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A UNIFIED THEORY ON SOLID-LIQUID SEPARATION: FILTRATION, EXPRESSION, SEDIMENTATION, FILTRATION BY CENTRIFUGAL FORCE, AND CROSS FLOW FILTRATION

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Abstract – Based upon a new conception that the solid compressive pressure on a cake surface is not null, almost of all solid-liquid separation operations have been re-examined. For cake filtration, the phenomenon caused by the solid compressive pressure on a cake surface is discussed for thin cake. New expression and hindered sedimentation theories are developed by above new conception using Darcy's equation. Application of the new conception to centrifugal filtration and tangential filtration is also discussed. Above results lead to the conclusion that cake filtration, expression, hindered sedimentation, centrifugal filtration and tangential filtration can be described with a unified theory, and the main difference between the operations is only the boundary condition of cake.

Key words: Solid-liquid Separation, Cake, Cake Filtration, Average Specific Resistance, Solid Compressive Pressure, Expression, Hindered Sedimentation, Centrifugal Filtration, Tangential Filtration

INTRODUCTION

Until now cake filtration, expression and hindered sedimentation have been considered as independent domains, and thus being analyzed by their individual theories. The internal mechanism of a cake in filtration has been studied based on the equations of Kozeny-Carman and Ruth for almost 50 years by Tiller [Tiller, 1953; Tiller and Hsyung, 1995]. The expression theory based on Terzaghi's equation was developed by Shirato [Shirato et al., 1967], and has been used as a unique mean to design the expression machines. It has been regarded that there seems to be no relation between those two theories. For hindered sedimentation, Kynch [1952] began theoretical study using the conception of volumetric fraction of solids in slurry, and the way of study is quite different from that of above two.

Yim and Ben Aim [1986] suggested the possibility to combine the theories on cake filtration and expression with Darcy's equation and new boundary condition. In this study, the same conception is applied to hindered sedimentation, centrifugal filtration and cross-flow filtration, and more detailed analyses for cake filtration and expression are also presented.

BOUNDARY CONDITION OF A CAKE SURFACE

The solid compressive pressure (p_s) of filter cake surface, i.e. the first solid layer of cake next to suspension, was considered as zero for a long time in cake filtration [Tiller, 1953]. This may be true if we define the starting point of a cake as the front of the particles in the first solid layer as shown in Fig. 1. But at the contact point of the particles in the first solid layer and the neighboring particles in the second solid layer, the solid compressive pressure should increase by as much as the drag force acting on the first solid layer plus the gravitational force acting on the first solid layer. This solid compressive pressure does not act on the variation of cake porosity and the increase of specific cake resistance for the first solid layer. For expression, however a part of mechanical force exerted by piston acts directly on the front of first solid layer.

The difference of solid compressive pressure across the first solid layer may not play an important role in cake filtration in most cases. But in case of thin cake which induces fast flow enough to exert large drag force for the first solid layer, for example the initial period of filtration and tangential filtration, the difference of solid compressive pressure on either sides of the first solid layer may not be omitted. The solid compressive pressure on the first solid layer for expression is not zero because of the direct mechanical force to sol-

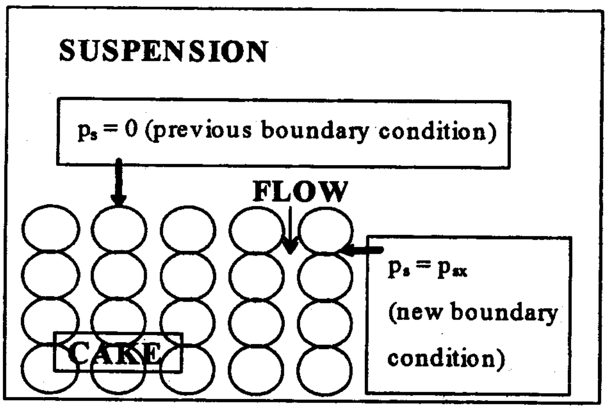

Fig. 1. Boundary condition of cake.

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id from piston as mentioned above. For centrifugal filtration, the first solid layer exerts large centrifugal force to second solid layer. To include above various conditions, the starting point of the boundary condition for a cake is proposed as the contact point of the first and second solid layer for filtration, centrifugal filtration, and tangential filtration.

By the new boundary conditions, the average specific resistance of a cake α_{av} can be written as follows:

$$\alpha_{av} = \frac{\Delta p - p_{sx}}{\int_{p_{sx}}^{\Delta P} \frac{dp_s}{\alpha}} \tag{1}$$

The limits of integration are given by the solid compressive pressure of the first solid layer p_{sx} and the filtration pressure Δp. The portion of the integration from zero to p_{sx} in traditional boundary condition is omitted at above equation. The omission is caused by the solid compressive pressure of the first solid layer p_{sx} which does not act for the modification of a cake as mentioned above.

The second assumption of this study is that the power function which defines the relation between specific cake resistance and solid compressive pressure is valid until very low pressure as shown by Eq. (2). It means that the conception p_i proposed by Tiller [1955] is not used in this study.

$$\alpha = a p_s^n \quad \text{for } p_s \geq 0 \tag{2}$$

Here, n is compressibility and a is the constant defined by this equation. Above relation was justified experimentally down to about 100 Pa by Shirato et al. [1985] and about 1 Pa by Yim and Ben Aim [1986].

