

## MODIFIED RESPONSE SURFACE METHODOLOGY (MRSM) FOR PHASE EQUILIBRIUM-APPLICATION

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**Abstract**—Attempts are being made to predict multicomponent azeotropic mixtures from the physical property of pure component and compositions of the constituting binary combination pairs. A modified response surface methodology (MRSM) model has been proposed which correlates boiling temperatures of binary, ternary and quaternary mixtures directly with the compositions of vapor and liquid phases. The generalized MRSM-2 models for liquid and vapor phases are proposed as follows:

$$T = \sum_{i=1}^N T_i X_i + \sum_{i \neq j}^{N-1, N} \Sigma A_{ij} X_i X_j + \sum_{i \neq j}^{N-1, N} \Sigma B_{ij} X_i X_j (X_i - X_j) + \sum_{i \neq j}^{N-1, N} \Sigma C_{ij} X_i X_j (X_i - X_j)^2 \quad (\text{for liquid phase})$$

$$T = \sum_{i=1}^N T_i Y_i + \sum_{i \neq j}^{N-1, N} \Sigma A_{ij} Y_i Y_j + \sum_{i \neq j}^{N-1, N} \Sigma B_{ij} Y_i Y_j (Y_i - Y_j) + \sum_{i \neq j}^{N-1, N} \Sigma C_{ij} Y_i Y_j (Y_i - Y_j)^2 \quad (\text{for vapor phase})$$

These models require normal boiling point of the pure components,  $T_i$  and group-group parameters  $A_{ij}$ ,  $B_{ij}$  &  $C_{ij}$  which can be estimated by the group-group concepts of the constituent components. Therefore, this methodology is applied for the system of three and four components by the computer simulation. No experimental data is required for seeking of composition and temperature of the multicomponent azeotropic mixtures. By means of this methodology, MRSM, it is possible to depict an isothermal line map, temperature contours of the individual phase of the constituting ternaries for each quaternary system. Furthermore, it is possible to predict the azeotropic behaviors, maximum, minimum, saddle or any other type of azeotropic mixtures by examining the graphic contours obtained by computer graphics in the triangular coordinate for ternary and tetrahedron for quaternary. The proposed methodology (MRSM model) has been tested and compared successfully with previously reported azeotropic data in various journals for several ternary and quaternary multicomponent systems. Two azeotropic mixtures are newly found for each of two different quaternary tetrahedrons. The composition, temperature and type of the newly found azeotropes are reported.

**Keywords:** Group-group Parameter, Interaction Parameter, Modified Response Surface Methodology (MRSM), Quaternary Azeotropic Mixture, Two Parameters and Two Terms (TPTT) Equations

### INTRODUCTION

The proposed MRSM-2 model is able to correlate directly the relationship between the boiling temperatures at binary, ternary and quaternary systems and the compositions of vapor and liquid phases in equilibrium. Also, this methodology should provide an information about the solution azeotrope (type, temperature & composition) without experimental data in ternary & quaternary systems by means of the proposed MRSM-2.

Since no adequate method is currently available for the quaternary systems, this simulation method demonstrated a possibility of predicting these complex azeotropic behaviors of quaternary system which is rather difficult to seek even by the sophisticated experimental method.

Furthermore, this methodology shows the possibility of estimation not only the azeotropic behavior but also the properties of various fluids.

### MODIFIED RESPONSE SURFACE METHODOLOGY FOR TERNARY AND QUATERNARY

#### 1. Quaternary Systems Selected

In order to investigate adaptability of MRSM methodology for ternary and quaternary systems, the following three quaternary systems have been selected [1-8, 16]. Since these systems have been previously published in the well-accepted journals [1-3], accuracy of experimental work is reasonably reliable.

Also, entire ternary and binary systems of each constituting components of the quaternaries are available in the published journals [1-3] {exception of two ternary systems: MEK(1)-cyclohexane(3)-2-propanol(4) and methyl acetate(1)-methanol(3)-benzene(4)}.

System I: Methyl ethyl ketone(1)-Benzene(2)-Cyclohexane(3)-2-Propanol(4) Constituting ternary systems

1. Methyl ethyl ketone(1)-Benzene(2)-Cyclohexane(3)
2. Methyl ethyl ketone(1)-Benzene(2)-2-Propanol(4)
3. Methyl ethyl ketone(1)-Cyclohexane(3)-2-Propanol(4)
4. Benzene(2)-Cyclohexane(3)-2-Propanol(4)

System II: Water(1)-Formic acid(2)-Acetic acid(3)-Propionic acid(4) Constituting ternary systems

1. Water(1)-Formic acid(2)-Acetic acid(3)
2. Water(1)-Formic acid(2)-Propionic acid(4)

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3. Water(1)-Acetic acid(3)-Propionic acid(4)
4. Formic acid(2)-Acetic acid(3)-Propionic acid(4)

System III: Methyl acetate(1)-Chloroform(2)-Methanol(3)-Benzene(4) Constituting ternary systems

1. Methyl acetate(1)-Chloroform(2)-Methanol(3)
2. Methyl acetate(1)-Chloroform(2)-Benzene(4)
3. Methyl acetate(1)-Methanol(3)-Benzene(4)
4. Chloroform(2)-Methanol(3)-Benzene(4)

## 2. Binary Interaction Parameters

In order to estimate the phase temperature  $T$  from phase composition  $x_1, x_2, \dots$  for the multicomponent system by means of the generalized MRSM equation the binary interaction parameters  $A_{ij}, B_{ij}, C_{ij}$  have to be known [7-11].

A slight modification of the previously suggested estimation method of the critical temperature of mixtures was utilized for this purpose [12].

The suggested equations to estimate the interaction parameters  $A_{ij}, B_{ij}$ , and  $C_{ij}$  are as follows:

$$A_{ij} = (1/2)\phi_T^A(T_i + T_j) \quad (1)$$

$$B_{ij} = (1/2)\phi_T^B(T_i + T_j) \quad (2)$$

$$C_{ij} = (1/2)\phi_T^C(T_i + T_j) \quad (3)$$

$\phi_T^A, \phi_T^B, \phi_T^C$  of the above equations are also estimated by Eqs. (4)-(6) [7-11].

$$\phi_T^A = \alpha + (\beta/\theta_T) + \gamma(\ln\theta_T) + \delta\theta_T + \varepsilon\theta_T^2 + \eta\theta_T^3 \quad (4)$$

$$\phi_T^B = \alpha' + (\beta'/\theta_T) + \gamma'(\ln\theta_T) + \delta'\theta_T + \varepsilon'\theta_T^2 + \eta'\theta_T^3 \quad (5)$$

$$\phi_T^C = \alpha'' + (\beta''/\theta_T) + \gamma''(\ln\theta_T) + \delta''\theta_T + \varepsilon''\theta_T^2 + \eta''\theta_T^3 \quad (6)$$

$\theta_T$  of the Eqs. (4)-(6) is calculated from the normal boiling points of components  $i$  and  $j$  of the selected group-group.

$$\theta_T = \left| \frac{T_i - T_j}{T_i + T_j} \right| \quad (7)$$

where  $\alpha, \beta, \gamma, \delta, \varepsilon, \eta,$

$\alpha', \beta', \gamma', \delta', \varepsilon', \eta',$

$\alpha'', \beta'', \gamma'', \delta'', \varepsilon'', \eta''$  are the group-group parameters.

