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DYNAMIC OPERATING POLICIES FOR PERIODIC PROCESSES SUBJECT TO EQUIPMENT FAILURES

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Abstract—The role of storage in buffering a process train from the effects of periodically reoccuring equipment failure is studied. The effects of deterministic and stochastic variations in the failure frequency and recovery time on the required size of intermediate storage are investigated. A dynamic operating policy is proposed to accommodate process variations in concern with the use of intermediate storage. Dynamic operation offers an increase in the average production rate of the process while using only modest levels of intermediate storage.

Key words: *Intermediate Storage, Periodic Process, Equipment Failure, Dynamic Operation*

INTRODUCTION

Batch chemical plants are often operated using extended campaigns of repeated production of batches of the same product. As with any manufacturing system, such plants are subject to failures of process equipment and failures of batches of materials to satisfy product specifications [Buzacott and Hanifin, 1978; El-sayed and Turley, 1980]. Such failures can distrupt batch product schedules and can introduce additional cascading losses in production time. The propagation of the effects of such failures can be mitigated by judicious placement of intermediate storage tanks of appropriate capacity [Henley and Hoshino, 1977; Oi et al., 1979; Takamatsu et al., 1984]. However the frequency of failures and the time required to recover from them are in general stochastic variables. Thus, regardless of the amount of assigned storage, there will always be a non-zero probability that the regular operation of the process must be interrupted because storage tanks run dry or overflow.

In this paper the use of systematic adjustments in the processing rates of subtrains of process equipment to increase the probability of uninterrupted operation of the batch production schedule is studied. In previous work, Lee and Reklaitis developed relations for determining the storage requirements for the building block system (the 1-1 system) consisting of one process subtrain, a storage tank, followed by another process subtrain [Lee and Reklaitis, 1989a, 1989b]. This analysis relies on the representation of the 1-1 system as a periodic process, an approximation which is tenable under the campaigning strategy employed in many batch plants. In the batch failure and the equipment failure cases, both average and stochastic failure frequencies could be accommodated via this analysis. In the present work, a dynamic operating policy which employs adjustments in the processing rates of the process subtrains up- and down-stream of the storage tank is presented. It is shown that a simple policy involving operation of such a subtrain at two rates with well-defined switch times substantially increases the probability of uninterrupted operation. The results are illustrated with suitable examples.

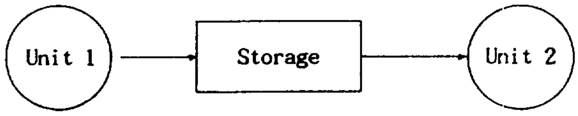

Fig. 1. 1-1 system.

PROCESS MODEL FOR 1-1 SYSTEM

1. Assumptions

A schematic diagram of the process under consideration is shown in Fig. 1. This system, denoted as a 1-1 system, consists of one upstream stage and one downstream stage, both employing batch/semicontinuous units. Let V_1 denote the upstream volume over the cycle time ω_1, and V_2 the downstream volume over the cycle time ω_2. The basic assumptions are as follows.

(1) Batch units operate with fixed batch sizes and cycle times

(2) In the normal case (without failure), the productivities of both stages are equal

(3) When flowrates into or out of the intermediate storage tank occur, they are at constant rates

(4) The required size of the storage tank is equal to the maximum hold up in the tank

(5) The frequency of failure (γ_i) and repair time of unit i (d_i) are fixed

(6) The first equipment failure (k_i^1) will occur at the γ_ith batch from time $t=0$.

(7) The greatest common measure of the cycle time of the hypothetical upstream and downstream flows is 1. i.e. GCM (ω_1^*, ω_2^*)$=1$.

2. Formulation

The cycle time of a batch unit is the sum of the filling time (t_f), processing time (t_p), discharge time (t_d) and cleaning time (t_c). For the 1-1 system, the fractional time of inflow to the storage tank is given by $x_1=t_d/\omega_1$ and the fractional time of outflow from the storage tank is given by $x_2=t_f/\omega_2$. Let the starting time of the inflow from the upstream unit be denoted as t_{1o} and the outflow to the downstream unit as t_{2o}. Furthermore, let y_1, y_2 denote the ratio of these times to their respective cycle times, that is,

$y_1 = t_{1o}/\omega_1$ and $y_2 = t_{2o}/\omega_2$.

In the general case involving N upstream and downstream batch units, the holdup volume of the intermediate storage tank can be expressed as follows [Karimi and Reklaitis, 1983].

$$V(t) = \sum_{i=1}^{N} \int_0^t c_i F_i(\tau - t_{io}) d\tau = V(0) + I(t)$$

$$= V(0) + \sum_{i=1}^{N} \left[c_i \left(\frac{V_i}{\omega_i} \right) t + \frac{1}{2} c_i U_i \omega_i h(u_i, y_i, z_i) \right] \quad (1)$$

where

$$h(u_i, y_i, z_i) = \sum_{n=1}^{\infty} \frac{1}{n^2 \Pi^2} [\cos 2n\Pi(u_i - z_i) - \cos 2n\Pi(z_i)$$

$$- \cos 2n\Pi(u_i - y_i) + \cos 2n\Pi y_i] \quad (2)$$

$$= \frac{1}{2} [\, |u_i - y_i| - |u_i - z_i| + (2u_i - 1)(y_i - z_i)] \quad (3)$$

where $u_i = \mathrm{mod}\left(\frac{t}{\omega_i}, 1\right)$, $z_i = \mathrm{mod}(x_i + y_i, 1)$

