to the associated process units. The effects of failures are assumed to be transmitted through the up-stream flows 1, 2,$\cdots$, l_1 and the down-stream flows 1, 2,$\cdots$, l_2 are assumed to be used to impose intentional shut-downs in order to prevent the storage facility from achieving overflow or depletion. Because arbitrary type of failures are difficult to analyze analytically, the failure patterns that this study will accommodate are confined to the following assumptions:

1. Failure Properties

(1) Failures are assumed to be unknown a priori and therefore, shut-downs are conducted after the initiation of failures.

(2) Failures occur once in each stream during the time period of interest.

(3) The starting moments of failures, the shut-down initiation times, the finishing moments of failures and the finishing moments of shut-down are assumed to occur in an ordered sequence and this sequence is assumed to be preserved.

Failure property (3) serves to make the hold-up profile convex during the transient phase, which enables us to predict the minimum/maximum values of the hold-up during that time period.

Part (a) of Fig. 2 shows the definition of our batch flow variables in the absence of failure, following the conventions introduced in Yi (1992). The subscript i=1 denotes up-stream flows and i=2 down-stream flows. The streams j=1, 2,$\cdots$, L_i include both those with failure and those subject to shut-down. The associated variables are defined in part (b) of Fig. 2. Failures or shut-downs are defined in terms of two sets of variables: the Failure/Shut-down Initiation Time t_{ij} and the Failure/Shut-down Duration Δt_{ij}. Failure/shut-down initiation times t_{ij} are restricted to integer multiples of the cycle time beginning with the starting moment of the first cycle of each stream.

$$t_{ij}=\alpha_{ij}\omega_{ij}+y_{ij}\omega_{ij} \tag{1}$$

for j=1, 2,$\cdots$, l_i, i=1, 2 and $\alpha_{ij}=0, 1, 2, \cdots$

The intermediate variables H_{ij} is defined as the integration over time of the batch flow before failure or shut-down.

$$H_{ij}=\frac{B_{ij}(t_{ij}-y_{ij}\omega_{ij})}{\omega_{ij}} \tag{2}$$

By using these defined variables, the problem and failure types can be clearly defined in the case of the forward propagation of long term failures through MIMO storage, where forward propagation means the flow direction of material and failures are the same.

Problem Definition: To determine t_{2j} and Δt_{2j}, to delay shut-down initiation as much as possible and to keep the total shut-down duration at a minimum for the given t_{1j} and Δt_{1j} under the assumptions;

$$\max_j\{t_{1j}\}\le t_{2k}\le \min_j\{t_{1j}+\Delta t_{1l}\} \tag{3}$$

$$\max_j\{t_{1j}+\Delta t_{1j}\}\le t_{2k}+\Delta t_{2k} \text{ for } k=1, 2, \cdots, l_2 \tag{4}$$

where Eq. (3) and (4) are the mathematical representation of failure property (3). Thereafter, we will omit the subscript j under the max operator for simplicity.

The backward propagation case can be stated by exchanging subscript 1 and 2 in the above definition. The analysis for the backward propagation case can be easily adopted from the results for the forward case.

Storage is susceptible to depletion in the forward propagation case. Shut-downs should therefore be conducted carefully for storage hold-up not to reach the bottom level during the transient phase and at the inclusion of all failures and shut-downs. Therefore, it is necessary to theoretically calculate the minimum hold-up or at least the lower bound of the minimum hold-up during the transient phase and after all the failures and shut-downs have terminated. Because there are many incoming and outgoing flows with different cycle times, it is almost impossible to describe the maximum/minimum of transient hold-up profile in general fashion. However, it is possible to obtain the upper/lower bound of hold-up after all the failures and shut-downs have terminated. The assumptions on the failure and shut-down sequence given in failure property (3) make the transient hold-up profile convex in shape. Consequently, the minimum hold-up during transient phase can be predicted under this assumption. Conceptually, two hold-up shapes, right-skewed and left-skewed, can occur during the transient phase and their associated minimum hold-up occurs at greatly different times. Fig. 3 shows the two cases; (A) the case in which the average hold-up profile from the last initiation of all of the shut-downs to the first finishing moment of all of the failures decreases and (B) the case in which the average hold-up increases. We will call case (A) The Failure Throttled Case and case (B) the Failure Spread Case. For convenience, the case in which the average hold-up profile from the last initiation of all of the shut-downs to the first finishing moment of all of the failures stays flat, called the Failure Balanced Case, is included in case (A). The mathematical conditions are as follows;

(A) Failure Throttled Case: $\Delta P\ge 0$

(B) Failure Spread Case: $\Delta P<0$

where

$$\Delta P=\sum_{j=l_2+1}^{L_2}\frac{B_{2j}}{\omega_{2j}}-\sum_{j=l_1+1}^{L_1}\frac{B_{1j}}{\omega_{1j}} \tag{5}$$

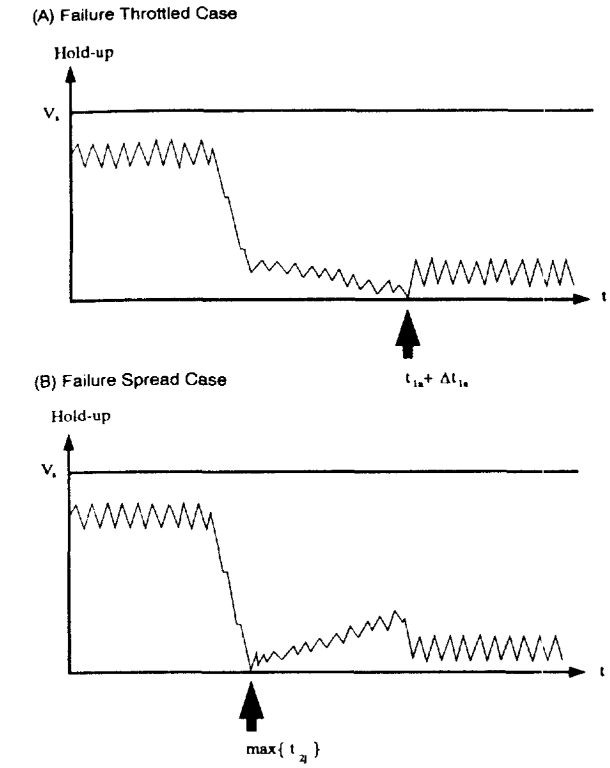

Fig. 3. Two cases of hold-up profile with MIMO storage.