## 3. Group-Group Parameters

Estimation of the binary interaction parameters  $A_{ij}, B_{ij}, C_{ij}$  from Eqs. (1)-(3) & (4)-(6) and (7) require the values of six group-group parameters for each interaction parameters [7-11, 16-18].

$A_{ij}: \alpha, \beta, \gamma, \delta, \varepsilon, \eta$

$B_{ij}: \alpha', \beta', \gamma', \delta', \varepsilon', \eta'$

$C_{ij}: \alpha'', \beta'', \gamma'', \delta'', \varepsilon'', \eta''$

This method, which has been established to predict the temperature of mixtures, is not limited to specific mixtures but intended to apply to the whole non-electrolytes except electrolytes and polymer solutions.

For this purpose, organic compounds of the selected three quaternary systems for this study are classified into eight chemical functional groups; paraffin, aromatic, alcohol, halogen, ester, ketone, acid, and water. By combining these eight chemical functional groups for binary group-group; thirteen binary group-groups were obtained, as shown on Table 1.

In order to obtain a reliable value of each individual group-group parameters 100 experimental data sets (1958 data points)

**Table 1. Number of data points and data sets for the selected binary group-group systems [1-3]**

Group-group systems	No. of data sets	No. of points	Group-group systems	No. of data sets	No. of points
Aromatic-Paraffin	11	259	Alcohol-Ester	8	126
Alcohol-Paraffin	9	164	Ketone-Paraffin	6	82
Alcohol-Aromatic	8	207	Alcohol-Ketone	11	182
Aromatic-Halogen	7	148	Acid-Water	5	122
Aromatic-Ester	7	126	Acid-Acid	6	102
Alcohol-Halogen	9	165	Aromatic-Ketone	6	122
Ester-Halogen	7	153			
			Total	100	1958

**Table 2. The binary group-group parameters of aromatic-paraffin system for calculating binary interaction parameter**

		Liquid phase					
		$A_{12}$	$B_{12}$		$C_{12}$		
Aromatic-Paraffin	$\alpha$	0.468280	$\alpha'$	1.733502	$\alpha''$	-3.189149	
	$\beta$	0.000157	$\beta'$	0.000489	$\beta''$	-0.000952	
	$\gamma$	0.096055	$\gamma'$	0.322962	$\gamma''$	-0.601712	
	$\delta$	-9.227683	$\delta'$	-34.599942	$\delta''$	60.895388	
	$\varepsilon$	139.845901	$\varepsilon'$	596.875557	$\varepsilon''$	-992.435205	
	$\eta$	-1175.835696	$\eta'$	-3943.541078	$\eta''$	6512.989281	
			Vapor phase				
		$A_{12}$	$B_{12}$		$C_{12}$		
	$\alpha$	-1.884406	$\alpha'$	0.635579	$\alpha''$	0.365289	
	$\beta$	-0.000530	$\beta'$	0.000168	$\beta''$	0.000063	
	$\gamma$	-0.345598	$\gamma'$	0.116650	$\gamma''$	0.059826	
	$\delta$	37.569492	$\delta'$	-14.886959	$\delta''$	-10.645614	
	$\varepsilon$	-608.237086	$\varepsilon'$	279.500767	$\varepsilon''$	206.924048	
	$\eta$	4244.155933	$\eta'$	-1950.782501	$\eta''$	-1457.255270	

of vapor liquid equilibrium of the binary systems previously published and collected on the references were utilized [1-3].

For individual group-group parameter, several reliable data sets are selected from the above reference, then the parameters are estimated by a multiple regression for liquid and vapor phases [4, 13-15].

As an example of aromatic-paraffin system, 259 data points of the 11 sets were utilized for this computation. For each selected binary group-group system, a summary of number of data sets and number of points utilized for each computation are summarized in Table 1.

Table 2 shows the results of the six computed values, binary group-group parameters,  $\alpha, \beta, \gamma, \delta, \varepsilon, \eta$  for each three binary interaction parameters  $A_{ij}, B_{ij}, C_{ij}$  for liquid and vapor phases of the aromatic-paraffin system (the rest of 12 of 13 systems are shown on the Appendix) [7-9, 17].

## REPRESENTATION OF ISOTHERMAL LINES BY COMPUTER GRAPHICS

### 1. Temperature Contour Map, Triangular and Tetrahedron

VLE behavior of the binary systems can be understood from the correlation between temperature and composition. However phenomena of phase surface is not able to be observed and visualized accurately for ternary and quaternary systems by vapor-

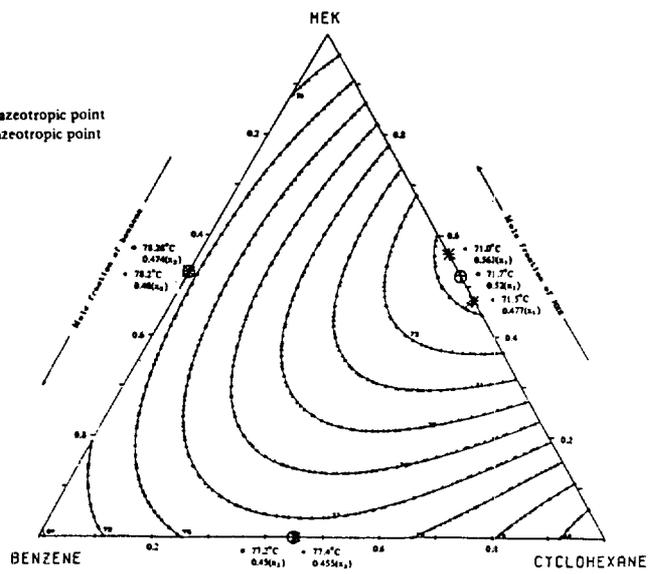
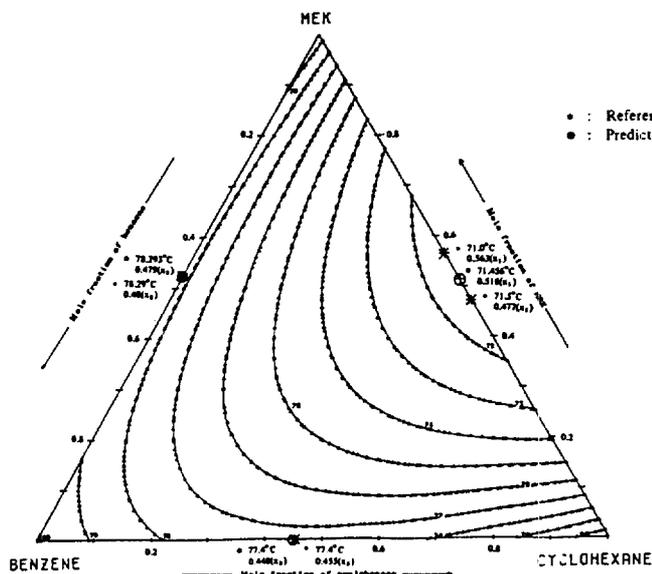


Fig. 1. Isothermal line of MEK-benzene-cyclohexane system predicted by MRSM-2 model (liquid phase).