The intermediate storage tank holdup volume and required initial holdup volume will be given by

$$V^* = \max_t I(t) + V(0) \quad (4)$$

$$V(0) \geq - \min_t I(t)$$

HOLDUP FOR DETERMINISTIC CASE

1. Approach

For the 1-1 system with batch failure, assume that there exists a hypothetical downstream flow (the failure stream), in addition to the normal flow of material from the upstream and to the downstream units. Then, the sum of the upstream and failure streams will represent the actual composite upstream flow. In the batch failure mode, while failure in the train upstream of the storage tank will affect the required capacity of the storage tank, failures in the downstream train have no such effect. In the equipment failure case, let γ_1 denote the frequency of upstream unit failure and d_1 the repair time of the failed unit. Since d_1 is, in general, different from ω_1, the cycle time of the actual stream flow in the upstream unit is not the same throughout the campaign. Therefore nominal holdup volume expression given in (1) cannot be applied directly. Instead, assume that there exists a set of hypothetical upstream flows as shown in Fig. 2. The sum of these hypothetical upstream flows will represent the actual composite upstream flow. If denote by ω_i^* the cycle time of the hypothetical flow of unit i, by x_i^* the fractional time of inflow to the storage tank or outflow from the storage tank by the hypothetic flow of unit i. Then

$$\omega_i^* = (\gamma_i - 1)\ \omega_i + d_i \quad (5)$$

$$x_i^* = x_i \left(\frac{\omega_i}{\omega_i^*} \right) \quad (6)$$

For the 1-1 system, let i=1 refer to the upstream unit and i=2 refer to the downstream unit and superscript j refer to the j^{th} hypothetical flow. Let also denote by V_1^* the hypothetical stream volume over the cycle time ω_1^*. Then, from the given input and output flowrates U_1 and U_2,

$$V_1 = U_1\ \omega_1\ x_1 \quad (7)$$

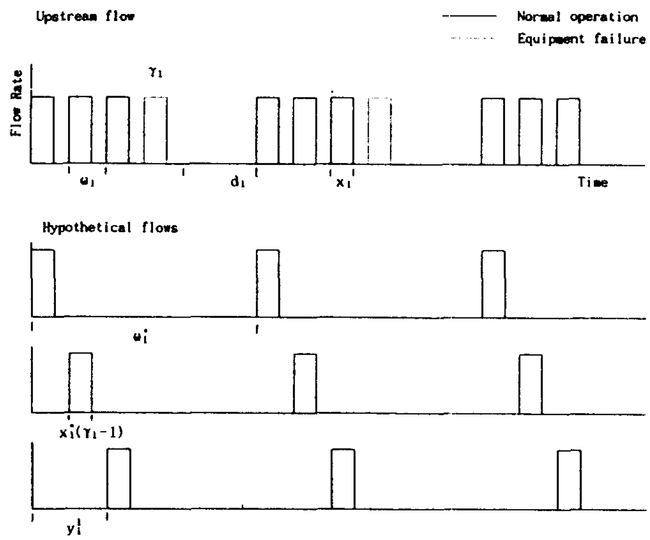

Fig. 2. Hypothetical flows.

$$V_2 = U_2\ \omega_2\ x_2 \quad (8)$$

$$V_1^* = U_1\ \omega_1^*\ x_1^* = V_1 \quad (9)$$

2. Throttling

From assumption 2, the productivity of the units will be equal under normal operation that is, $V_1/\omega_1 = V_2/\omega_2$. Then from (7) and (8), it follows that the average processing rate R will be given by

$$R = U_1 x_1 = U_2 x_2 \quad (10)$$

In the presence of process failures, a loss of the flow from the failed unit will cause a processing rate imbalance. In this case, we need throttling of the flow of the non-failed unit to compensate for the deficit in the flow from the failed unit. This can be accomplished by reducing the flow rate or reducing the filling fraction. For the upstream failure case, the downstream flow rate has to be throttled, while for the downstream equipment failure case, the upstream flow rate has to be throttled. For the combined up and downstream failure case, either or both of the upstream or downstream flow rates can be throttled. Let denote by $\bar{U}_i$ the throttled flow rate of i unit. Then the throttled flow and holdup volume expressions for the equipment failure case are as follows:

For upstream equipment failure,

$$R_u = \bar{U}_2 x_2 = U_1 x_1^* (\gamma_1 - 1) \quad (11)$$

$$I(t) = \frac{U_1 \omega_1^*}{2} \sum_{j=0}^{\gamma_2 - 2} h(u_1^j, y_1^j, z_1^j) - \frac{\bar{U}_2 \omega_2}{2} h(u_2, y_2, z_2) \quad (12)$$

For downstream equipment failure,

$$R_d = \bar{U}_1 x_1 = U_2 (\gamma_2 - 1) x_2^* \quad (13)$$

$$I(t) = \frac{\bar{U}_1 \omega_1}{2} h(u_1, y_1, z_1) - \frac{U_2 \omega_2^*}{2} \sum_{j=0}^{\gamma_2 - 2} h(u_2^j, y_2^j, z_2^j) \quad (14)$$