The mathematical description of the condition of no depletion after all the failures and shut-downs have terminated is common to both cases. The differential material balance around the storage after all the failures and shut-downs have terminated consists of a simple ordinary differential equation. The integration of the material balance equation can be carried out in the same way as in Yi (1992).

$$\begin{aligned} V(t)=V(0)&+\sum_{j=1}^{l_1} H_{1j}-\sum_{j=1}^{l_2} H_{2j} \\ &+\sum_{j=1}^{L_1} B_{1j}\left[\mathrm{int}\left(\frac{t-y'_{1j}\omega_{1j}}{\omega_{1j}}\right)+\min\left\{1,\ \frac{1}{x_{1j}}\mathrm{res}\left(\frac{t-y'_{1j}\omega_{1j}}{\omega_{1j}}\right)\right\}\right] \\ &-\sum_{j=1}^{L_2} B_{2j}\left[\mathrm{int}\left(\frac{t-y'_{2j}\omega_{2j}}{\omega_{2j}}\right)+\min\left\{1,\ \frac{1}{x_{2j}}\mathrm{res}\left(\frac{t-y'_{2j}\omega_{2j}}{\omega_{2j}}\right)\right\}\right] \end{aligned} \tag{6}$$

where

$$\begin{aligned} y'_{ij}&=\frac{t_{ij}+\Delta t_{ij}}{\omega_{ij}} \quad \text{for } i=1,\ 2 \text{ and } j=1,\ 2,\cdots,\ l_i \\ y'_{ij}&=y_{ij} \quad \text{for } j=l_i+1,\ l_i+2,\cdots,\ L_i \end{aligned} \tag{7}$$

Even though we can not locate the exact maximum or minimum of Eq. (6). The upper and lower bound of Eq. (6) are easily available using the Technical Lemma in Yi (1992).

$$\begin{aligned} V_{ub}=V(0)&+\sum_{j=1}^{l_1} H_{1j}-\sum_{j=1}^{l_2} H_{2j}-\sum_{j=1}^{l_1} B_{1j}(y'_{1j}+x_{1j}-1) \\ &-\sum_{j=l_1+1}^{L_1} B_{1j}(y_{1j}+x_{1j}-1)+\sum_{j=1}^{l_2} B_{2j}y'_{2j}+\sum_{j=l_2+1}^{L_2} B_{2j}y_{2j} \end{aligned} \tag{8}$$

$$\begin{aligned} V_{lb}=V(0)&+\sum_{j=1}^{l_1} H_{1j}-\sum_{j=1}^{l_2} H_{2j}-\sum_{j=1}^{l_1} B_{1j}y'_{1j}-\sum_{j=l_1+1}^{L_1} B_{1j}y_{1j} \\ &+\sum_{j=1}^{l_2} B_{2j}(y'_{2j}+x_{2j}-1)+\sum_{j=l_2+1}^{L_2} B_{2j}(y_{2j}+x_{2j}-1) \end{aligned} \tag{9}$$

The sufficient conditions for storage not to be depleted and for storage not to overflow are;

$$0+\varepsilon\le V_{lb}<V_{ub}\le V_s-\varepsilon \tag{10}$$

Eq. (10) can be rewritten with respect to operating variables Δt_{2j} such as;

$$S_{lb}\le\sum_{j=1}^{l_2}\frac{B_{2j}\Delta t_{2j}}{\omega_{2j}}\le S_{ub}+V_s \tag{11}$$

where

$$S_{lb}=\sum_{j=1}^{l_1}\frac{B_{1j}\Delta t_{1j}}{\omega_{1j}}-V_\varepsilon(0)+\sum_{j=1}^{L_1} B_{1j}y_{1j}-\sum_{j=1}^{L_2} B_{2j}(y_{2j}+x_{2j}-1) \tag{12}$$

$$S_{ub}=\sum_{j=1}^{l_1}\frac{B_{1j}\Delta t_{1j}}{\omega_{1j}}-V^\varepsilon(0)+\sum_{j=1}^{L_1} B_{1j}(y_{1j}+x_{1j}-1)-\sum_{j=1}^{L_2} B_{2j}y_{2j} \tag{13}$$

Let us introduce Shut-down Distribution Parameters θ_{2j} which satisfy the following relationship:

$$\Delta t_{2j}=\Delta t_2\theta_{2j} \tag{14}$$

Since storage is subject to depletion, the minimum amount of total shut-down duration occurs when the first inequality in Eq. (11) becomes active. The active side of Eq. (11) can be rewritten to;

$$\Delta t_2=\frac{S_{lb}}{\displaystyle\sum_{j=1}^{l_2}\frac{B_{2j}\theta_{2j}}{\omega_{2j}}} \tag{15}$$

The shut-down distribution parameters serve the role of weighting factors according to Eq. (14). By inserting Eqs. (14) and (15) into Eq. (4), we obtain constraints about shut-down distribution parameters.

$$(\max[t_{1j}+\Delta t_{1j}]-t_{2k})\sum_{j=1}^{l_2}\frac{B_{2j}\theta_{2j}}{\omega_{2j}}<S_{lb}\theta_{2k} \tag{16}$$

A sufficient condition for satisfactory operation which can be derived from Eq. (16) is;

$$\frac{\max[t_{1j}+\Delta t_{1j}]-t_{2k}}{S_{lb}}\sum_{j=1}^{l_2}\frac{B_{2j}}{\omega_{2j}}<\frac{\theta_{2k}}{\bar{\theta}_2}<1 \tag{17}$$

where $\bar{\theta}_2$ is the upper bound of θ_{2k}.

Eq. (17) gives two useful expressions. One is the restriction on the shut-down initiation times in order to guarantee the effectiveness of our result and the other is an explicit expression for the shut-down distribution parameters.

$$t_{2k}>\max\{t_{1j}+\Delta t_{1j}\}-\frac{S_{lb}}{\displaystyle\sum_{j=1}^{l_2}\frac{B_{2j}}{\omega_{2j}}} \tag{18}$$

$$\theta_{2k}=\frac{[\max\{t_{1j}+\Delta t_{1j}\}-t_{2k}]\displaystyle\sum_{j=1}^{l_2}\frac{B_{2j}}{\omega_{2j}}}{S_{lb}}+\varepsilon' \tag{19}$$

where $\varepsilon'\ll 1$ and $\theta_{2k}<1$.

Eq. (19) can be obtained by setting $\bar{\theta}_{2k}=1$, without loss of generality.