Fig. 2. Isothermal line of MEK-benzene-cyclohexane system predicted by MRSM-2 model (vapor phase).

liquid equilibrium data alone. Especially, in the case of azeotropic mixtures of saddle or any other complex type, prediction of azeotropic point, temperature and composition, is rather difficult even by experimental work. It has been known that the discrepancy among the reported values of the literatures was not rare [1-3].

This approach is to show that visual analysis is feasible by plotting isothermal lines of phase surface of ternary and quaternary systems using MRSM model and to predict existence of azeotropes, type, composition and temperature. Furthermore, the phenomena of multicomponent vapor-liquid equilibrium phase surface can be studied by examining shifting locus of azeotropic point, observation of temperature distribution and composition of the phase surface [5-11, 16-18].

For ternary system, the temperature contours with change of composition of the liquid and vapor phases can be represented on a plane (isotherm map). However isotherms of quaternary system can be depicted stereographically only in three-dimensional space. Therefore geometry of tetrahedron has been adapted to depict isotherms stereographically in three-dimensional space so that the sum of each composition is unit at any arbitrary point. Isothermal lines are illustrated in Fig. 1 & 2, which is one of four ternaries of vapor and liquid phases\* of the selected quaternary systems MEK(1)-benzene(2)-cyclohexane(3)-2-propanol(4) [8, 9, 16, 18].

Each phase of four tetrahedron and 20, 40, 60 and 80% planes\*\* (Fig. 3) of the top components  $x_4$ , and  $y_4$ , are drawn individually by the computer graphics. Then, these eight planes are reassembled to construct one of each tetrahedron corn for liquid and vapor phases [7-9].

Each phase of three tetrahedron except for the bottom-facets are shown by rotating  $120^{\circ}$  as for the liquid phase (Fig. 4) and vapor phase (Fig. 5).

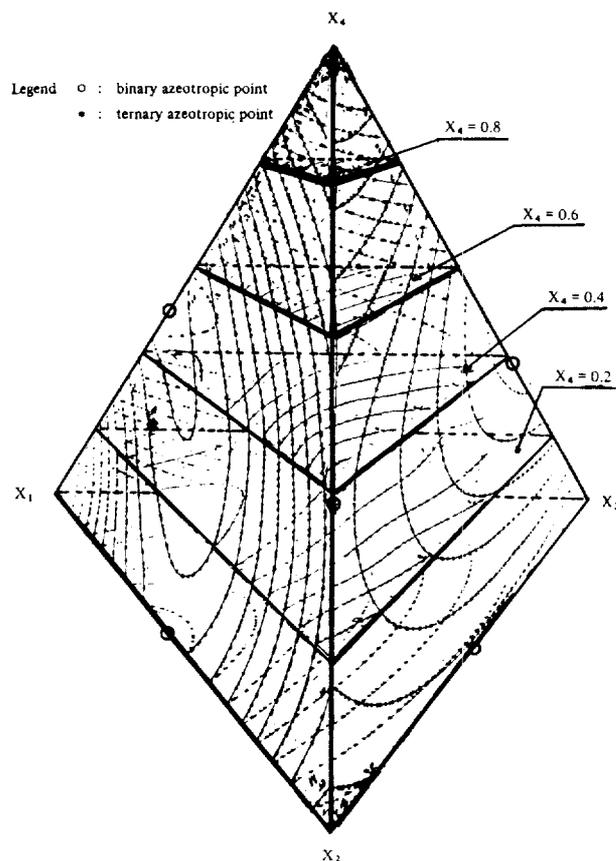


Fig. 3. Tetrahedron of quaternary isotherm with planes of  $x_4=0, 0.2, 0.4, 0.6$  and  $0.8$  mole fractions.

It has been observed that individual isothermal line of each phase is precisely matched at every point of phase intersection. **2. Algorithms of the Isotherm Map**

In order to plot the isothermal lines by computer graphics, two steps are required. That is, computing various compositions for

\*Other six ternary isotherm maps of three vapor and three liquid phases can be obtained from the author by request.

\*\*Ternary isotherm maps can be obtained from the author by request.

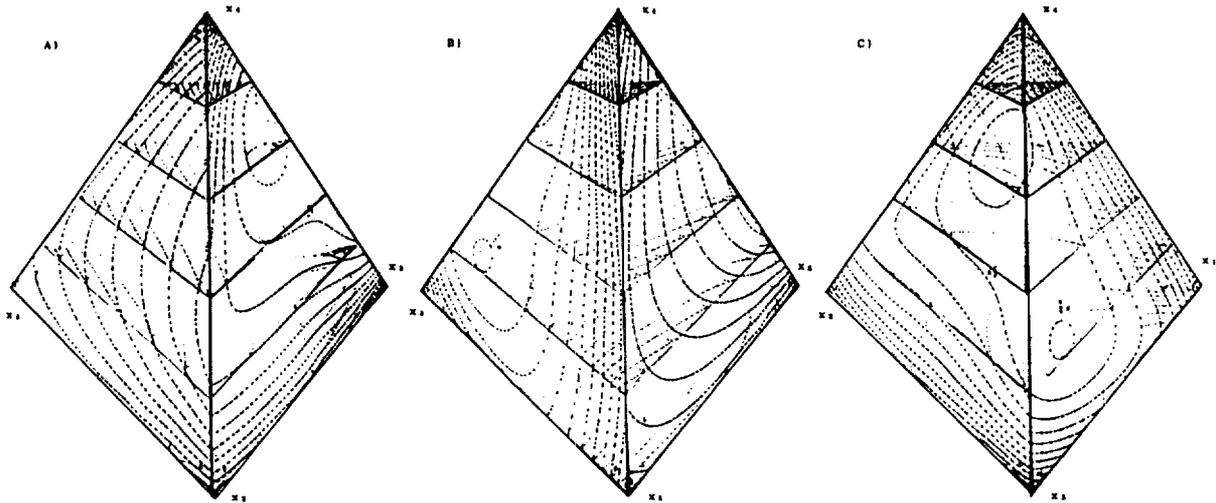


Fig. 4. Isothermal lines of quaternary tetrahedron, system MEK(1)-benzene(2)-cyclohexane(3)-2-propanol(4). At three different angle of views (liquid phase).

