Short Communication

DATA RECONCILIATION FOR INPUT-OUTPUT MODELS IN LINEAR DYNAMIC SYSTEMS

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Abstract — Sequential data reconciliation algorithms have been developed for input-output models in linear dynamic systems. Existing filtering methods do not treat the case where there are measurement errors in the input variables. In our approach, the measurement errors in the input variables are optimally handled by the least squares method. This method shows good performance for input-output models.

INTRODUCTION

Data reconciliation has received considerable attention to resolve inconsistencies between plant measurements and balance equations. A number of commercial data reconciliation software packages are available for steady-state processes [Ayral, 1994]. In order to enhance robustness of the calculations, the data reconciliation step can be accompanied by gross error detection [Kim et al., 1995].

Most previous research on data reconciliaiton has focused on linear steady-state systems. During the last several years, there have been several approaches proposed for nonlinear dynamic systems. Data reconciliation for nonlinear dynamic systems was treated by Kim et al. [1991], Liebman et al. [1992] and Ramamurthi and Bequette [1993]. Darouach and Zasadzinski [1991], and Rollins and Devanathan [1993] suggested data reconciliation algorithms for linear dynamic systems. A nonlinear data reconciliation program has been applied to reconcile actual plant data from a feed-blend tank of an Exxon Chemicals plant [McBrayer, 1994]. In this paper we employ a model used in Darouach and Zasadzinski, which has different discrete equations of the form $EX_{k+1} = BX_k$. This is called a singular or generalized dynamic model, because the matrix E is singular and therefore the standard Kalman filter cannot be applied.

In this article, sequential data reconciliation algorithms are developed for input-output models in linear dynamic systems. Unlike other filtering methods, the measurement errors in the input variables are optimally handled by least squares.

PROBLEM STATEMENTS

1. Linear Difference Equation

Consider a dynamical system with input signal $\{u(t)\}$ and output signal $\{y(t)\}$. Suppose that these signals are sampled in discrete time $t = 1,2,3,\cdots$ and that the sampled values can be related through a linear difference equation. The general nth order difference equation relating the input $u(k)$ and output $y(k)$ is

$$y(k) + a_1 y(k-1) + \cdots + a_n y(k-n) = b_o u(k) + b_1 u(k-1) + \cdots + b_n u(k-n) \tag{1a}$$

$$\text{or } y(k) + \sum_{j=1}^{n} a_j y(k-j) = \sum_{j=0}^{n} b_j u(k-j) \tag{1b}$$

where k is the integer time index, and a_j and b_j are the constant coefficients.

The representation of Eq. (1) can be extended to systems that have multiple inputs and multiple outputs. Suppose there are m inputs and r outputs, and that the vectors U(k) and Y(k) are defined as

$$U(k) = \begin{bmatrix} u_1(k) \\ \cdot \\ \cdot \\ \cdot \\ u_m(k) \end{bmatrix} \qquad Y(k) = \begin{bmatrix} y_1(k) \\ \cdot \\ \cdot \\ \cdot \\ y_r(k) \end{bmatrix}$$

The system can then be represented by the vector difference equation

$$Y(k) + \sum_{j=1}^{n} A_j Y(k-j) = \sum_{j=0}^{n} B_j U(k-j) \tag{2}$$

in which A_j and B_j are constant coefficient matrices of dimension $r \times r$ and $r \times m$ respectively.

2. Data Reconciliation in Linear Dynamic Systems

Here we consider the problem of estimating the vector X_i at time instants $i = 1,2,\cdots,k+1$. From Eqs. (1) and (2) we can collect the $(k+1)$ measurements and the k constraints as follows [Darouach and Zasadzinski, 1991]:

$$Z = X^* + \varepsilon \tag{3}$$

$$\Phi_k X^* = 0 \tag{4}$$

where X^* is a vector of true values of the variables and ε is a vector of normally distributed random measurement noise with zero mean and known covariance matrix.