FAILURE THROTTLED CASE

As can be seen from part (A) in Fig. 3, the minimum hold-up during the transient phase occurs at the finishing moment of all failures. Suppose that the n-th resumption of the up-stream flow among the failed up-stream flows completely compensates the material flow difference ΔP and suppose that the up-stream flows are labeled in increasing order of $t_{1j}+\Delta t_{1j}$. The up-stream index n should satisfy the following inequalities;

$$\sum_{k=1}^{n-1} \frac{B_{1k}}{\omega_{1k}} < \Delta P \leq \sum_{k=1}^{n} \frac{B_{1k}}{\omega_{1k}} \tag{20}$$

Then, the minimum hold-up occurs in the neighborhood of $t_{1n}+\Delta t_{1n}$ and the lower bound on the hold-up, which occurs at $t_{1n}+\Delta t_{1n}$, should be manipulated to be greater than zero by adjusting t_{2j}. The hold-up at $t_{1n}+\Delta t_{1n}$ is;

$$\begin{aligned} V(t_{1n}+\Delta t_{1n}) = V(0) &+ \sum_{j=1}^{l_1} H_{1j} - \sum_{j=1}^{l_2} H_{2j} \\ &+ \sum_{j=l_1+1}^{L_1} B_{1j}\left[\mathrm{int}\left(\frac{t_{1n}+\Delta t_{1n}-y_{1j}\omega_{1j}}{\omega_{1j}}\right)\right. \\ &\left.+\min\left\{1, \frac{1}{x_{1j}}\mathrm{res}\left(\frac{t_{1n}+\Delta t_{1n}-y_{1j}\omega_{1j}}{\omega_{1j}}\right)\right\}\right] \\ &- \sum_{j=l_2+1}^{L_2} B_{2j}\left[\mathrm{int}\left(\frac{t_{1n}+\Delta t_{1n}-y_{2j}\omega_{2j}}{\omega_{2j}}\right)\right. \\ &\left.+\min\left\{1, \frac{1}{x_{2j}}\mathrm{res}\left(\frac{t_{1n}+\Delta t_{1n}-y_{2j}\omega_{2j}}{\omega_{2j}}\right)\right\}\right] \\ &+ \sum_{k=1}^{n-1} B_{1k}\left[\mathrm{int}\left(\frac{t_{1n}+\Delta t_{1n}-t_{1k}-\Delta t_{1k}}{\omega_{1k}}\right)\right. \\ &\left.+\min\left\{1, \frac{1}{x_{1k}}\mathrm{res}\left(\frac{t_{1n}+\Delta t_{1n}-t_{1k}-\Delta t_{1k}}{\omega_{1k}}\right)\right\}\right] \end{aligned} \tag{21}$$

The condition that the lower bound of Eq. (21) must be greater than zero gives the following equation with the aid of Eq. (1) and (2):

$$\sum_{j=1}^{l_2} B_{2j}\alpha_{2j} \leq R_\varepsilon \tag{22}$$

where

$$\begin{aligned} R_\varepsilon = V_\varepsilon(0) &+ \sum_{j=1}^{l_1} \frac{B_{1j}(t_{1j}-y_{1j}\omega_{1j})}{\omega_{1j}} + \sum_{j=l_1+1}^{L_1} \frac{B_{1j}(t_{1n}+\Delta t_{1n}-y_{1j}\omega_{1j})}{\omega_{1j}} \\ &+ \sum_{j=l_2+1}^{L_2} B_{2j}\left(\frac{t_{1n}+\Delta t_{1n}-y_{2j}\omega_{2j}}{\omega_{2j}}+1-x_{2j}\right) \\ &+ \sum_{k=1}^{n-1} \frac{B_{1k}(t_{1n}+\Delta t_{1n}-t_{1k}-\Delta t_{1k})}{\omega_{1k}} \end{aligned} \tag{23}$$

We seek to maximize the left side of Eq. (22) with respect to integer variables α_{2j} because the closer the minimum hold-up approach to the lower storage limit, the more the storage inventory is utilized. This constitutes an integer linear programming where the constraints are Eq. (3) and (18). Since the solution of the integer linear programming is too computationally involved for real-time application, we introduce an alternate explicit procedure. Suppose that α_{2m} is the maximum among the $\{\alpha_{2j}\}$. We will propose the following Shut-down Initiation Rule:

$$t_{2k} = \mathrm{int}\left(\frac{t_{2m}-y_{2k}\omega_{2k}}{\omega_{2k}}\right)\omega_{2k} + y_{2k}\omega_{2k} \tag{24}$$

where $k=1, 2, \cdots, m-1, m+1, \cdots, l_2$.

Eq. (24) can be interpreted to mean that we prefer to conduct the shut-downs at almost the same time and that the m-th down-stream flow is the last one to be shut-down. By inserting Eq. (24) into Eq. (22) and taking the maximum of the left hand side of Eq. (22), the explicit expression for α_{2m} can be obtained.

$$\alpha_{2m} = \mathrm{int}\left[\frac{\dfrac{R_\varepsilon + \sum_{j=1}^{l_2} B_{2j}y_{2j}}{\sum_{j=1}^{l_2} \dfrac{B_{2j}}{\omega_{2j}}} - y_{2m}\omega_{2m}}{\omega_{2m}}\right] \tag{25}$$

At this point, we have developed all of the equations for the failure throttled, forward propagation case for MIMO storage. Before we move to the other case, two more conditions which should be satisfied to guarantee the effectiveness of our solution can be developed from Eq. (3), (18) and (22), namely,

$$V(0) > \max[t_{1j}] \sum_{j=1}^{l_2} \frac{B_{2j}}{\omega_{2j}} + \{V(0)-R_\varepsilon\} - \sum_{j=1}^{l_2} B_{2j}y_{2j} \tag{26}$$

$$\max[t_{1j}+\Delta t_{1j}] \sum_{j=1}^{l_2} \frac{B_{2j}}{\omega_{2j}} < R_\varepsilon + S_{lb} + \sum_{j=1}^{l_2} B_{2j}y_{2j} \tag{27}$$

Eq. (26) is composed of initial inventory, failure variables and system parameters and can be interpreted to mean that the initial inventory should be greater than the right side of Eq. (26) in order to guarantee satisfactory system performance. Eq. (27), which is composed of failure variables and system parameters, indicates that our results are restricted to a class of failure types that satisfies Eq. (27). Both Eq. (26) and (27) do not include the shut-down variables. Therefore, these equations are uncontrollable conditions by our operating algorithms.