For the combined equipment failure case,

$$R_c = \bar{U}_1 x_1^* (\gamma_1 - 1) = \bar{U}_2 x_2^* (\gamma_2 - 1) \quad (15)$$

$$I(t) = \frac{\bar{U}_1 \omega_1^*}{2} \sum_{j=0}^{\gamma_1 - 2} h(u_1^j, y_1^j, z_1^j) - \frac{\bar{U}_2 \omega_2^*}{2} \sum_{j=0}^{\gamma_2 - 2} h(u_2^j, y_2^j, z_2^j) \quad (16)$$

(For details, refer to [Lee and Reklaitis, 1989b; Lee, 1988])

3. Minimum and Maximum Holdup

For the case $U_1 \geq U_2$, the increases or decreases in the holdup volume are determined by the beginning and ending times of flow of the two units. That is, I(t) is a piecewise linear function with multiple local optimal all of which are corner points. Also the hold up in the intermediate storage tank, V(t) is a periodic function with period $\Omega = LCM(\omega_1^*, \omega_2^*)$. As a consequence of this periodicity, the times at which local maximum and minimum values of the holdup volume can occur can be expressed as follows,

$$t_{max} = \alpha_1 \omega_i^* + \alpha_2 \omega_i + x_i^* \omega_i^*$$
$$t_{min} = \alpha_1 \omega_i^* + \alpha_2 \omega_i$$

where α_1 and α_2: integers

Then from the given value of the cycle time, the frequency of failure, the repair time, the local optimal time expressions and the underlying restricting of the search domains, the analytical expressions of global maximum and minimum holdup can be derived [Lee and Reklaitis, 1989b].

4. Time Delay Effect

From the analytical solutions, the following properties of the hold up volume can be established in terms of the variables y_2 and k_1

(1) V^* is a periodic function with respect to y_2 with period $1/\omega_2$.

(2) The first equipment failure variable k_1 does not affect the size of intermediate storage.

(3) The effect of y_2 on holdup volume is the same as in the case analyzed by Karimi et al. [1985], that is, $V^*(y_2)$ takes on its maximum value at $y_2 = 1 - x_2 \pm ip$ and its minimum value at

$$y_2 = (1 - x_2) - p(1 - x_2) \pm ip \quad \text{when } U_1 \geq \bar{U}_2$$
$$y_2 = (1 - x_2) - p(1 - x_1) \pm ip \quad \text{when } U_1 < \bar{U}_2$$

DYNAMIC OPERATION

1. Process Variations

In actual process operations, fluctuations can arise in the various process parameters. For purpose of this work, confine our attention to the failure frequency and repair time variations and classify the failure parameter fluctuations into two cases: time penalty and no time penalty. No time penalty variations are said to occur when the frequency of failure is changed but its repair time is maintained as scheduled. On the other hand, time penalty variations will involve perturbation of both the frequency of failure and the repair time. In both cases, the variations are assumed to be temporary and non periodic, that is, variations which are introduced at any given time are not permanent. The process is assumed to return to its nominal values after each variation from nominal behavior has occurred.

1-1. Variation with No Time Penalty

Recall that under this type of variation, failure occurs not with the batch corresponding to the mean failure frequency γ_i but with some other batch. Moreover, it is assumed that the repair time (d_i) does not change. Let denote by Δk_i^k the amount of the k^{th} failure frequency deviation of unit i. Let n failure frequency deviations be represented by $\Delta k_i^n = \sum_{k=1}^{n} \Delta k_i^k$ and $k_i^n = k_i + \Delta k_i^n$. As noted in previous section, these deviations do not affect the size of intermediate storage tank but only affect the initial holdup by the amount $U_i x_i^* d_i \Delta k_i^n$ (See Appendix A). Then the hold up $V_n(t)$ after completion of the n^{th} deviation becomes,

$$V_n(t) = V_n(0) + I(t) \tag{17}$$

where $V_n(0) = V_0(0) + U_i x_i^* d_i \Delta k_i^n$ (18)

If denote by V_{min}^n the minimum holdup volume after n failure deviations, then $V_n(0) + V_{min}^n \geq 0$. As $V_{min}^n = V_{min} + U_i x_i^* d_i \Delta k_i^n$, the initial holdup must satisfy

$$V_n(0) \geq -V_{min} - U_i x_i^* d_i \Delta k_i^n \tag{19}$$

From (19), it can be found that the longer the delay of failure, the less initial hold up is required due to the volume accumulation term. If the failure delay is extended beyond a certain time period, overflow of the tank will occur. In that case either upstream processing must be interrupted or else a dynamic operation strategy is needed as explained in detail in a later section.