Using this notation, the dynamic data reconciliation problem can be formulated analogous to the steady-state case, that is,

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$$\text{Min}_{\hat{X}} \frac{1}{2}(\hat{X}-Z)^T V^{-1}(\hat{X}-Z) \tag{5}$$

subject to

$$\Phi_k \hat{X} = 0$$

where V is the variance-covariance matrix of measurements. The solution of this problem is given in many references [e.g., Darouach and Zasadzinski, 1991; Kuehn and Davidson, 1961]. For dynamic data reconciliation problems, a recursive solution based on the sequential method developed for steady-state case [Darouach et al., 1988] can be applied.

DERIVATION OF ESTIMATION ALGORITHMS

The dynamic data reconciliation problem can be solved using the steady-state sequential method [Darouach and Zasadzinski, 1991]. Their algorithms are applied to input-output models for linear dynamic systems.

1. First Order Model

$$y(k+1)+a_1y(k)=b_ou(k+1)+b_1u(k) \tag{6}$$

The above equation can be written

$$EX_{k+1}=BX_k \tag{7}$$

where

$$X_k = \begin{bmatrix} y(k) \\ u(k) \end{bmatrix}, \quad E=[1 -b_o] \quad \text{and} \quad B=[-a_1 \ b_1]$$

The reconciled estimates are calculated by the equations below.

$$\hat{X}_{k+1/k+1}=Z_{k+1}+VE^T\Omega_k(B\hat{X}_{k/k}-EZ_{k+1}) \tag{8a}$$

$$\hat{X}_{k/k+1}=\hat{X}_{k/k}-\Sigma_{kk}^k B^T\Omega_k(B\hat{X}_{k/k}-EZ_{k-1}) \tag{8b}$$

$$\Omega_k^{-1}=B\Sigma_{kk}^k B^T+EVE^T \tag{8c}$$

with

$$\Sigma_{kk}^k=V-VE^T\Omega_kEV \tag{8d}$$

$\hat{y}_{k+1/k-1}$ is the first element of $\hat{X}_{k+1/k+1}$ and $\hat{u}_{k+1/k+1}$ is the second element of $\hat{X}_{k+1/k+1}$. If $b_o=0$, $\hat{u}_{k+1/k+1}=u_{k+1}$ (measurement of u at k+1). Thus $y_{k+1/k+1}$ and $u_{k/k+1}$ will be updated at every time step. $\hat{u}_{k/k+1}$ can be calculated from the second element of $\hat{X}_{k/k+1}$. $\hat{y}_{k+1/k+1}$ is the output estimate at time instant k+1 given the measurements up to k+1 and $\hat{u}_{k/k+1}$ is the input estimate at time instant k given the measurements up to k+1.

2. Second Order Model

$$y(k+1)+a_1y(k)+a_2y(k-1)=b_ou(k+1)+b_1u(k)+b_2u(k-1) \tag{9}$$

The above equation can be written

$$EX_{k+1}=BX_k+AX_{k-1} \tag{10}$$

where

$$X_k=\begin{bmatrix} y(k) \\ u(k) \end{bmatrix}, \quad E=[1-b_o], \quad B=[-a_1 \ b_1] \quad \text{and} \quad A=[-a_2 \ b_2]$$

The estimates are calculated by the equations below.

$$\hat{X}_{k+1/k+1}=P_{21}\hat{X}_{k-1/k}+P_{22}\hat{X}_{k/k}+P_{23}Z_{k+1} \tag{11a}$$

$$\hat{X}_{k,k+1}=P_{11}\hat{X}_{k-1/k}+P_{12}\hat{X}_{k/k}+P_{13}Z_{k+1} \tag{11b}$$

$$\Omega_k^{-1}=A\Sigma_{(k-1)(k-1)}^k A^T+A\Sigma_{(k-1)k}^k B^T+B\Sigma_{k(k-1)}^k A^T+B\Sigma_{kk}^k B^T \\ +EVE^T \tag{11c}$$