FAILURE SPREAD CASE

As can be seen from part (B) of Fig. 3, the minimum hold-up during the transient phase occurs at the last shut-down initiation time, $\max\{t_{2j}\}$ in the failure spread case. Suppose that the last shut-down occurs at the m-th down-stream flow.

$$t_{2m} = \max\{t_{2j}\} \tag{28}$$

The hold-up at t_{2m} is;

$$\begin{aligned} V(t_{2m}) = V(0) &+ \sum_{j=1}^{l_1} \frac{B_{1j}(t_{1j}-y_{1j}\omega_{1j})}{\omega_{1j}} - \sum_{j=1}^{l_2} \frac{B_{2j}(t_{1j}-y_{2j}\omega_{2j})}{\omega_{2j}} \\ &+ \sum_{j=l_1+1}^{L_1} B_{1j}\left[\mathrm{int}\left(\frac{t_{2m}-y_{1j}\omega_{1j}}{\omega_{1j}}\right)+\min\left\{1, \frac{1}{x_{1j}}\mathrm{res}\left(\frac{t_{2m}-y_{1j}\omega_{1j}}{\omega_{1j}}\right)\right\}\right] \\ &- \sum_{j=l_2+1}^{L_2} B_{2j}\left[\mathrm{int}\left(\frac{t_{2m}-y_{2j}\omega_{2j}}{\omega_{2j}}\right)+\min\left\{1, \frac{1}{x_{2j}}\mathrm{res}\left(\frac{t_{2m}-y_{2j}\omega_{2j}}{\omega_{2j}}\right)\right\}\right] \end{aligned} \tag{29}$$

In the same way as for the failure throttled case, we can develop the condition under which the lower bound of Eq. (29) is greater than zero, namely,

$$\sum_{j=1, j\neq m}^{l_2} B_{2j}\alpha_{2j} + B'_{2m}\alpha_{2m} \leq R_\varepsilon \tag{30}$$

where

$$\begin{aligned} B'_{2m} &= B_{2m} + \omega_{2m}\Delta P \\ R_\varepsilon(0) &= V_\varepsilon(0) + \sum_{j=1}^{l_1} \frac{B_{1j}t_{1j}}{\omega_{1j}} - \sum_{j=1}^{L_1} B_{1j}y_{1j} + \sum_{j=l_2+1}^{L_2} B_{2j}(x_{2j}+y_{2j}-1) \\ &\quad - y_{2m}\omega_{2m}\Delta P \end{aligned} \tag{31}$$

The left side of Eq. (30) should be maximized with respect to integer variables α_{2j} subject to the Eq. (3), (18), (28) and (30).

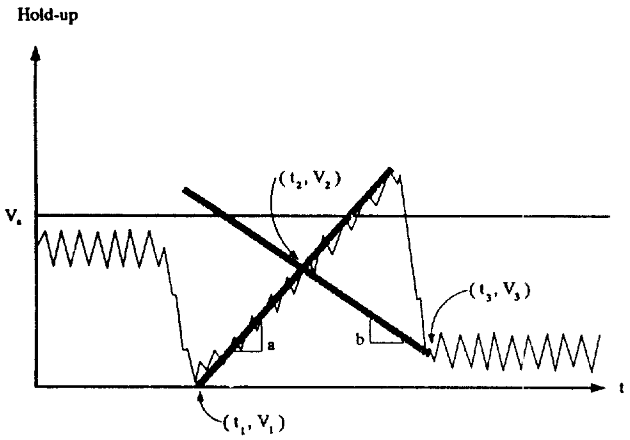

Fig. 4. Hold-up profile resulting from transient imbalance.

This integer linear programming problem is more difficult than that of the failure throttled case because of the additional constraint Eq. (28). We can develop another explicit procedure by following the shut-down initiation rule Eq. (24) although this will make our solution suboptimal. Eq. (24) and the following equation replace the integer linear programming problem.

$$t_{2m}=\frac{V(0)+\sum_{j=1}^{l_1}\frac{B_{1j}t_{1j}}{\omega_{1j}}+\sum_{j=1}^{L_1}B_{1j}y_{1j}-\sum_{j=1}^{L_2}B_{2j}y_{2j}-\sum_{j=l_2+1}^{L_2}B_{2j}(1-x_{2j})}{\sum_{j=1}^{l_1}\frac{B_{1j}}{\omega_{1j}}} \tag{32}$$

There are also parametric restrictions in the failure spread case. Specifically, Eq. (3), (18) and (30) give two inequalities which have the same meaning as Eq. (26) and (27) of the failure throttled case, namely,

$$V(0)>\max\{t_{1j}\}\sum_{j=1}^{l_1}\frac{B_{1j}}{\omega_{1j}}+\{V(0)-R_\varepsilon\}-\sum_{j=1}^{l_2}B_{2j}y_{2j} \tag{33}$$

$$\max[t_{1j}+\Delta t_{1j}]\sum_{j=1}^{l_1}\frac{B_{1j}}{\omega_{1j}}<R_\varepsilon+\frac{S_{lb}\sum_{j=1}^{l_1}\frac{B_{1j}}{\omega_{1j}}}{\sum_{j=1}^{l_2}\frac{B_{2j}}{\omega_{2j}}}+\sum_{j=1}^{l_2}B_{2j}y_{2j} \tag{34}$$

Eq. (33) and (34) define clear parametric domain which guarantee the effectiveness of these results for failure spread case. However, the failure spread case has another type of failure which these equations do not cover. Fig. 4 shows that when the unbalanced flow during the transient phase ($-\Delta P$) is too big, the transient hold-up can exceed the available storage capacity. The sufficient condition to ensure that this kind of failure, which is called the Transient Imbalance, does not occur can be obtained from the fact that the upper bound of the maximum hold-up during the transient phase, which occurs around $\min\{t_{1j}+\Delta t_{1j}\}$, should be less than the storage size V_s.

$$\begin{aligned}V(0)&+\sum_{j=1}^{l_1}\frac{B_{1j}(t_{1j}-y_{1j}\omega_{1j})}{\omega_{1j}}-\sum_{j=1}^{l_2}\frac{B_{2j}(t_{2j}-y_{2j}\omega_{2j})}{\omega_{2j}}\\&+\sum_{j=l_1+1}^{L_1}\frac{B_{1j}}{\omega_{1j}}\{\min[t_{1j}+\Delta t_{1j}]-y_{1j}\omega_{1j}+(1-x_{1j})\omega_{1j}\}\\&-\sum_{j=l_2+1}^{L_2}\frac{B_{2j}}{\omega_{2j}}\{\min[t_{1j}+\Delta t_{1j}]-y_{2j}\omega_{2j}\}<V_s-\varepsilon\end{aligned} \tag{35}$$