1-2. Variation with Time Penalty

In the more general case, the repair time may undergo variation with each occurrence of equipment failure. As with the failure frequency deviation, denote by Δd_i^k as the amount of the k^{th} repair time variation of unit i. Then $\Delta d_i^n = \sum_{k=1}^{n} \Delta d_i^k$ and $d_i^n = t_{io} + \Delta d_i^n$. For the combined up and downstream failure case, the holdup profile after the completion of the repair time variation $[V_n(t)]$ can be obtained by rearranging $V_n(t)$ and shifting the time origin to d_1^n resulting in the following expressions:

$$V_n(t) = V_n(0) + I(t)$$

where

$$V_n(0) = V_0(0) + U_1 x_1^* d_1 \Delta k_1^n - U_2 x_2^* d_2 \Delta k_2^n + \sum_{j=0}^{\gamma_i - 2} \int_0^{\Delta d_1^n - t_{2o}^j} c_2 F_2(\tau) d\tau \tag{20}$$

2. Static Operation

There are two modes of operation which can be used to absorb the process parameter variations, namely, static operation and dynamic operation. The static mode involves operation without operator intervention, that is, the flow rates are maintained as scheduled through the whole campaign in spite of the failure parameter variations. In this case, the intermediate storage volume also must suffice to absorb all of the effect of the parameter deviations. First examine the required size of the storage tank for this mode of operation for the 1-1 system. Denote by ΔV_1^n and ΔV_2^n the holdup deviation after n upstream and downstream unit deviations, respectively. Then

$$\Delta V_1^n = U_1 x_1^* d_1 \Delta k_1^n \tag{21}$$

$$\Delta V_2^n = U_2 x_2^* d_2 \Delta k_2^n \tag{22}$$

And the net volume change ΔV^n becomes;

$$\Delta V^n = \Delta V_2^n - \Delta V_1^n \tag{23}$$

The first important observation which can be made is that the effects of an advance (equipment failure occurs earlier than scheduled) and of a delay (equipment failure occurs later than scheduled) on intermediate storage tank are quite different. Advance deviations do affect the initial holdup requirement and, in turn, the intermediate storage size. However, delay deviations only affect the required intermediate storage size. Let assume that l delay deviations and m advance deviations occur during operation. If we seek the minimum initial holdup requirement, $V(0) = -V_{min}$, then, to absorb these deviations, the intermediate storage tank size and the initial holdup requirement have to be increased as follows;

$$V_{lm}^* = V^* + \Delta V^l + V(0) \tag{24}$$

$$V(0) = -V_{min} - \Delta V^m \quad (25)$$

For instance, for the upstream equipment failure case, the quantities ΔV^l and ΔV^m will be,

$$\Delta V^l = U_1 x_1^* d_1 l \quad (26)$$

$$\Delta V^m = U_1 x_1^* d_1 m \quad (27)$$

Under static operation, the process may never actually need to use the full capacity of the downstream unit due to the throttled flow rate. Moreover, a considerable increase in the intermediate storage volume is required. By applying a dynamic operating policy, we can avoid the volume increase and also can increase the average production rate.

3. Dynamic Operation

Suppose that the intermediate storage tank size is chosen to be large enough to accommodate l delay and m advance deviations. If delay deviations in excess of the selected design value actually take place, then volume accumulation will occur and eventually the tank will overflow. In this case, either the intermediate storage tank size must be increased or the downstream flow rate must be increased to prevent overflow. On the other hand, if advance deviations in excess of the design limit occur, then the initial holdup requirements will be larger than that provided and thus the intermediate storage tank will run dry sometime during operation. To prevent this from occurring, either more initial holdup must be provided or the downstream flow rate must be reduced. In other words, instead of increasing the intermediate storage tank size, variations in failure parameters which are beyond design limits can be absorbed by manipulating the downstream flow rate. This strategy let call dynamic operation.

3-1. Operating Mode Change

As noted earlier, there are three equipment failure cases of interest: upstream failure, downstream failure and combined failure. The average processing rate of each of these operating modes is given by Eqs. (11), (13) and (15) respectively. Compared to the average processing rate in the nominal operating mode, given by (10), the average processing rate of the combined case is the lowest while the other two cases are intermediate. The main object of using dynamic operation is to absorb the failure rate fluctuations without increasing intermediate storage capacity and also to increase the average processing rate to approach the nominal value. This can be done by altering the operation as follows. To improve the performance of the combined case to approach that of the upstream failure case, the output of the system has to be allowed to be uninterrupted. This can be accomplished either by providing extra capacity in the downstream subtrain or by operating without downstream equipment failure. Improvements in the combined case performance to approach that of the downstream failure case can be achieved by applying a policy similar to that described for the upstream unit case. Another possible way is to delay equipment failure. Operating the unit without failure is equivalent to operating with delay of equipment failure until the production campaign is finished. Therefore, delay of equipment failure for a certain time period is equivalent to a shifting of the performance from the combined case to the upstream or downstream failure modes for that period. In summary, in order to improve the performance of combined failure mode to approach that of the upstream failure mode, four operating policies can be used:

(1) Maintain the upstream equipment without failure (that is, achieve normal operation)

(2) Increase the initial holdup (static operation)

(3) Additionally throttle the downstream flow rate (dynamic operation)

(4) Increase the upstream flow rate (dynamic operation)

Among these operating policies, policy 1 obviously is the best. Under policy 2, we have to increase the intermediate storage tank size as shown earlier. In the case of policy 3, an increase in the intermediate storage tank size to absorb the failure fluctuations is not needed, moreover, the average processing rate is not increased in spite of the delay of downstream equipment failure. The last policy will increase the average production rate without increasing intermediate storage tank size and thus is the preferred dynamic operating policy.