with

$$\begin{bmatrix} P_{11} & P_{21} \\ P_{12} & P_{22} \\ P_{13} & P_{23} \end{bmatrix} = \begin{bmatrix} -\Sigma_{k(k-1)}^k A^T\Omega_kA - \Sigma_{kk}^k B^T\Omega_kA & VE^T\Omega_kA \\ I-\Sigma_{k(k-1)}^k A^T\Omega_kB - \Sigma_{kk}^k B^T\Omega_kB & VE^T\Omega_kB \\ \Sigma_{k(k-1)}^k A^T\Omega_kE + \Sigma_{kk}^k B^T\Omega_kE & I-VE^T\Omega_kE \end{bmatrix} \tag{11d}$$

$$\begin{bmatrix} \Sigma_{(k-1)(k-1)}^k & \Sigma_{(k-1)k}^k \\ \Sigma_{k(k-1)}^{k1} & \Sigma_{kk}^k \end{bmatrix} = \begin{bmatrix} P_{11}\Sigma_{(k-2)(k-1)}^{k-1}+P_{12}\Sigma_{(k-1)(k-1)}^{k-1} & P_{13}V \\ P_{21}\Sigma_{(k-2)(k-1)}^{k-1}+P_{22}\Sigma_{(k-1)(k-1)}^{k-1} & P_{23}V \end{bmatrix} \tag{11e}$$

$\hat{y}_{k+1/k+1}$ is the first element of $\hat{X}_{k+1/k+1}$ and $\hat{u}_{k+1/k+1}$ is the third element of $\hat{X}_{k+1/k+1}$. If $b_o=0$, $\hat{u}_{k/k+1}$ can be calculated from the third element of $\hat{X}_{k/k+1}$.

SIMULATION RESULTS

1. First Order Model

$$G(s)=\frac{1}{\tau s+1} \tag{12}$$

The first order model can be discretized exactly for a piecewise constant input [Seborg et al., 1989].

$$y(k+1)+a_1y(k)=b_1u(k) \tag{13}$$

The estimates can be calculated sequentially with the initial conditions $\Sigma_y^k=V_y$ and $\Sigma_u^k=V_u$.

$$\hat{y}_{k-1/k+1}=(1-V_y\Omega_k)y_{k+1}+V_y\Omega_k(-a_1\hat{y}_{k/k}+b_1u_k) \tag{14a}$$

$$\hat{u}_{k/k+1}=(1-b_1^2\Sigma_u^k\Omega_k)u_k+b_1\Sigma_u^k\Omega_k(a_1\hat{y}_{k/k}+y_{k+1}) \tag{14b}$$

$$\Omega_k=1/(a_1^2\Sigma_y^k+b_2^2\Sigma_u^k+V_y) \tag{14c}$$

where

$$\Sigma_y^k=V_y-V_y\Omega_kV_y \tag{14d}$$

$$\Sigma_u^k=V_u \tag{14e}$$

and Σ_u^k is the variance of the estimate $\hat{u}_{k/k}$ and Σ_y^k is the variance of $\hat{y}_{k/k}$. Since $\hat{u}_{k/k}$ is equal to u_k in this case, Σ_u^k is V_u.

The percentage reduction in total errors in the data is computed as follows:

Error Reduction in y $(\%)=[\{\sqrt{(y-y_{true})^2}-\sqrt{(\hat{y}-y_{true})^2}\}/$
$$\sqrt{(y-y_{true})^2}]\times100$$

Error Reduction in u $(\%)=[\{(\sqrt{(u-u_{true})^2}-\sqrt{(\hat{u}-u_{true})^2}\}/$
$$\sqrt{(u-u_{true})^2}]\times100$$