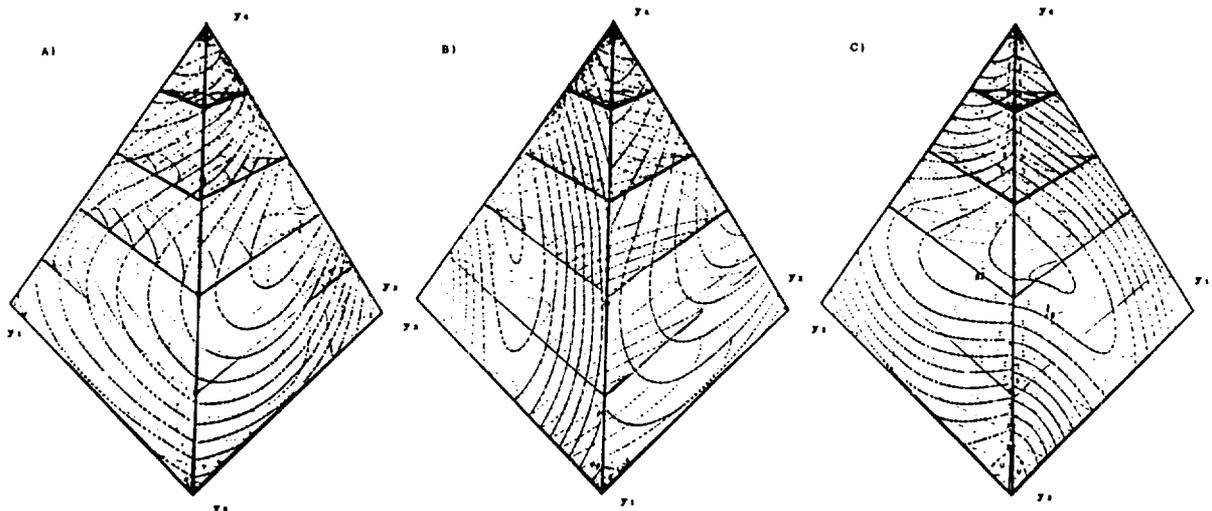


Fig. 5. Isothermal lines of quaternary tetrahedron system MEK(1)-benzene(2)-cyclohexane(3)-2-propanol(4). At three different angle of views (vapor phase).

given isotherm temperature and then plotting isothermal lines on paper on triangular coordinates. The Newton-Raphson algorithm with the formula of MRSM is utilized for the first step and coordination algorithm XX, YY with a size factor  $\alpha$  of plotting isotherms on triangular coordinate for the second step.

1st step:

Ternary system isotherm

$$x_{n+1} = x_n - FT(x_i)/FT'(x_i) \tag{8}$$

$$FT(x_i) = \sum_{i=1}^3 T_i x_i + \sum_{i \neq j}^{2,3} A_{ij} x_i x_j + \sum_{i \neq j}^{3,4} B_{ij} x_i x_j (x_i - x_j) + \sum_{i \neq j}^{2,3} C_{ij} x_i x_j (x_i - x_j)^2 - T = 0 \tag{9}$$

Quaternary system isotherm

$$x_{n+1} = x_n - FT(x_i)/FT'(x_i) \tag{10}$$

$$FT(x_i) = \sum_{i=1}^4 T_i x_i + \sum_{i \neq j}^{3,4} A_{ij} x_i x_j + \sum_{i \neq j}^{3,4} B_{ij} x_i x_j (x_i - x_j)$$

$$+ \sum_{i \neq j}^{3,4} C_{ij} x_i x_j (x_i - x_j)^2 - T = 0 \tag{11}$$

Whereas,  $FT(x_i)$  is the difference between the estimated temperature of the mixture  $T$  by the MRSM formula and the given temperature  $T$  of the isotherm.

$FT'(x_i)$  is a formula obtained from partial differentiation of  $FT(x_i)$  with respect to composition  $x_i$ . For ternary,  $\sum x_i = 1$  also  $x_2 = 1 - x_1 - x_3$ , then fixing temperature  $T$  and one of the composition  $x_1$ , one of the rest composition  $x_2$  or  $x_3$  is varied with a fixed internal (such as 0.02) for the Newton-Raphson computation. When absolute value of  $FT(x_i)$  has been obtained within the range of allowable error (such as 0.0001), the computed compositions can be accepted.

Subsequently, similar computation is repeated for the other composition. Then the third composition is computed from the predetermined first and second compositions. This composition is accepted as an equilibrium composition at the given temperature  $T$ . For quaternary, the similar computations are repeated for indivi-

**Table 3. Summary of mean A.P.E. of liquid and vapor phases for thirteen binary group-group, constituting three quaternary systems**

Group-Group systems	Mean A.P.E.	
	Liquid	Vapor
Aromatic-Paraffin	0.2960	0.3504
Alcohol-Paraffin	1.6673	1.6415
Alcohol-Aromatic	1.0902	1.0635
Aromatic-Halogen	0.1926	0.2346
Aromatic-Ester	0.8554	0.8535
Alcohol-Halogen	1.5602	1.3060
Ester-Halogen	1.3349	1.3071
Alcohol-Ester	0.4156	0.3703
Ketone-Paraffin	0.7379	0.6771
Alcohol-Ketone	1.0456	0.9944
Acid-Water	0.4259	0.4494
Acid-Acid	0.3531	0.3797
Aromatic-Ketone	0.3341	0.4004
Total mean A.P.E.	0.7930	0.7714

dual four triangular surfaces; after fixing the composition of  $x_4$  with an incremental interval of 0.2, 0.4, 0.6, 0.8, the isotherm composition of four intermediate triangular surfaces are also computed with the similar method [7, 8, 16].

2nd step: The computed compositions  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  are plotted on the triangular coordinate. The algorithm used for this purpose is:

$$XX = [x_1 \cos(\pi/3) + x_3] \alpha \quad (12)$$

$$YY = [x_1 \sin(\pi/3)] \alpha \quad (13)$$

Whereas,  $\alpha$  is size factor of the triangle. XX and YY are the coordinate number on the rectangular coordinate after transforming individual compositions, therefore the isothermal line is obtained by connecting respective coordinate values of XX, YY for a given temperature [7, 8, 16].

## TEST RESULTS AND COMPARATIVE ANALYSIS

### 1. Comparison of Phase Temperature

In order to evaluate a feasibility or goodness of the modified Response Surface Methodology for estimation of multicomponent vapor and liquid equilibrium temperatures and to predict a type, location and composition of various azeotropic mixtures, the previously reported experimental data on journals [1-3] and the estimated data by the MRSM were compared for binary, ternary and quaternary systems [7, 8, 16]. Average percentage error between two values are calculated and listed in Table 3 for binary, Table 4 for ternary and Table 5 for quaternary.

$$\text{A.P.E.} = \frac{\sum_i^n |T_{\text{pre}} - T_{\text{exp}}| / T_{\text{exp}}}{N} \times 100 \quad (14)$$

For thirteen binary group-group systems of this study constituting binary group-group systems of three quaternaries, the mean APE was 0.79 for liquid phase and 0.77 for vapor phase. Similarly, for ten ternary systems, the mean APE was 1.25 for liquid phase and 1.22 for vapor phase. Among these ten ternary systems, a system of chloroform-methanol-benzene showed an exceptionally high value 4.298 for liquid and 3.484 for vapor phases, which com-

**Table 4. Summary of mean A.P.E. of liquid and vapor phases for ten ternaries constituting three quaternary systems**

Systems	Phase	A.P.E.
MEK(1)-benzene(2)-cyclohexane(3)	Liquid	0.52
	Vapor	0.54
MEK(1)-benzene(2)-2-propanol(4)	Liquid	0.31
	Vapor	0.41
Benzene(2)-cyclohexane(3)-2-propanol(4)	Liquid	1.57
	Vapor	1.12
Water(1)-formic acid(2)-acetic acid(3)	Liquid	1.28
	Vapor	0.78
Water(1)-formic acid(2)-prop. acid(4)	Liquid	1.45
	Vapor	1.40
Water(1)-acetic acid(3)-prop. acid(4)	Liquid	0.78
	Vapor	1.16
Formic acid(2)-acetic acid(3)-prop. acid(4)	Liquid	0.64
	Vapor	0.58
Methyl acetate(1)-chloroform(2)-methanol(3)	Liquid	1.11
	Vapor	1.15
Methyl acetate(1)-chloroform(2)-benzene(4)	Liquid	0.54
	Vapor	1.60
Chloroform(2)-methanol(3)-benzene(4)	Liquid	4.30
	Vapor	3.48