Eq. (35) gives another constraint on shut-down initiation time t_{2j} which is not easy to compromise with other constraints involving shut-down initiation time so as to fully utilize the storage inventory. As long as the shut-down initiation times are already fixed, the only way to handle the transient imbalance is to manipulate the shut-down distribution. If some of the shut-down streams are resumed earlier, the increasing transient hold-up can be reduced to decreasing direction before it hits the storage limit. We thus introduce the Triangular Modification procedure named after the construction shown in Fig. 4. The coordinates (t_k, V_k) where k=1, 2 and 3 constitute the triangle in Fig. 4. The slope [a] is naturally the average flow rate, $-\Delta P$. We intend to cut down the average flow along the line (t_2, V_2)-(t_3, V_3) with slope [b]. This can be accomplished by resuming the operation of n down-stream flows from among the l_2 shut-down streams at time t_2, where n is determined so as to satisfy the following inequalities;

$$\sum_{k=1}^{n-1}\frac{B_{2k}}{\omega_{2k}}\le-\Delta P<\sum_{k=1}^{n}\frac{B_{2k}}{\omega_{2k}} \tag{36}$$

Then, the slope [b] becomes;

$$b=-\Delta P-\sum_{k=1}^{n}\frac{B_{2k}}{\omega_{2k}} \tag{37}$$

Obviously, the resumed n down-stream flows will result in smaller shut-down durations than that calculated without triangular modification. The specific down-stream flows should be selected in order in which the user prefer to minimize the shut-down duration.

Without loss of generality, we assume that t_1 is $\max\{t_{2j}\}$, t_3 is $\min\{t_{2j}+\Delta t_{2j}\}$ and V_1 equals V_3. The earlier resumption time t_2, which becomes $t_{2k}+\Delta t_{2k}$ where k=1,2,$\cdots$, n, can be easily determined from the construction of the triangle.

$$t_{2k}+\Delta t_{2k}=\frac{(\Delta P)\max\{t_{2j}\}+(b)\min\{t_{1j}+\Delta t_{1j}\}}{\Delta P+b} \tag{38}$$

where k=1, 2,$\cdots$, n. In order to determine the other shut-down distributions from Eq. (19), S_{lb} should be adjusted to remove the effects of Δt_{2k} where k=1, 2,$\cdots$, n which are already calculated by Eq. (38).

$$S'_{lb}=S_{lb}-\sum_{k=1}^{n}\frac{B_{2k}\Delta t_{2k}}{\omega_{2k}} \tag{39}$$

The shut-down durations for stream n+1, n+2,$\cdots$, l_2 are determined by replacing S'_{lb} with S_{lb} in Eq. (15) and (19) where subscript j runs from n+1 to l_2.

MIMO STORAGE OPERATING ALGORITHM

In the previous three sections, we developed the equations for the shut-down durations, the shut-down distribution parameters and the shut-down initiation times for the long term failure flows through a MIMO storage. All the derivations are based on the MIMO storage model, the failure type assumptions and the physical constraints with the intention of minimizing the effect of failures. In spite of the efforts to achieve a rigorous analysis, our procedures do not specify the choice of the stream to shut down from among all the down-stream flows and the choice of the last shut-down initiation, m-th down-stream, which obviously influence system performance. Although there is clearly scope for future research, the following operating algorithm is suggested based on the above results.

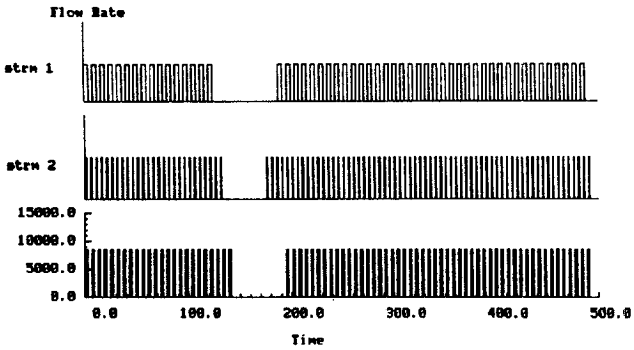

Fig. 5. Up-stream flow patterns of example with balanced failures.

(1) Identify failure variables t_{1j} and Δt_{1j} where subscript j is sequenced according in the increasing order of $t_{1j}+\Delta t_{1j}$.

(2) Determine shut-down streams $j=1, 2,\cdots, l_2$ in the order of preference to minimizing shut-down duration and the last shut-down initiation, m-th stream.

(3) Calculate ΔP by Eq. (5) and
if $\Delta P \geq 0$, refer to failure throttled case
if $\Delta P < 0$, refer to failure spread case.

(4) Check the inequalities Eqs. (26) and (27) for failure throttled case or Eqs. (33) and (34) for failure spread case. If the failure variables do not satisfy any of the above inequalities, we can not guarantee satisfactory performance of this algorithm.

Failure Throttled Case

(5) Identify n-th up-stream flow by Eq. (20). Calculate R_ε by Eq. (23), α_{2m} by Eq. (25), t_{2m} by Eq. (1) and the shut-down initiation times t_{2k} by Eq. (24).

(6) Calculate S_{lb} by Eq. (12), θ_{2j} by Eq. (19), Δt_2 by Eq. (15) and the shut-down duration Δt_{2j} by Eq. (14).

– End of Algorithm –

Failure Spread Case

(5) Calculate t_{2m} by Eq. (32) and shut-down initiation time t_{2j} by Eq. (24).

(6) Check the inequality Eq. (35) and if it satisfies, go to step (6) in failure throttled case. Otherwise, continue with (7).

(7) Identify n-th down-stream by Eq. (36). Calculate [b] by Eq. (37), Δt_{2k} $(k=1, 2,\cdots, n)$ by Eq. (38) and S'_{lb} by Eq. (39).

(8) In order to calculate Δt_{2j} $(j=n+1, n+2,\cdots l_2)$, follow step (6) in failure throttled case after replacing S_{lb} with S'_{lb}.

DISCUSSION WITH SIMULATION EXAMPLES

In the following examples, the MIMO storage vessel is assumed to be connected to 3 up-stream and 4 down-stream units. Three failure types; balanced failures, throttled failures and spread failures and their corresponding operating algorithms were simulated in order to demonstrate the performance of the operating algorithms for the MIMO storage system.

At first, long term failures were assumed to have occurred with the 3 up-stream units and to propagate to the 4 down-stream units. Fig. 5 shows the patterns of the up-stream flows. Fig. 6 and 7 shows the resulting hold-up profile and down-stream flows when a careless operating method has been applied. The inventory was not fully utilized to reduce the shut-down length. Moreover, the shut-down maldistribution leads to the depletion of the storage inventory during the transient state. Fig. 8 and 9 show the results of our proposed MIMO storage operating algorithm.