The data were simulated by a random number generator. By the use of the method proposed here the input and output variables can be filtered simultaneously as shown in Fig. 1 (76% error reduction in y) and Fig. 2 (64% error reduction in u). Tables 1 and 2 shows the error reductions of input and output variables as a function of the ratio of standard deviations. As σ_y/σ_u is decreased from 20 to 0.05, the error reduction in y decreases exponentially from 92.3% to 1.7%, and the error reduction in u is increased exponentially from 0.3% to 73%. As shown in Fig. 3, the measurement errors of the output variable are reduced exponentially as σ_y/σ_u is increased. In other filtering problems, the input variables are assumed to be error-free. But in practical situations, measurements may include random and/or gross errors. In the worst case, there could be small errors in output variables, but large errors in input measurements. If large errors in the input variables are ignored, the filtered estimates from other methods lose the reliability. Fig. 4 shows the reconciled estimates of the input variable when the input is assumed to be error-free, even though there exists a relatively large error in the input var-
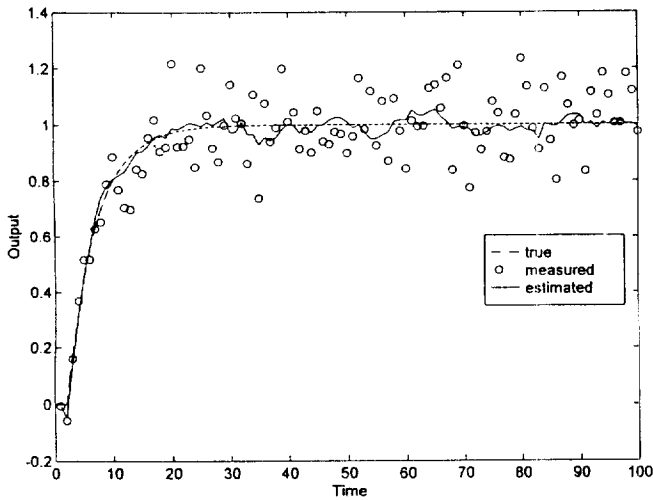
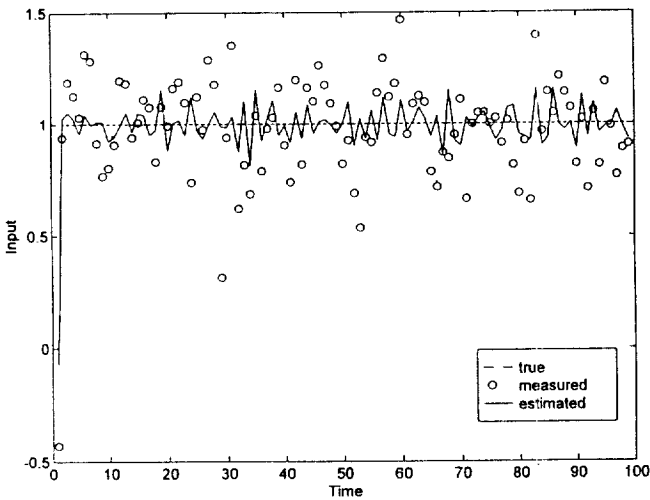**Fig. 1. Output reconciliation of G(s)= 1/(5s+ 1) with σ_y=0.1 and σ_u= 0.1.**

Fig. 2. Input reconciliation of G(s)= 1/(5s+ 1) with σ_y=0.01 and σ_u = 0.2.

Table 1. Error reduction of input and output variables in the first order model, G(s)= 1/(5s+ 1)

Std Y /Std U	Err. red. Y %	Err. red. U %	Std Y /Std U	Err. red. Y %	Err. red. U %
1/1	76.0	2.9	1/1	76.0	2.9
1/2	60.5	7.0	2/1	86.4	1.3
1/5	35.4	20.5	5/1	93.4	0.5
1/10	17.4	40.5	10/1	95.5	0.2
1/20	6.7	64.0	20/1	96.3	0.1

iable. The actual standard deviation of the input is 0.02, but the estimates are calculated with $\sigma_u = 0.001$. The estimates are worse than the measured data.

This method is more effective for fast processes which have small time constants. Table 3 indicates that for fast processes one should consider the errors in input variables more carefully.