3-2. Flow Rate Change

If the flow rate difference between normal operation and operation under failure is $\Delta U_i = U_i - \bar{U}_i$, then for a delay in the failure occurrence beyond the design allowance, the flow rate must be increased from $\bar{U}_i$ to U_i. For an advance in failure occurrence, the flow rate must be decreased from $\bar{U}_i$ to $\bar{U}_i - \Delta U_i$, that is the flow rate becomes $2\bar{U}_i - U_i$. Thus, in the upstream failure and the downstream failure cases, it is needed to switch the upstream and the downstream flow rates from throttled flow rate to normal flow rate according to the failure mode. For the combined case, from (15), either the upstream flow rate or the downstream flow rate can be throttled. Then from (15) with (5), the basic throttling policy is to throttle the downstream flow rate ($\bar{U}_2$) and maintain normal flow rate for upstream flow (U_1) if $d_1/(\gamma_1 - 1)\omega_1 \geq d_2/(\gamma_2 - 1)\omega_2$. Otherwise, the upstream flow rate must be throttled. Typically, for the combined case, flow rates have to be increased beyond the normal flow rate when applying the dynamic operation for delay of failure (See Appendix B). For the 1-1 system, this does not affect the system design. However, in the more general serial case, such increases in flow rates beyond the normal value will require a corresponding increase in the size of the downstream batch units. Such increased capacity requirements may serve as a useful upper design limit for these process units.

3-3. Recovery Time (δ)

When applying the dynamic operating policy, the volume change due to process failure parameter deviations can be absorbed by changing flow rates. In general, the volume change is not absorbed instantly rather it requires some recovery time (δ). For upstream failure case, the holdup volume change due to a failure delay of one cycle is $\Delta V = U_1 x_1^* d_1$. The rate at which this extra holdup will be removed under dynamic operation is $(U_2 - \bar{U}_2)$, where U_2 and $\bar{U}_2$ are given at Eqs. (10) and (11), respectively. The time δ required to dissipate ΔV will therefore be given by

$$\delta = \frac{\Delta V}{U_2 - \bar{U}_2} = \frac{U_1 x_1^* d_1}{U_1 x_1 d_1 / x_2 \omega_1^*} = \omega_1 x_2$$

The recovery times for the downstream failure and combined cases with upstream deviation and downstream deviation can be obtain in an analogous fashion. The expressions are summarized in Table 1.

3-4. Application to Failure Delay Case

Next consider how dynamic operation will affect the intermediate storage size when upstream equipment failure is delayed by one batch cycle in the purely upstream failure case. In this case, suppose the downstream flow rate is switched from $\bar{U}_2$ to

Table 1. Recovery time

Failure mode	Recovery time
Upstream failure	$\omega_1 x_2$
Downstream failure	$\omega_2 x_1$
Combined case	
Upstream deviation	$\omega_1 x_2^*(\gamma_2 - 1)$
Downstream deviation	$\omega_2 x_1^*(\gamma_1 - 1)$

U_2 for an interval $\delta = \omega_1 x_2$. If the failure is delayed by only one batch cycle and then operation returns to normal, after applying the dynamic operating policy for the δ interval, the overall system operation will correspond to the deterministic case. First examine how the recovery time can be accommodated within the period of the holdup of function, $\Omega = \mathrm{LCM}(\omega_1, \omega_2)$. The total number of downstream batches processed within this period is Ω/ω_2 and the flow time to the downstream unit in each cycle is $\omega_1 x_2$. Therefore, the total downstream flow time during Ω becomes $(\Omega/\omega_2)\omega_2 x_2 = \Omega x_2$. This also can be expressed as $(\Omega/\omega_1)\omega_1 x_2$ where Ω/ω_1 refers to the total number of upstream batches within Ω and $\omega_1 x_2$ refers to the total flow time of the downstream unit. Thus, the mean flow time of the downstream unit within upstream cycle time ω_1 is $\omega_1 x_2$. Let t_2 denote a particular instance of the flow time of the downstream unit over one upstream cycle. Denote by x_2' the fractional flow time of the downstream unit over one upstream cycle. Then $x_2' = t_2/\omega_1$ and, as the mean fractional flow of downstream unit within a ω_1 interval is x_2, it follows that,

$$\frac{\sum_{\Omega} (n x_2')}{n} = x_2$$

where n is the number of upstream batches within the period Ω. In other words, use of the increased flow rate for one cycle of upstream flow is enough to absorb the volume deviation. Moreover, consecutive delays will also be compensated for one at a time. For example, assume that two delays occur consecutively. When the first delay occurs, apply the dynamic policy and the associated volume deviation will be compensated within the first cycle providing $x_2' \geq x_2$. If the second deviation occurs right after the first deviation, the volume change for the first deviation will have already been accommodated and therefore no additional storage volume is needed for the second deviation. Again, by applying the dynamic policy, the second deviation will be compensated for and the volume deviation becomes the same as in the deterministic case. Thus the following proposition can be derived:

Proposition I :

Assuming $x_2' \geq x_2$, the effects of a failure delay of one cycle time will be compensated in one cycle time by setting

$\bar{U}_2 = U_2$ for the upstream failure case

$\bar{U}_1 = U_1$ for the downstream failure case

As a result, for the case $x_2' \geq x_2$ the provision of additional storage for a delay of one cycle time is enough to cover all other delays.