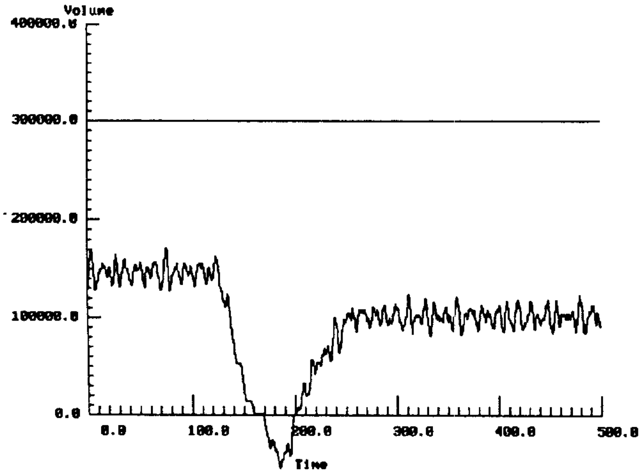

Fig. 6. Hold-up profile under balanced failures resulting from poor operating decisions.

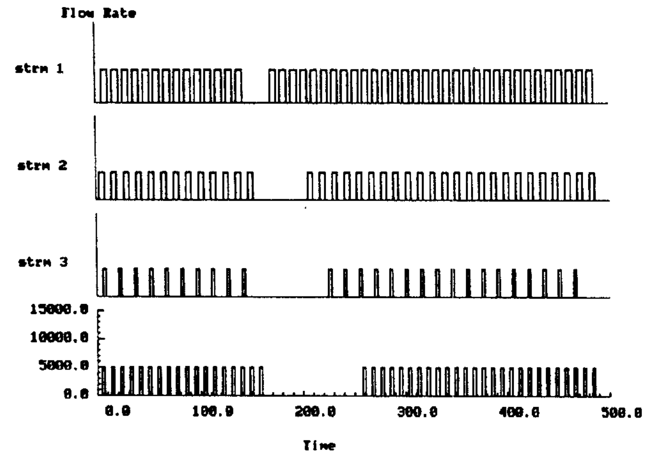

Fig. 7. Shut-downs corresponding to Fig. 6.

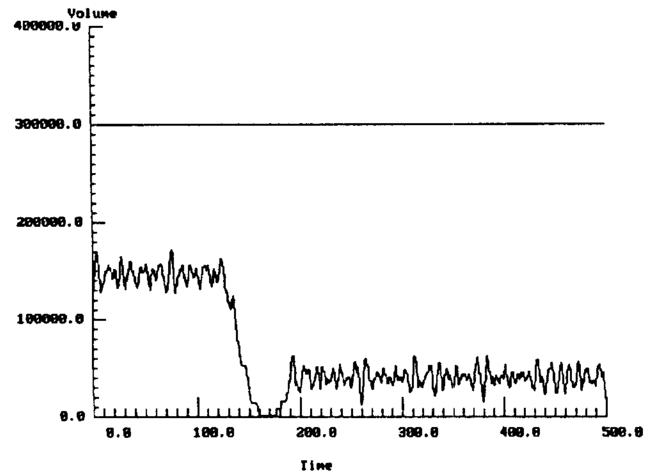

Fig. 8. Hold-up profile under balanced failures using MIMO storage operating algorithm.

Table 1 summarizes the values of system parameters and calculated data. The shut-down distribution parameters do result in a hold-up profile that follow the lower level of the storage. The

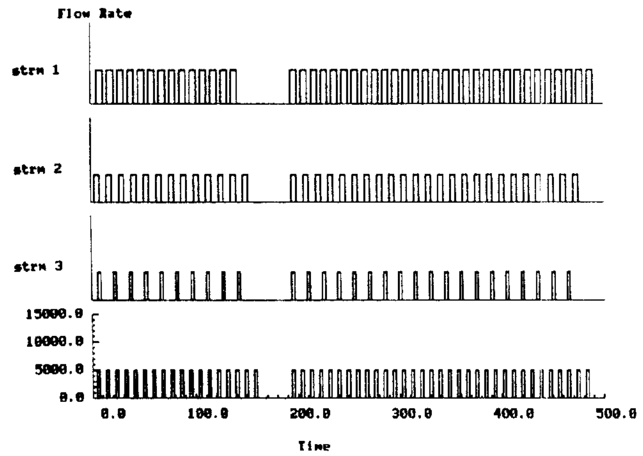

Fig. 9. Shut-downs corresponding to Fig. 8.

Table 1. Input and output data for the simulation of balanced failure case

	Up-streams			Down-streams			
	1	2	3	1	2	3	4
B_{ii}	32000	7500	15000	35000	24000	15000	13500
ω_{ii}	8	5	6	10	12	15	9
x_{ii}	0.6	0.2	0.3	0.6	0.4	0.2	0.3
y_{ii}	0.0	0.2	0.3	0.6	0.3	0.4	0.5
α_{1i}	16	27	24				
t_{1i}	128.0	136.0	145.8				
Δt_{1i}	60.0	40.0	50.0				
θ_{2i}				1.2	0.9	1.0	0.7
U_{ii}	6667	7500	8333	5833	5000	5000	5000
α_{2i}				14	13	10	18
t_{2i}				146.0	159.6	156.0	166.5
Δt_{2i}				48.1	35.0	38.5	28.3

V(0): 121000
V_s: 300000
S_{1b}: 319400
Δt_2: 149.9

minimum hold-up after all failures and shut-downs were terminated maintained above 0 (about 10% of initial hold-up) since the lower bound was used rather than the precise minimum value. According to Eq. (19), θ_{2k} should be less than 1. However, one of the θ_{2k} in this example is greater than 1 which means that the t_{2k} violated the constraints Eq. (4). Eq. (4) derived from failure property (3) which makes the hold-up profile convex during the transient phase. Because it is not sure that the hold-up profile during the transient is convex, we could not guarantee satisfactory performance of the proposed algorithm Nevertheless, proposed algorithm performed quite well.

As a second example, two of the up-stream units were assumed to be subject to long term failures that storage alone could not absorb. Fig. 10 shows the simulated failures of average duration of 120 time units. Two of down-stream flows were chosen as manipulated streams. Because the average flow rate of the two manipulated down-stream flows was less than that of the two up-stream flows with failure, this was the failure throttled case. The system parameters and failure variables satisfy the basic requirements such as Eq. (26) and (27) and the operating algorithm showed good performance. The resulting hold-up and down-stream flows are shown at Fig. 11 and 12 respectively.