**Table 5. Summary of mean A.P.E. of liquid and vapor phases for three quaternary systems**

Systems	I		II		III	
	MEK(1)- Benzene(2)- Cyclohexane(3)- 2-Propanol(4)		Water(1)- Formic acid(2)- Acetic acid(3)- Propionic acid(4)		Methyl acetate(1)- Chloroform(2)- Methanol(3)- Benzene(4)	
Phase	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor
mean A.P.E.	0.69	0.94	0.55	2.01	2.80	2.92

pare with the rest of the nine systems whose average value is 0.910 for liquid and 0.969 for vapor phases. A reason of high APE for chloroform-methanol-benzene system could be caused with a large deviation of experimental data of this particular system. The simulated results are in good agreement with experimental data previously published on the various journals within the statistically reasonable experimental errors.

For three quaternary systems, liquid and vapor phases of system I and liquid phase of system II show APE less than 1.0%, ranging 0.94-0.64, while vapor phase of system II, liquid and vapor phases of system III show about 2.0-2.92%.

### 2. Azeotropes

Comparison between the reported and graphically predicted values of azeotropic mixture at 760mmHg for three quaternaries, and subsequently constituting ternaries and binaries are listed in Table 6, 7 and 8.

In general, types of the azeotropic mixture; maximum, minimum or saddle types agree well with previously reported types of binaries and ternaries. However, temperatures and compositions of azeotropic points showed a slight difference for some cases.

Particular importance of this study is that the special type of azeotropic mixtures for quaternary system are newly found for system I, MEK(1)-benzene(2)-cyclohexane(3)-2-propanol(4) and

**Table 6. Comparison between the reported data and graphically predicted values of the azeotropic point at 760 mmHg for system I, MEK(1)-benzene(2)-cyclohexane(3)-2-propanol(4), based on MRSM**

## (1) Quaternary system

Systems Components	Reported data		MRSM-2 model		Type of azeotrope
	mole	T, °C	Liquid mole T, °C	Vapor mole T, °C	
I MEK(1)	none		0.009 69.77	0.009 69.77	cocoon
Benzen(2)			0.012	0.012	
Cyclohexane(3)			0.529	0.529	
2-Propanol(4)			0.450	0.450	

## (2) Ternary systems

Systems Components	Reported data		MRSM-2 model		Type of azeotrope
	mole	T, °C	Liquid mole T, °C	Vapor mole T, °C	
I-3-1 MEK(1)	none		none		
Benzen(2)					
Cyclohexane(3)					
I-3-2 MEK(1)	none		none		
Benzen(2)					
2-Propanol(4)					
I-3-3 MEK(1)	0.174	68.9	0.140 68.9	uncertain	min.
Cyclohexane(3)	0.535		0.620		
2-Propanol(4)	0.291		0.240		
I-3-4 Benzene(2)	0.142	69.1	0.100 68.5	uncertain	min.
Cyclohexane(3)	0.474		0.260		
2-Propanol(4)	0.384		0.640		

## (3) Binary systems

Systems Components	Reported data		MRSM-2 model		Type of azeotrope
	mole	T, °C	Liquid mole T, °C	Vapor mole T, °C	
I-2-1 MEK(1)	0.520	78.29	0.521 78.29	0.526 78.28	min.
Benzen(2)	0.480		0.479	0.474	
I-2-2 MEK(1)	0.477	71.5	0.516 71.46	0.520 71.7	min.
Cyclohexane(3)	0.523		0.484	0.480	
I-2-3 MEK(1)	0.618	77.7	0.616 77.35	0.643 77.2	min.
2-Propanol(4)	0.382		0.384	0.357	
I-2-4 Benzene(2)	0.545	77.4	0.552 77.4	0.550 77.2	min.
Cyclohexane(3)	0.455		0.448	0.450	
I-2-5 Benzene(2)	0.605	71.92	0.681 71.5	0.600 72.5	min.
2-Propanol(4)	0.395		0.319	0.400	
I-2-6 Cyclohexane(3)	0.595	69.6	0.282 68.72	0.500 69.5	min.
2-Propanol(4)	0.405		0.718	0.500	

mole: mole fraction, T: temperature, min.: minimum azeotrope

system II, water(1)-formic acid(2)-acetic acid(3)-propionic acid(4) (refer to Table 6, 7 & 8). These quaternary azeotropes are not reported or appeared on any reference previously. A cocoon type isotherm contour in the tetrahedron of system I is a particularly worthy phenomena to mention [8, 9, 18].

### CONCLUSIONS

1. Due to various assumptions and simplifications associated with TPTT and MPMT equations, the MRSM model was proposed as one of the new methods of estimating temperatures of phase

**Table 7. Comparison between the reported data and graphically predicted values of the azeotropic point at 760 mmHg for system II, water(1)-formic acid(2)-acetic acid(3)-propionic acid(4), based on MRSM**

## (1) Quaternary system

Systems Components	Reported data		MRSM-2 model		Type of azeotrope
	mole	T, °C	Liquid mole T, °C	Vapor mole T, °C	
II Water(1)	none		0.309 106.3	0.360 107.7	saddle
Formic acid(2)			0.426	0.494	
Acetic acid(3)			0.245	0.126	
Prop. acid(4)			0.020	0.020	

## (2) Ternary systems

Systems Components	Reported data		MRSM-2 model		Type of azeotrope
	mole	T, °C	Liquid mole T, °C	Vapor mole T, °C	
II-3-1 Water(1)	0.363	107.0	0.300 106.3	0.370 107.2	saddle
Formic acid(2)	0.469		0.430	0.480	
Acetic acid(3)	0.168		0.270	0.150	
II-3-2 Water(1)	0.420	107.5	0.350 107.1	0.427 107.4	saddle
Formic acid(2)	0.540		0.540	0.563	
Prop. acid(4)	0.040		0.110	0.010	
II-3-3 Water(1)	none		none		
Acetic acid(3)					
Prop. acid(4)					
II-3-4 Formic acid(2)	none		none		
Acetic acid(3)					
Prop. acid(4)					

## (3) Binary systems

Systems Components	Reported data		MRSM-2 model		Type of azeotrope
	mole	T, °C	Liquid mole T, °C	Vapor mole T, °C	
II-2-1 Water(1)	0.424	107.6	0.421 107.52	0.388 107.61	min.
Formic acid(2)	0.576		0.579	0.612	
II-2-2 Water(1)	none		none		
Acetic acid(3)					
II-2-3 Water(1)	0.950	99.8	0.934 99.58	0.987 99.96	min.
Prop. acid(4)	0.050		0.066	0.013	
II-2-4 Formic acid(2)	none		none		
Acetic acid(3)					
II-2-5 Formic acid(2)	none		none		
Prop. acid(4)					
II-2-6 Acetic acid(2)	none		none		
Prop. acid(4)					

equilibrium systems.