For the case $x_2' < x_2$, we need a time period of $\omega_1(x_2 - x_2')$ beyond the next cycle time to compensate for the deviation. Let n' denote the number of batches for which $x_2' < x_2$ and let $\Delta x_2' = x_2 - x_2'$. Then the n' and $\Delta x_2'$ values can be selected via worst case analysis.

Proposition II :

For the case $x_2' < x_2$, the effects of a delay of one cycle will be compensated for in one cycle plus $\omega_1 \Delta x_2'$ and will require an adjustment in V^*

If apply the dynamic operation policy for delay deviation only, the intermediate storage tank size and the initial holdup requirements become;

$$V_{lm}^* = V^* + U_1 x_1^* d_1 (1 + \sum_{n'} \omega_1 \Delta x_2') + V(0) \tag{28}$$

$$V(0) = -V_{min} - \Delta V^m \tag{29}$$

From Eq. (24) to (29), the dynamic operating policy has the following advantages.

(1) Intermediate storage size is smaller than in the static case.

(2) It is applicable to any pattern of delay deviations.

(3) If the dynamic policy is used for 1 delays, the cumulative production will increase by as much as $(U_2 - \bar{U}_2)\omega_1 x_2 l$.

3-5. Application to Combined Delay and Advance Case

The effect of dynamic operation on advances in failures is different from the delay case. Failures which occur earlier then γ_i batches can not be compensated for in a consecutive fashion. If an advance occurs by as much as m_1', it has to be provided enough initial holdup to absorb that deviation. However by applying the dynamic policy, it can be compensated for the cummulative additional initial holdup requirement due to a series of advances failure. Under such policy, after completion of the compensation for each advance, further advances can be accommodated providing that the next advance is of less than m_1' cycles.

Proposition III :

Effects of consecutive advances can be accommodated by designing intermediate storage for the maximum advance in the failure occurrence measured in number of cycles and applying the dynamic policy.

When applying the dynamic policy for the combined delay and advance case, the intermediate storage tank size and the initial holdup requirement become;

$$V_{lm}^* = V^* + U_1 x_1^* d_1 (1 + \sum_{n'} \omega_1 \Delta x_2') + V(0) \tag{30}$$

$$V(0) = -V_{min} - U_1 x_1^* d_1 (m_1' + \sum_{n''} \omega_1 \Delta x_2') \tag{31}$$

where m_1' is the maximum allowable number of cycles of advance deviation. The selection of n'' and $\Delta x_2'$ is the same as in the delay case. By applying the dynamic policy to advances, the cumulative effects of advance deviations ($m_1' < m$) can be avoided and, in turn, the required initial holdup is reduced. If 1 delay deviations and m of advance deviations have occurred, the net change in the cumulative production will be $(U_2 - \bar{U}_2)\omega_1 x_2 (n - m)$.

For a deviation in the downstream failure frequency, a delay deviation will affect the initial holdup and an advance deviation will affect the storage size. Let m_2' denote the allowable advance deviation of the downstream unit. Then the storage tank size and the initial holdup required to absorb the downstream equipment deviation become,

$$V_{lm}^* = V^* + U_2 x_2^* d_2 (m_2' + \sum_{n} \omega_2 \Delta x_1') + V(0)$$

$$V(0) = -V_{min} - U_2 x_2^* d_2 (1 + \sum_{n} \omega_2 \Delta x_1')$$

For the combined case, to accommodate the upstream and the downstream deviations via dynamic operation,

$$V_{lm}^* = V^* + \Delta V_{lm} + V(0) \tag{32}$$

$$V(0) = -V_{min} - \Delta V(0) \tag{33}$$

Where

$$\Delta V_{lm}=\max[U_1x_1{}^*d_1(1+\sum_{n'}\omega_1\Delta x_2'),\ U_2x_2{}^*d_2(m_2'+\sum_{n'}\omega_2\Delta x_1')]$$

$$\Delta V(0)=\max[U_1x_1{}^*d_1(m_1'+\sum_{n''}\omega_1\Delta x_2'),\ U_2x_2{}^*d_2(1+\sum_{n''}\omega_2\Delta x_1')]$$

3-6. Stochastic Variation

If the failure frequency variations and the repair time variations are described by stochastic distributions, then the $V_n(t)$ relation must incorporate these stochastic models. Assume that failure and repair time variations are described by normal distribution functions, in which X_1 represents the equipment failure variation variable and X_2 the repair time variation variable. Clearly, the sum of the equipment failure and repair time variations will also follow normal distribution functions. Suppose that the failure variations and the repair time variations are normally distributed random variables with mean zero and standard deviations $\sigma_{\Delta k_1^n}$ and $\sigma_{\Delta d_1^n}$, respectively. Given these values and the desired probability of uninterrupted operation, P_{op}, the allowable lower and upper bounds of Δk_1^n and Δd_1^n can be established via the conditions:

$$P_r[|X_1|\le\eta(Z)]=P_{op}$$
$$P_r[|X_2|\le\overline{\eta}(Z)]=P_{op}$$

From normal distribution tables, it can be readily obtained:

$$Z_{\Delta k_1^n}=\frac{|X_1|}{\sigma_{\Delta k_1^n}}$$

As Δk_1^n is an integer variable, pick the largest adjacent integer value of $|X_1|$ and readjust the P_{op} value, according to the chosen Δk_1^n value, to say, $\overline{P}_{op}$. Then