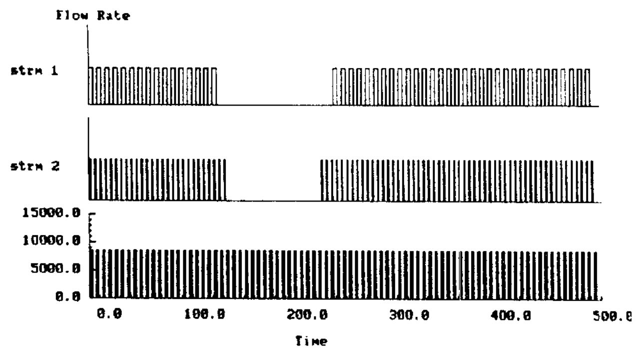

Fig. 10. Example of throttled long term failures.

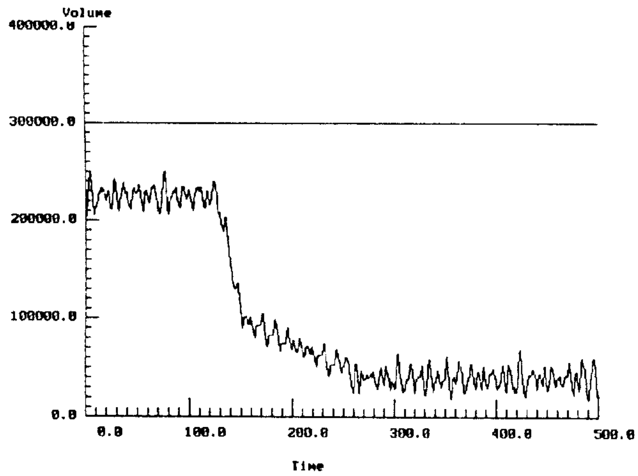

Fig. 11. Hold-up profile under throttled failures using MIMO storage operating algorithm.

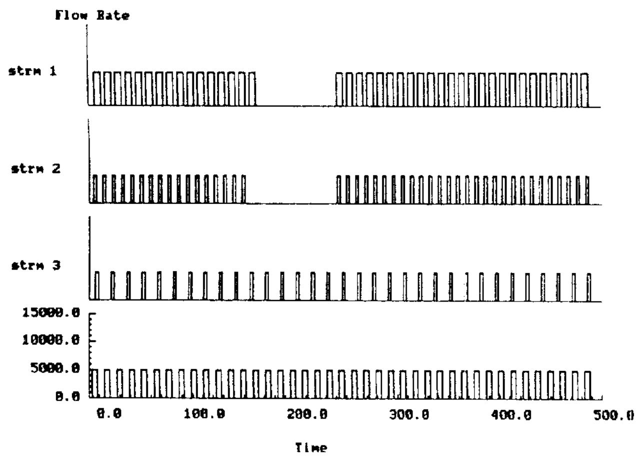

Fig. 12. Shut-downs corresponding to Fig. 11.

In order to test the ability of operating algorithm for the failure spread case, two of the up-stream units were supposed to include the simulated failures of average duration of 230 time units. Three of down-stream flows whose average flow rate was greater than that of the two up-stream flows with failures were chosen as manipulated streams, (see Fig. 13). As is shown at Fig. 14 and 15, when we did not apply the triangular modification procedure, the

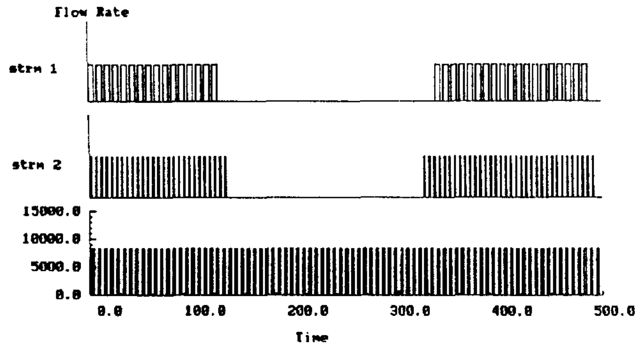

Fig. 13. Example of spread long term failures.

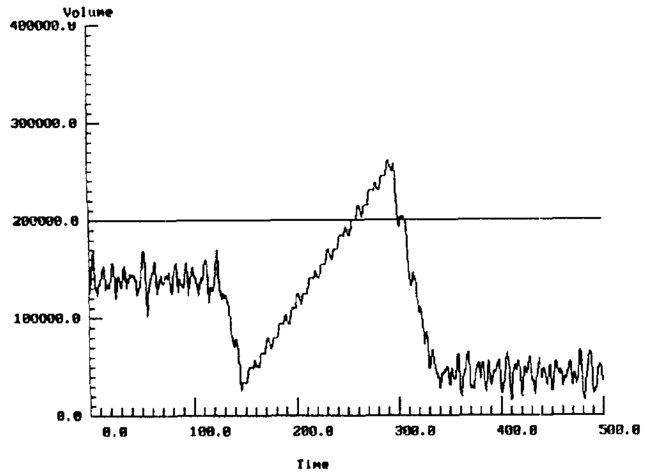

Fig. 14. Hold-up profile showing transient imbalance.

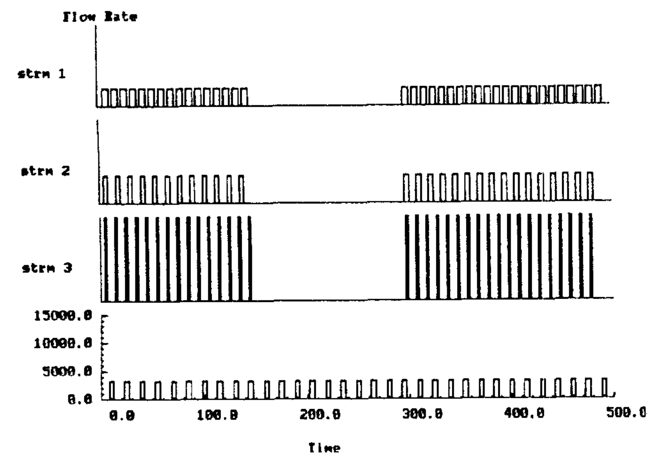

Fig. 15. Shut-downs corresponding to Fig. 14.

transient hold-up reached the storage limit which is the transient imbalance case. Fig. 16 and 17 show the hold-up profile and downstream flows obtained when the triangular modification was applied. The operating algorithm started up only the first stream earlier in order to cut the transient hold-up down.