2. Majority of estimating methods for phase equilibrium compositions is to predict activity coefficient first; subsequently, equilibrium phase composition was estimated. However, a newly proposed method, MRSM model, makes it possible to estimate phase temperature and vapor phase composition directly from a given liquid phase composition without any experimental work or data.

3. MRSM model, similar to the TCR equation, which is useful for practical engineering has been proposed, and a generalized form for N-component system is as follows:

**Table 8. Comparison between the reported data and graphically predicted values of the azeotropic point at 760 mmHg for system III, methyl acetate(1)-chloroform(2)-methanol(3)-benzene(4), based on MRSM**

## (1) Quaternary system

Systems	Components	Reported data		MRSM-2 model		Type of azeotrope
		mole	T, °C	Liquid mole T, °C	Vapor mole T, °C	
III	Methyl acetate(1) Chloroform(2) Methanol(3) Benzene(4)	none		none		

## (2) Ternary systems

Systems	Components	Reported data		MRSM-2 model		Type of azeotrope
		mole	T, °C	Liquid mole T, °C	Vapor mole T, °C	
III-3-1	Methyl acetate(1) Chloroform(2) Methanol(3)	0.263	56.4	0.260	55.7	0.320 57.5 saddle
III-3-2	Methyl acetate(1) Chloroform(2) Benzene(4)	none		none		
III-3-3	Methyl acetate(1) Methanol(3) Benzene(4)	none		none		
III-3-4	Chloroform(2) Methanol(3) Benzene(4)	none		none		

## (3) Binary systems

Systems	Components	Reported data		MRSM-2 model		Type of azeotrope
		mole	T, °C	Liquid mole T, °C	Vapor mole T, °C	
III-2-1	Methyl acetate(1) Chloroform(2)	0.355	64.8	0.345 65.08	0.339 64.96	max.
III-2-2	Methyl acetate(1) Methanol(3)	0.672	53.8	0.741 54.69	0.690 54.9	min.
III-2-3	Methyl acetate(1) Benzene(4)	none		none		
III-2-4	Chloroform(2) Methanol(3)	0.650	53.5	0.696 52.95	0.650 54.0	min.
III-2-5	Chloroform(2) Benzene(4)	none		none		
III-2-6	Methanol(3) Benzene(4)	0.610	58.3	0.793 60.34	0.720 60.9	min.

max: maximum azeotrope

$$T = \sum_{i=1}^N T_i x_i + \sum_{i < j}^{N-1, N} \sum A_{ij} x_i x_j + \sum_{i < j}^{N-1, N} \sum B_{ij} x_i x_j (x_i - x_j) + \sum_{i < j}^{N-1, N} \sum C_{ij} x_i x_j (x_i - x_j)^2 \text{ (for liquid phase)}$$

$$T = \sum_{i=1}^N T_i y_i + \sum_{i < j}^{N-1, N} \sum A_{ij} y_i y_j + \sum_{i < j}^{N-1, N} \sum B_{ij} y_i y_j (y_i - y_j) + \sum_{i < j}^{N-1, N} \sum C_{ij} y_i y_j (y_i - y_j)^2 \text{ (for vapor phase)}$$

4. By means of MRSM model, it is possible to predict the temperature of binary, ternary and quaternary system with only nor-

mal boiling point of pure components and the binary group-group parameters listed on this paper.

5. Calculated results are in good agreement with experimental data for the selected quaternary systems and their constituent ternaries and binaries. A little deviation was noted in the estimated values for highly nonideal solutions containing the alcohol or volatile organic compound.

6. MRSM methodology, provides a simple procedure for examining existence of azeotropic mixture, type of azeotrope, azeotropic temperature and composition by observing vapor-liquid isothermal contours of ternary and quaternary systems depicted from computer graphics. Two azeotropic mixtures (type, composition, temperature) for two quaternary systems were newly found by this methodology.

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## NOMENCLATURE

$A_{12}, A_{13}, A_{14}, A_{23}, A_{24}, A_{34}, B_{12}, B_{13}, B_{14}, B_{23}, B_{24}, B_{34}, C_{12}, C_{13}, C_{14}, C_{23}, C_{24}, C_{34}$ : binary interaction parameters  
 $A_{ij}, B_{ij}, C_{ij}$ : binary interaction parameters of component  $i$  &  $j$  defined by Eqs. (1)-(3)  
 $N$ : number of data  
 $T$ : boiling temperature of mixture  
 $T_i, T_j, T_1, T_2, T_3, T_4$ : boiling temperature of pure component  $i, j, 1, 2, 3, 4$   
 $X_i, X_j, X_1, X_2, X_3, X_4$ : mole fraction of component  $i, j, 1, 2, 3, 4$  in liquid phase  
 $Y_i, Y_j, Y_1, Y_2, Y_3, Y_4$ : mole fraction of component  $i, j, 1, 2, 3, 4$  in vapor phase  
 $XX, YY$ : coordinate number on the rectangular coordinate

## Greek Letters

$\alpha$ : size factor of the triangle by Eqs.(12), (13)  
 $\alpha, \beta, \gamma, \delta, \epsilon, \eta$ : binary group-group parameter for  $A_{ij}$   
 $\alpha', \beta', \gamma', \delta', \epsilon', \eta'$ : binary group-group parameter for  $B_{ij}$   
 $\alpha'', \beta'', \gamma'', \delta'', \epsilon'', \eta''$ : binary group-group parameter for  $C_{ij}$   
 $\theta_T$ : parameter defined by Eq. (7)  
 $\phi_T^A, \phi_T^B, \phi_T^C$ : parameter defined by Eqs. (4), (5) and (6)

## Subscripts

exp: exponential  
 $i$ : component  $i$   
 $j$ : component  $j$   
 $i-j, i-j-k$ :  $i-j$  pair or  $i-j-k$  pair  
max.: maximum  
min.: minimum  
 $n, n+1$ : number of iteration  
pre.: predicted

## REFERENCES

- Gmehling, J., Onken, U. and Arlt, W.: "Vapor-Liquid Equilibrium Data Collection", DACHEMA, Frankfurt Germany (1981).
- Horsely, L. E.: "Azeotropic Data III", Am. Chem. Soc. Washi-

- ngton D.C. (1973).
- Reid, R. C., Prausnitz, J. M. and Poling, B. E.: "The Properties of Gases and Liquids", 4th ed., McGraw-Hill, N.Y. (1987).
  - Park, J. C.: *PACHEC Proceeding*, **2**, 340 (1983).
  - Park, J. C.: "Estimation of Multicomponent Phase Surface by Means of Response Surface Methodology and Comparison of Fitting for 3 & 4 Component Systems", Kyung Hee Univ. Publication, Seoul, Korea (1987).
  - Park, J. C.: "Representation of Multiple Phase Response Surface by Means of Computer Graphics", Kyung Hee Univ. Publication, Seoul, Korea (1987).
  - Ha, D. M.: Thesis, Kyung Hee University, Seoul, Korea (1989).
  - Kim, M. G.: Thesis, Kyung Hee University, Seoul, Korea (1989).
  - Park, J. C., Ha, D. M. and Kim, M. G.: APCChE Congress, Kuala Lumpur, Malaysia (1990).
  - Park, J. C.: KICHe Annual Meeting, Seoul, Korea (1991).
  - Park, J. C.: 68th Annual Meeting of the Korean Chemical Society, Changwon, Korea (1991).
  - Cheuh, P. L. and Prausnitz, J. M.: *AIChE J.*, **13**(6), 1107 (1967).
  - Park, J. C.: The 183rd Meeting of European Federation of Chem. Engineers at Helsinki, Finland, June (1977).
  - Park, J. C.: Theses Collection, Kyung Hee Univ., 15, 239, Seoul, Korea (1986).
  - Park, J. C. and Kim, M. G.: *HWAHAK KONGHAK*, **26**, 194 (1988).
  - Park, J. C.: "Mandang, Park, Jong-Chul, Ph.D., Full Professor, 60th Birthday Anniversary Theses Collection", Munyangsa, Seoul, Korea (1988).
  - Ha, D. M. and Park, J. C.: *HWAHAK KONGHAK*, **29**, 284 (1991).
  - Park, J. C.: "Mandang, Park, Jong-Chul, Ph.D., Honorable Retirement Commemoration Thesis Collection", Munyangsa, Seoul, Korea (1992).