2. Second Order Model

Table 2. Error reduction of input and output variables in the first order model, G(s)= 1/(15s+ 1)

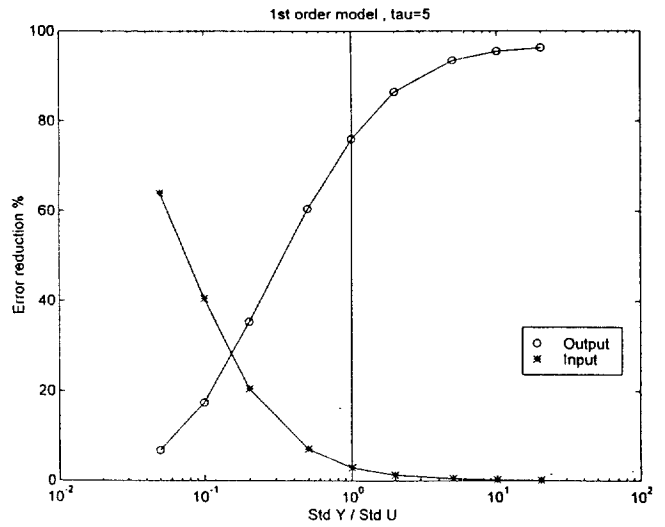| Std Y /Std U	Err. red. Y %	Err. red. U %	Std Y /Std U	Err. red. Y %	Err. red. U %
1/1	87.9	0.8	1/1	87.9	0.8
1/2	79.0	1.7	2/1	92.3	0.4
1/5	60.2	5.3	5/1	94.2	0.1
1/10	43.0	12.7	10/1	94.6	0.0
1/20	24.9	27.6	20/1	94.6	0.0

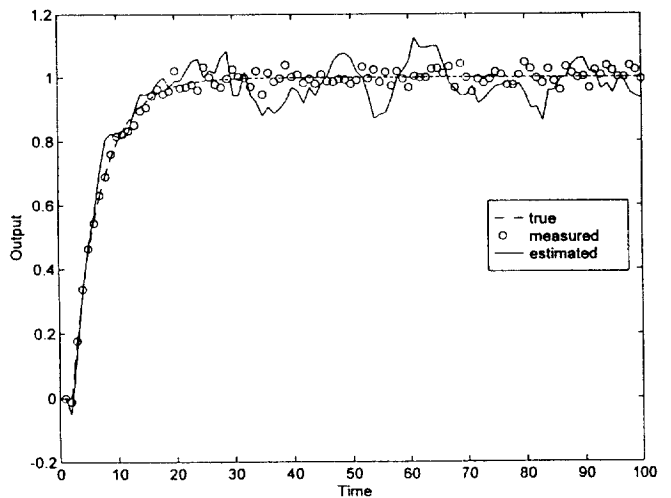**Fig. 3. Error reduction as a function of σ_y/σ_u, G(s)= 1/(5s+ 1).**

Fig. 4. Input reconciliation of G(s)= 1/(5s+ 1) with σ_y=0.02 and σ_u= 0.001 when the actual value of σ_u is 0.2.

$$G(s) = \frac{1}{\tau^2 s^2 + 2\tau\zeta s + 1} \quad (15)$$

$$G(s) = \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (16)$$

Two types of the second order model, Eqs. (15)-(16), are considered. The first one is for an underdamped case and the second one is for an overdamped case. Both types of models are discre-

Table 3. Error reduction of input and output variables in the first order model with respect to time constant for $\sigma_Y/\sigma_U = 1$

Time constant	Err. red. Y	Err. red. U
τ	%	%
2	62.1	8.8
5	76.0	2.9
10	84.4	1.2
20	89.4	0.6

Table 4. Error reduction of input and output variables in the second order model, $G(s) = 1/(s^2 + s + 1)$

Std Y /Std U	Err. red. Y %	Err. red. U %	Std Y /Std U	Err. red. Y %	Err. red. U %
1/1	50.6	10.7	1/1	50.6	10.7
1/2	31.4	20.2	2/1	68.0	5.2
1/5	11.6	43.3	5/1	83.9	1.7
1/10	4.4	62.3	10/1	89.9	0.7
1/20	1.7	73.0	20/1	92.3	0.3

Table 5. Error reduction of input and output variables in the second order model, $G(s) = 1/(2s + 1)(0.5s + 1)$