$$\Delta k_1^n=\sum_{k=1}^{n}(\Delta k_1^k)=\pm[|X_1|]$$

The allowable delay and advance deviations becomes;

$$l'=(k_1^n)_{upper}=k_1+[|X_1|] \tag{34}$$

$$m_1'=(k_1^n)_{lower}=k_1-[|X_1|] \tag{35}$$

Following the same procedure as with the $\overline{P}_{op}$ value, the upper and lower limits on (Δd_1^n) becomes;

$$\Delta d_1^n=\sum_{k=1}^{n}(\Delta d_1^k)=\pm[|X_2|]$$

$$(d_1^n)_{upper}=+|X_2| \tag{36}$$

$$(d_1^n)_{lower}=-|X_2| \tag{37}$$

Thus, Eqs. (34) to (37) with (20) and (32), (33), can be used to obtain the intermediate storage size and the initial holdup requirement for dynamic operation and to compare these with static operation via a worst case analysis.

EXAMPLE

1. 1-1 system

Assume that for the upstream unit, the time for filling with raw material is 2 hr, the processing time is 6 hr and the discharge time is 1 hr. Discharge from storage to the downstream unit starts 1.5hr after discharge from the upstream unit. Assume it takes 4 hr for discharge to the downstream unit and after a 1 hr interval, discharge will start again. Suppose that the upstream unit will fail on average every 8th batch and it will take 15 hr to repair that unit. For the downstream unit, the equipment will fail on average every 7th batch and it will take 7 hr to repair. Finally suppose the flow rate of the upstream batch transfer pump will be 1500 kg/hr. Then, from the given data, $B_1=1500$ kg, $\omega_1=9$ hr, $\omega_2=5$ hr, $y_2=0.3$, $x_1=1/9$, $x_2=0.8$. If there is no failure, the flow rate of the downstream unit will become $U_2=U_1(x_1/x_2)=208$ kg/hr. Then the size of the intermediate storage tank and initial holdup requirements for various operation modes can be obtained from the equations given in previous section. The results are given in Table 2.

Table 2. Intermediate storage size for various operating modes

	V^*	V(0)	Flow rates
Normal operation	1,458	0	$U_2=208$
Batch failure	2,589	0	$\overline{U}_2=182$
Upstream failure	3,200	0	$\overline{U}_2=168$
Downstream failure	2,100	840	$\overline{U}_1=1,216$
Combined failure	4,100	840	$\overline{U}_2=208$

2. Dynamic Operation

Suppose that in the upstream failure case, failure is delayed as much as 5 batches. To absorb this deviation under static operation, the intermediate storage volume must be increased by $U_1x_1{}^*d_1l=1440$ kg. Alternatively, the delay can be accommodated by using the dynamic strategy as follows. When equipment failure does not occur with the 7th batch, change the downstream flow rate from the throttled flow rate ($\overline{U}_2=168$ kg/hr) to the normal flow rate ($U_2=208$ kg) until failure occurs. After the next failure occurs, return to operation with the throttled flow rate. In this case, via worst case analysis, $\sum_{n}\omega_1\Delta x_2'=0.8$ and the additional storage volume increase will only be $U_1x_1{}^*d_1(1+\sum_{n'}\omega_1\Delta x_2')=520$ kg, which is one third of that required in the static case. Moreover, with this switching strategy we can absorb all succeeding delay variations and can also increase the average production rate by as much as 23%. The total production increase due to dynamic operation becomes $(U_2-\overline{U}_2)w_1x_2l=1440$ kg.

3. Stochastic Model

Suppose that, from operating experience, the standard deviations of the equipment failure and the repair time are 0.48 and 0.23 respectively. Moreover, suppose that these variables are normally distributed with mean zero. We would like to design the storage volume for operation with 98% confidence that $\Delta k_1{}^n$ and $\Delta d_1{}^n$ deviations can be accommodated. Using the given variances, it can be found from standard normal distribution tables that:

$$P_r(-1\le\Delta k_1{}^n\le1)=0.98$$
$$P_r(-0.536\le\Delta d_1{}^n\le0.536)=0.98$$

Therefore, the lower bounds become $(k_1{}^n)_L=k_1-1=7$ and $(d_1{}^n)_L=t_{2o}+\Delta d_1{}^n=0.964$ while the upper bounds are $(k_1{}^n)_U=k_1+1=9$ and $(d_1{}^n)_U=2.036$. For the upstream failure mode, the maximum size of storage tank will occur when $x_2+y_2=1.0$. As $x_2=0.8$, the maximum size will occur for $d_1{}^n=1.0$. Also from the worst case analysis, $\sum_{n}\omega_1\Delta x_2'=0.8$ and $\sum_{n''}\omega_1\Delta x_2'=0$. Then from Eq. (30) and (31), with $l'=7$ and $m'=1$, the tank volume and initial holdup become $V^*=4030$ kg and $V(0)=318$ kg.