## APPENDIX

Summary of group-group parameters ( $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\epsilon$ ,  $\eta$ ;  $\alpha'$ ,  $\beta'$ ,  $\gamma'$ ,  $\delta'$ ,  $\epsilon'$ ,  $\eta'$ ;  $\alpha''$ ,  $\beta''$ ,  $\gamma''$ ,  $\delta''$ ,  $\epsilon''$ ,  $\eta''$ ) for liquid phase and vapor phase of each binary interaction parameters  $A_{12}$ ,  $B_{12}$  and  $C_{12}$  for the selected 13 group-groups [2-5, 9].

	Liquid phase						Vapor phase					
		$A_{12}$		$B_{12}$		$C_{12}$		$A_{12}$		$B_{12}$		$C_{12}$
Alcohol-	$\alpha$	-0.560006	$\alpha'$	0.389042	$\alpha''$	-1.194521	$\alpha$	0.119848	$\alpha'$	-0.472245	$\alpha''$	5.671192
Paraffin	$\beta$	-0.000144	$\beta'$	0.000029	$\beta''$	-0.000090	$\beta$	0.000120	$\beta'$	-0.000265	$\beta''$	0.002191
	$\gamma$	-0.078169	$\gamma'$	0.057533	$\gamma''$	-0.160644	$\gamma$	0.047971	$\gamma'$	-0.099602	$\gamma''$	1.020639
	$\delta$	6.741378	$\delta'$	-13.576128	$\delta''$	20.381796	$\delta$	-7.845495	$\delta'$	-3.292597	$\delta''$	-133.191886
	$\epsilon$	-55.817410	$\epsilon'$	497.626159	$\epsilon''$	-407.894259	$\epsilon$	309.901588	$\epsilon'$	178.583286	$\epsilon''$	2364.731487
	$\eta$	-271.671706	$\eta'$	-4559.943809	$\eta''$	3641.282525	$\eta$	-2491.91289	$\eta'$	-1065.202043	$\eta''$	-16428.47658
Alcohol	$\alpha$	-0.001258	$\alpha'$	5.241742	$\alpha''$	-3.276837	$\alpha$	-2.328973	$\alpha'$	8.401823	$\alpha''$	3.278223
Aromatic	$\beta$	-0.000405	$\beta'$	0.003159	$\beta''$	-0.002720	$\beta$	-0.001473	$\beta'$	0.004652	$\beta''$	0.002423
	$\gamma$	-0.011556	$\gamma'$	1.041554	$\gamma''$	-0.682469	$\gamma$	-0.449948	$\gamma'$	1.642821	$\gamma''$	0.672137
	$\delta$	-12.987846	$\delta'$	-88.723888	$\delta''$	38.812280	$\delta$	38.331885	$\delta'$	-155.735577	$\delta''$	-44.685984
	$\epsilon$	329.312188	$\epsilon'$	1613.740505	$\epsilon''$	-608.968938	$\epsilon$	-689.921990	$\epsilon'$	2423.973167	$\epsilon''$	542.250984
	$\eta$	-2717.096736	$\eta'$	-13500.390799	$\eta''$	6436.647152	$\eta$	6862.752454	$\eta'$	-15015.240829	$\eta''$	-4319.573902
Aromatic-	$\alpha$	6.043312	$\alpha'$	-2.014979	$\alpha''$	-2.716525	$\alpha$	5.497483	$\alpha'$	6.434403	$\alpha''$	2.079667
Halogen	$\beta$	0.004703	$\beta'$	-0.001376	$\beta''$	-0.002364	$\beta$	0.004220	$\beta'$	0.005212	$\beta''$	0.001392
	$\gamma$	1.241364	$\gamma'$	-0.405160	$\gamma''$	-0.566624	$\gamma$	1.126473	$\gamma'$	1.328898	$\gamma''$	0.418017
	$\delta$	-88.317003	$\delta'$	31.316354	$\delta''$	40.220833	$\delta$	-81.043505	$\delta'$	-94.080174	$\delta''$	-31.153216
	$\epsilon$	1048.150816	$\epsilon'$	-396.189621	$\epsilon''$	-531.572162	$\epsilon$	1012.224132	$\epsilon'$	1163.563061	$\epsilon''$	360.806335
	$\eta$	-5318.760411	$\eta'$	2117.753953	$\eta''$	2939.979662	$\eta$	-5050.559690	$\eta'$	-5835.203167	$\eta''$	-1522.471855
Aromatic-	$\alpha$	-23.270161	$\alpha'$	16.533978	$\alpha''$	-17.343981	$\alpha$	-22.739837	$\alpha'$	-17.133884	$\alpha''$	-17.073289
Ester	$\beta$	-0.022291	$\beta'$	0.015622	$\beta''$	-0.017335	$\beta$	-0.022014	$\beta'$	-0.016236	$\beta''$	-0.016392
	$\gamma$	-4.992249	$\gamma'$	3.536463	$\gamma''$	-3.760785	$\gamma$	-4.890405	$\gamma'$	-3.666853	$\gamma''$	-3.667330
	$\delta$	306.969041	$\delta'$	-221.220911	$\delta''$	217.955777	$\delta$	297.035083	$\delta'$	229.215147	$\delta''$	223.219430
	$\epsilon$	-3611.800910	$\epsilon'$	2646.717743	$\epsilon''$	-2368.432185	$\epsilon$	-3403.512190	$\epsilon'$	-2747.741690	$\epsilon''$	-2572.816911
	$\eta$	19068.714694	$\eta'$	-14303.307958	$\eta''$	11548.097784	$\eta$	17881.630114	$\eta'$	15193.774260	$\eta''$	13594.956758
Alcohol-	$\alpha$	-7.371867	$\alpha'$	121.016739	$\alpha''$	-162.119342	$\alpha$	-37.069008	$\alpha'$	11.402612	$\alpha''$	98.269519
Halogen	$\beta$	-0.003860	$\beta'$	0.073431	$\beta''$	-0.096927	$\beta$	-0.021658	$\beta'$	0.007139	$\beta''$	0.058388
	$\gamma$	-1.386557	$\gamma'$	23.501386	$\gamma''$	-31.384270	$\gamma$	-7.134088	$\gamma'$	2.241605	$\gamma''$	18.989363
	$\delta$	153.312734	$\delta'$	-2471.139685	$\delta''$	3339.032910	$\delta$	775.956775	$\delta'$	-222.674618	$\delta''$	-2056.853281
	$\epsilon$	-3068.151735	$\epsilon'$	51394.970039	$\epsilon''$	-69715.272320	$\epsilon$	-16539.062923	$\epsilon'$	4359.283865	$\epsilon''$	44105.025307
	$\eta$	24851.275632	$\eta'$	-482902.424325	$\eta''$	654287.064739	$\eta$	159342.604938	$\eta'$	-40869.445159	$\eta''$	-429040.213721
Alcohol-	$\alpha$	11.936197	$\alpha'$	-5.472907	$\alpha''$	16.372178	$\alpha$	12.142564	$\alpha'$	5.590163	$\alpha''$	-1.718955
Ester	$\beta$	0.003368	$\beta'$	-0.001669	$\beta''$	0.004524	$\beta$	0.003443	$\beta'$	0.001566	$\beta''$	-0.000610
	$\gamma$	2.100945	$\gamma'$	-0.971952	$\gamma''$	2.863188	$\gamma$	2.139050	$\gamma'$	0.979568	$\gamma''$	-0.319868
	$\delta$	-358.274894	$\delta'$	153.578868	$\delta''$	-495.472336	$\delta$	-362.246630	$\delta'$	-170.703730	$\delta''$	38.426639
	$\epsilon$	10640.159930	$\epsilon'$	-4055.253707	$\epsilon''$	14691.697118	$\epsilon$	10704.345212	$\epsilon'$	5210.972515	$\epsilon''$	-674.850816
	$\eta$	-132516.712695	$\eta'$	44070.891427	$\eta''$	-179470.870500	$\eta$	-130845.205050	$\eta'$	-67181.718649	$\eta''$	2459.435345