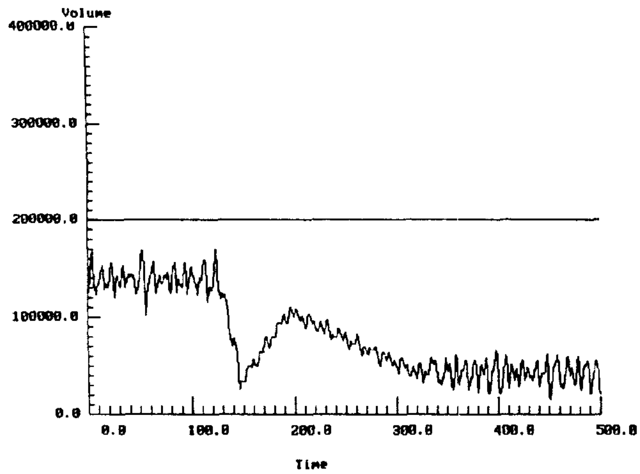

Fig. 16. Hold-up profile using triangular modification for transient imbalance.

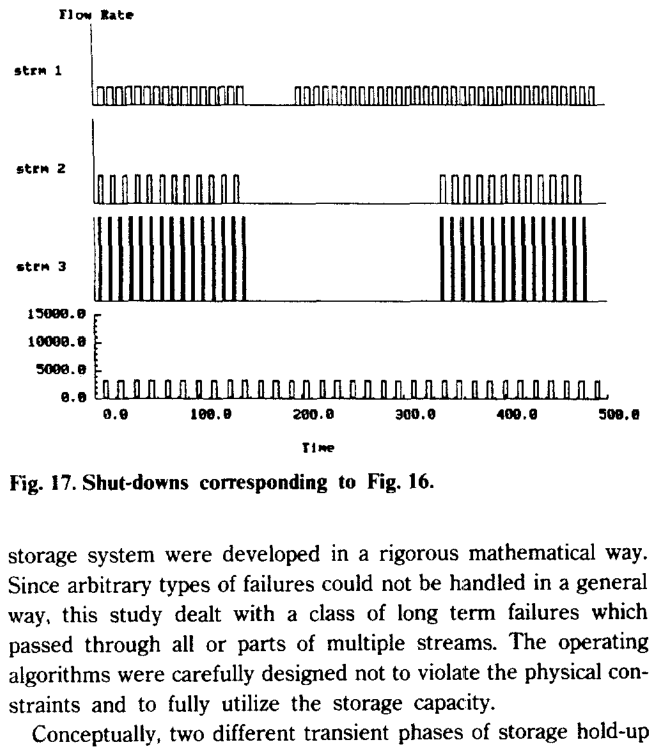

Fig. 17. Shut-downs corresponding to Fig. 16.

CONCLUSIONS

In this article, operating algorithms for a failure prone MIMO storage system were developed in a rigorous mathematical way. Since arbitrary types of failures could not be handled in a general way, this study dealt with a class of long term failures which passed through all or parts of multiple streams. The operating algorithms were carefully designed not to violate the physical constraints and to fully utilize the storage capacity.

Conceptually, two different transient phases of storage hold-up could occur; the case in which the transient hold-up goes down (Failure Throttled Case) and the opposite case (Failure Spread Case) where Balanced Failure Case, the case in which the transient hold-up stays flat, was included within failure throttled case. The two cases had the same form of equations for determining shut-down time length and shut-down distribution rule but they had quite a different procedure for determining shut-down initiation time because the minimum hold-up during transient phase occurred at different time. The failure spread case included another type of failure such that the transient hold-up could reach the storage roof (the Transient Imbalance). In order to cut down the increasing hold-up, a heuristical way to advance the resuming of parts of shut-down streams was suggested (Triangular Modification).

In order to test the effectiveness of our approach, three simulating failures; balanced failures, throttled failures and spread failures, were prepared. The simulation showed that the minimum hold-up after failures and shut-downs differed by only 10% of the initial inventory from the optimum value. This difference is the result of modelling approximation. A small violation in the shut-down initiation condition could be compensated by judicious shut-down distribution rule. The algorithm for failure throttled case manipulated relatively small number of shut-down streams but required a large volume of initial inventory and the opposite was true for failure spread case. Therefore, the algorithm for failure spread case had more powerful operating ability than failure throttled case while it spreads the failure influence.

In spite of all the efforts devoted to rigorous mathematical analysis, the proposed algorithms did not provide how to determine shut-down streams which might be resort to the user experience. These are related with global plant performance rather than one stage of MIMO storage system and therefore requires further study.

NOMENCLATURE

b : slope defined by Eq. (37)
B_{1j} : up-stream batch size of j-th stream
B_{2j} : down-stream batch size of j-th stream
H_{ij} : integration of batch flow before failure or shut-down
L_1 : number of up-stream flows
l_1 : number of up-streams including failure
L_2 : number of down-stream flows
l_2 : number of down-streams including shut-down
ΔP : magnitude of unbalanced flow
R_ε : constant to indicate the amount of feeding defined by Eq. (23) and (31)
S_{lb} : constant to indicate the material deficient due to failures defined by Eq. (12)
S'_{lb} : constant to indicate the material deficient due to failures defined by Eq. (39)
t_{ij} : failure initiation times or shut-down initiation times
Δt_{ij} : failure or shut-down time length of streams
Δt_2 : shut-down time length for forward propagation
$V(t)$: hold-up function
$V(0)$: initial hold-up
$V^\varepsilon(0)$: $V(0)+\varepsilon$
$V_\varepsilon(0)$: $V(0)-\varepsilon$
V_s : storage size
V_{ub} : upper bound of hold-up
V_{lb} : lower bound of hold-up
x_{ij} : transportation time fraction
y_{ij} : initial time delay fraction
y'_{ij} : initial time delay fraction after failure

Greek Letters

α_{ij} : integer search variable corresponding to t_{ij}
ε : safety margin for storage
ε' : small positive number
θ_{2k} : shut-down time length distribution parameters
$\bar{\theta}_{2k}$: upper bound of θ_{2k}
ω_{2k} : cycle times

Subscripts

i : 1 for up-streams and 2 for down-streams
j : stream number
k : stream number
m : stream index of last shut-down initiation
n : stream index defined by Eq. (21) and (37)

Special Functions

int[.] : truncation function to make integer
res[.] : residual function to be truncated

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