Let compare the dynamic operation policy with a fixed time horizon to static operation. From the given standard deviation of failure variations, it can be found from standard normal distribution tables that $P_r(|\Delta k_1{}^n|\le0.1\ \gamma)=0.95$. i.e. batch failure variation $\Delta k_1{}^n$ will vary within 10% of γ with probability 95%. Then for a fixed time horizon let assume that in the worst case, a delay of batch failure will occur with every failed batch by as much as 10% of γ. As 20 batches will fail during the two months of operation, $\Delta k_1^{20}=16$. The tank volume and initial holdup with no time penalty become $V^*=7800$ kg and $V(0)=4600$ kg. Thus, the required intermediate storage volume is increased by almost a factor of 5 compared with normal operation and by a factor of 2 compared with upstream equipment failure when the dynamic operating policy is employed.

CONCLUSION

The analysis of the effects of variations in process parameters was performed for two cases: that in which the failed equipment resume operation after nominal fixed recovery time and the case in which the recovery time period is variable. It was shown that, in the former case, variations in the failure occurrence frequency have no effect on storage size but only affect the initial holdup while, in the latter case, both are affected. For the stochastic case with normally distributed variations in the failure frequency and recovery times, estimates were developed for the volume required for a selected confidence level of satisfactory operation. The dynamic operating policy proposed, allowed storage size to be selected less conservatively by exploiting the partial control of storage accumulation afforded by the adjustment in the processing rate of the upstream or downstream unit.

APPENDICIES

Appendix A; Initial holdup change

Typically, for upstream failure, from Eqs. (5), (6) and (11), the throttled downstream flow rate will be,

$$\bar{U}_2=U_1\frac{x_1}{x_2}\left[\frac{1}{1+\frac{d_1}{(\gamma_1-1)\omega_1}}\right]$$

For normal operation, the flow rate is given by (10). Then, the volume accumulation due to upstream equipment failure deviation becomes,

$$\Delta V=(U_2-\bar{U}_2)x_2t=U_1\frac{x_1}{x_2}\left[\frac{d_1}{(\gamma_1-1)\omega_1+d_1}\right]x_2t$$

For a deviation of Δk_1^n cycles, $t=\omega_1\Delta k_1^n$. Then

$$\Delta V=U_1\omega_1x_1\frac{d_1}{(\gamma_1-1)\omega_1+d_1}\Delta k_1^n=U_1x_1^*d_1\Delta k_1^n$$

Appendix B; Flow rate change for the combined case

From Eqs. (15) with (5), the flow rates for the combined case become;

$$\bar{U}_1x_1=\bar{U}_2x_2\left[\frac{1+\frac{d_1}{(\gamma_1-1)\omega_1}}{1+\frac{d_2}{(\gamma_2-1)\omega_2}}\right]$$

Then, for the $\frac{d_1}{(\gamma_1-1)}\leq\frac{d_2}{(\gamma_2-1)\omega_2}$, take $\bar{U}_2=U_2$ and throttle the upstream flowrate. Thus,

$$\bar{U}_1=U_2\frac{x_2^*(\gamma_2-1)}{x_1^*(\gamma_1-1)}$$

To apply the dynamic operating policy, the new downstream flow rate becomes [refer to Eq. (13)];

$$U_2'=\bar{U}_1x_1\frac{1}{x_2^*(\gamma_2-1)}$$

Therefore using the $\bar{U}_1$ expressions given above, the new flow rate for use under dynamic operation becomes;

$$U_2'=U_2\frac{x_1}{x_1^*(\gamma_1-1)}$$

Note that as $x_1\geq x_1^*(\gamma_1-1)$, U_2' is greater than normal flow rate.

NOMENCLATURE

B_i : Batch size of unit i
d_i : repair time of unit i
$h(u_i, y_i, z_i)$: function defined in Eq. (2)
$I(t)$: function defined in Eq. (1)
k_1 : index of 1st batch failure flow in upstream flow
k_1^u : index of 1st equipment failure flow in upstream flow
l : number of batches of delay deviations
m : ω_2/ω_1, number of batches of advance deviations
m′ : allowable number of batches of advance deviations
n′ : number of batches which is $x_2'<x_2$ in delay deviations.
n″ : number of batches which is $x_2'<x_2$ in advance deviations.
P_{op} : probability of uninterrupted operation
t_{2i} : starting time of outflow to the downstream
U_i : input or output flow rate of a unit i
$\bar{U}_i$: throttled flow rate of a unit i
u_i : mod $(t/\omega_i, 1)$
u_i^j : variable of hypothetical j^{th} stream defined as mod $(t/\omega_i^*, 1)$
V_i : batch size of a unit i
V^* : intermediate storage size
$V^n(0)$: initial holdup required for n batch failure variation
$V_n(t)$: holdup profile after n^{th} batch failure variation defined in Eq. (17)
X_1 : batch failure variation variable
X_2 : cycle time variation variable
x_i : fractional flow time of unit i
x_i^* : fractional flow of hypothetical streams of unit i
x_2' : fractional flow time of downstream unit with respect to upstream cycle time
y_i : fractional delay time of a unit i
y_i^j : fractional delay time of hypothetical j^{th} stream for unit i
z_i : mod $(x_i+y_i, 1)$
z_i^j : variable of hypothetical j^{th} stream defined as mod $(x_i^*+y_i^j, 1)$

Greek Letters

γ_i : frequency of failure of unit i
ΔV^m : volume change after m deviations
δ : recovery time
ζ : γ-k_1
η : lower and upper bound of P_{op}
Ω : period of holdup variations
ω_i : cycle time of unit i
ω_i^* : cycle time of hypothetical streams of unit i

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