Std Y /Std U	Err. red. Y %	Err. red. U %	Std Y /Std U	Err. red. Y %	Err. red. U %
1/1	66.3	4.4	1/1	66.3	4.4
1/2	49.7	9.1	2/1	79.6	1.9
1/5	26.1	21.2	5/1	89.4	0.7
1/10	12.2	37.9	10/1	92.2	0.3
1/20	4.7	57.3	20/1	93.2	0.2
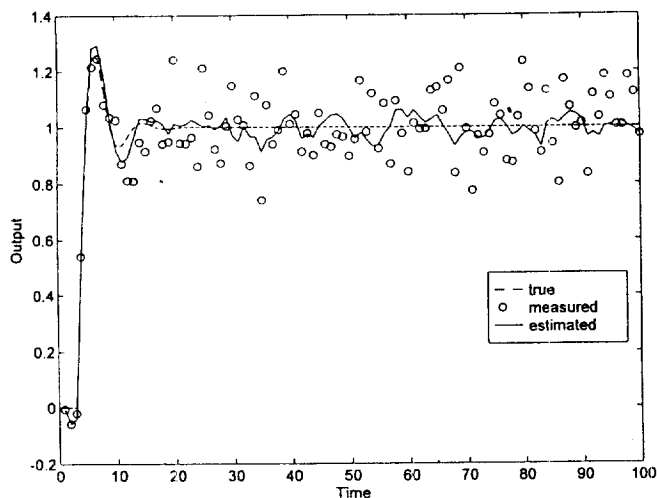
Fig. 5. Output reconciliation of $G(s) = 1/(s^2 + s + 1)$ with $\sigma_y = 0.1$ and $\sigma_u = 0.05$.

tized to the following linear difference equation.

$$y(k+1) + a_1 y(k) + a_2 y(k-1) = b_1 u(k) + b_2 u(k-1) \tag{17}$$

Tables 4 and 5 show the error reduction percentage as a function of the ratio of standard deviation of input and output variables. The results are similar to those of the first order model.
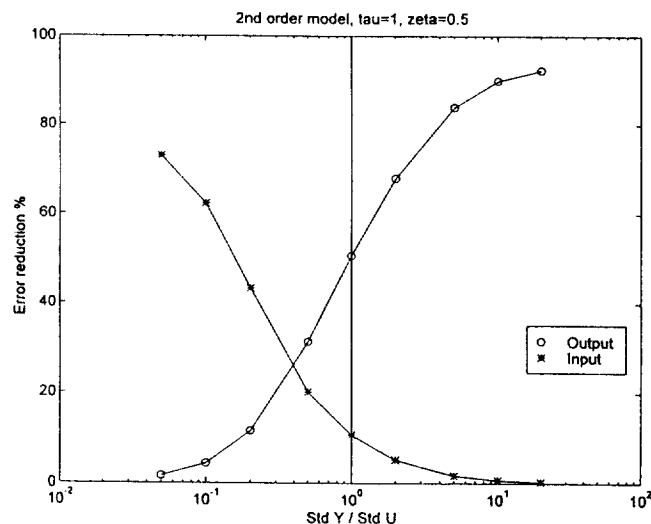

Fig. 6. Error reduction as a function of σ_y/σ_u, $G(s) = 1/(s^2 + s + 1)$.

Fig. 5 shows the reconciled estimates for an underdamped process with $\tau = 1$ and $\zeta = 0.5$. The measured data are reconciled well (68% error reduction in y). Fig. 6 shows the error reduction as a function of the ratio of standard deviations of output and input variables. When the input is corrupted by relatively large errors, the method here should be applied to reconcile the plant data.

CONCLUSION

A data reconciliation method, which can treat input and output variables simultaneously, is proposed for the input-output models in linear dynamic systems. Unlike other filtering methods, the measurement errors in the input variables are optimally handled by the least squares method. If relatively large errors occur in the input variables, the data reconciliation method here should be applied.

ACKNOWLEDGMENT

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