		Liquid phase						Vapor phase					
		A <sub>12</sub>		B <sub>12</sub>		C <sub>12</sub>		A <sub>12</sub>		B <sub>12</sub>		C <sub>12</sub>	
Ester-Halogen	α	-2.214814	α'	0.836089	α''	1.217449	α	-1.320981	α'	-0.832012	α''	-1.179817	
	β	-0.000511	β'	0.000191	β''	0.000248	β	-0.000346	β'	-0.000150	β''	-0.000239	
	γ	-0.420097	γ'	0.159467	γ''	0.224299	γ	-0.260319	γ'	-0.147326	γ''	-0.216898	
	δ	38.396000	δ'	-14.961493	δ''	-21.923779	δ	19.509103	δ'	16.750497	δ''	24.351437	
	ε	-544.833607	ε'	217.570626	ε''	263.874609	ε	-201.97854	ε'	-223.020515	ε''	-404.723337	
Ketone-Paraffin	η	2898.226246	η'	-1213.992292	η''	-1203.813023	η	1040.52807	η'	1310.520417	η''	2807.778648	
	α	2.252156	α'	1.202463	α''	6.399489	α	0.261475	α'	-16.333964	α''	44.662958	
	β	0.000786	β'	0.000268	β''	0.002240	β	0.000298	β'	-0.004345	β''	0.011604	
	γ	0.428306	γ'	0.205095	γ''	1.185395	γ	0.083826	γ'	-2.862595	γ''	7.789163	
	δ	-57.179896	δ'	-30.693728	δ''	-146.054589	δ	-4.427577	δ'	403.308153	δ''	-1119.335413	
Alcohol-Ketone	ε	1114.640602	ε'	816.292109	ε''	2581.079683	ε	37.198508	ε'	-8279.227263	ε''	22753.320020	
	η	-8534.764723	η'	-7119.005162	η''	-17474.242162	η	712.955805	η'	64370.835248	η''	-175615.978361	
	α	0.570674	α'	-1.833134	α''	1.728054	α	0.881896	α'	0.024608	α''	1.259059	
	β	0.000104	β'	-0.000465	β''	0.000347	β	0.000185	β'	0.000151	β''	0.000314	
	γ	0.103122	γ'	-0.319363	γ''	0.296216	γ	0.157403	γ'	0.015819	γ''	0.217526	
Acid-Water	δ	-17.804209	δ'	50.288269	δ''	-50.395503	δ	-25.927962	δ'	3.203899	δ''	-34.588183	
	ε	388.408640	ε'	-1122.188140	ε''	1216.362831	ε	623.537286	ε'	-184.753782	ε''	823.766202	
	η	-3538.697831	η'	10143.611867	η''	-11632.335726	η	-5198.595439	η'	2709.718961	η''	-8012.800671	
	α	17.213003	α'	-30.058261	α''	-25.647617	α	-35.356711	α'	26.205931	α''	-31.539656	
	β	-0.001068	β'	0.001872	β''	0.001603	β	0.002216	β'	-0.001640	β''	0.001970	
Acid-Acid	γ	2.243839	γ'	-3.933000	γ''	-3.356846	γ	-4.640853	γ'	3.434064	γ''	-4.136298	
	δ	-592.557989	δ'	1030.496043	δ''	875.119903	δ	1202.707804	δ'	-892.587002	δ''	1077.595168	
	ε	11008.257318	ε'	-19221.746816	ε''	-16320.029020	ε	-22387.453621	ε'	16569.883657	ε''	-20130.344115	
	η	-68646.758141	η'	120018.569963	η''	101096.732677	η	139427.899938	η'	-102716.817618	η''	124890.267751	
	α	-0.164828	α'	-0.489583	α''	0.357241	α	-0.074118	α'	-0.656170	α''	-0.438936	
Aromatic-Ketone	β	0.015102	β'	-0.068693	β''	-0.301961	β	-0.061686	β'	0.366780	β''	-0.950399	
	γ	0.333217	γ'	-1.594204	γ''	-6.412884	γ	-1.287837	γ'	7.787857	γ''	-20.516087	
	δ	53.224578	δ'	-176.650632	δ''	-775.269257	δ	-142.051653	δ'	957.617505	δ''	-2423.574711	
	ε	-1072.521760	ε'	3317.351808	ε''	13812.246524	ε	2462.809643	ε'	-17169.239659	ε''	43402.431497	
	η	6505.565775	η'	-18765.090126	η''	-74993.546648	η	-12811.672380	η'	93799.266957	η''	-235925.691570	
Aromatic-Ketone	α	1.957143	α'	-2.930690	α''	0.336688	α	3.609554	α'	3.313805	α''	-1.630760	
	β	0.000943	β'	-0.001496	β''	-0.000773	β	0.002007	β'	0.001148	β''	-0.001973	
	γ	0.453881	γ'	-0.692709	γ''	-0.108452	γ	0.888666	γ'	0.675400	γ''	-0.612586	
	δ	-21.141032	δ'	28.586822	δ''	-52.308759	δ	-23.036905	δ'	-50.965906	δ''	-39.872122	
Aromatic-Ketone	ε	225.354263	ε'	-323.467808	ε''	1209.078114	ε	81.920693	ε'	697.359058	ε''	1096.945208	
	η	-1491.198423	η'	2302.776844	η''	-8486.155588	η	62.988730	η'	-3906.035039	η''	-7317.377570	