APPLICATION OF NEW CONCEPTION FOR CAKE FILTRATION

For filtration by moderately compressible cake, the flow rate is very slow during almost of all operation period except for the initial period. The slow flow rate does not give enough drag force to cause significant solid compressive pressure to the first solid layer. So the new conception does not play an important role for ordinary cake filtration. The two examples below are the exceptions to above comment.

1. Initial Period of Filtration

Phenomena during the initial period of filtration could not be easily analyzed by experimental means because of the fast rate of flow and the short duration time. According to Tiller and Cooper [1960], the average specific resistance of a cake at the initial period of filtration is smaller than the equilibrium value because of the sharing of applied pressure between cake and filter media. This theory has not been verified experimentally because the calculated duration time by above theory was too short to be detected by actual filtration. According to our theory, the fast flow at the start of filtration gives larger drag force, i.e. higher p_{sx}, on the first solid layer. The calculation by Eq. (1) for high p_{sx} gives larger value of average specific resistance even though Δp of cake becomes smaller by the pressure sharing with septum. This is an opposite result that expected from the theory of Tiller and Cooper [1960]. An experimental method named filtration-permeation proposed by the author [1990] makes it possible that forming a cake of desired thickness (or mass) and sustain the state for a long time by permeating filtrate. By this experimental method the average specific resistance of thin cake, which is observed only at the initial period of filtration and thus its average specific resistance cannot be easily measured by filtration, can be calculated from permeating velocity which is constant during permeation period. The filtration-permeation of calcium carbonate ($CaCO_3$) suspension for various cake mass gave the results as below. All of the filtration experiments were conducted at 0.5 atm.

W (kg/m^2)	1.62	0.87	0.49	0.22
α_{av} (m/kg)	1.72×10^{11}	1.88×10^{11}	2.68×10^{11}	4.57×10^{11}

The average specific resistance of thin cake, which has essentially small value of W, is obviously larger than thick cake. It could be assumed that the fast velocity of fluid at the initial period of filtration gives large p_{sx}, and the large p_{sx} leads to the large average specific resistance. But the value of p_{sx} is large only at the beginning of filtration and decreases rapidly with time. So the influence of p_{sx} to the entire filtration is very small.

2. Sedimented Floc Filtration

The average specific cake resistance of bentonite floc previously sedimented was 8.9×10^{11} m/kg at 1.0 atm. The bentonite floc was made from bentonite suspension and polymer flocculant Super Floc SC 581. The value of p_{sx} calculated by Eq. (1) with average specific resistance and constants by Compression-Permeability Cell (CPC; n is 1.13 and a is 2.87×10^7) is 36 Pa. The p_{sx} calculated by the sedimented floc was 20 Pa. In this case, the difference of the two pressures could be assumed as the solid compressive pressure originated from the drag force.

To have a conclusive evidence for the existence of p_{sx} on filtration is not easy, and the role of p_{sx} for filtration is not certain, but there still exist a possibility that the p_{sx} could be an important factor for filtration by thin cake or of sedimented slurry.

NEW EXPRESSION THEORY

As shown in Fig. 2, new expression model begins with the hypothesis that the solid compressive pressure just under a piston p_{sx} is neither zero nor constant but increases during expression process from the pressure p_{si}, initial solid compressive pressure of the first solid layer which can be determined by filtration result, to the expression pressure Δp.

Neglecting R_m, which is relatively small compared to the

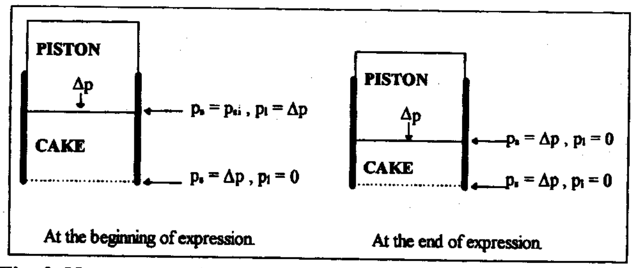

Fig. 2. New conception of expression.

total cake resistance during expression, Darcy's filtration equation can be written as Eq. (3).

$$\frac{dv}{dt}=\frac{dx}{dt}=\frac{\Delta p-p_{sx}}{\mu\alpha_{av}W} \tag{3}$$

Here, v is filtrate volume, more exactly the expressed liquid volume per unit filter area, and this value coincides with the piston displacement, x. The expression pressure Δp, liquid viscosity μ, and cake mass per unit area of expression W are constant during expression. The average specific resistance α_{av} increases as expression proceeds because of the p_{sx} increment by piston. Combining Eqs. (1), (2) and (3), we can get Eq. (4).

$$\frac{dx}{dt}=\frac{\Delta p^{1-n}-p_{sx}^{1-n}}{\mu W a(1-n)} \tag{4}$$

The piston velocity dx/dt is calculated by p_{sx} at any cake thickness. Since an operation of expression is finished when the p_{sx} reaches Δp, at this moment the piston velocity becomes zero by above equation.

The p_{sx} at a certain cake thickness L is calculated by the definition of average cake porosity as below.

$$\begin{aligned}1-\varepsilon_{av}&\equiv\frac{\text{solid volume}}{\text{total cake volume}}\\&=\frac{\text{(total solid mass in the cake)/(solid density)}}{\text{(filter area) (cake thickness)}}\\&=W\frac{1}{L\rho_s}\end{aligned} \tag{5}$$

Where, L is the cake thickness and ρ_s is the particle density. The equation which describes the average porosity proposed by Tiller and Cooper [1962] could be modified to represent p_{sx} as below:

$$1-\varepsilon_{av}=\frac{\int_{p_{sx}}^{\Delta P}\frac{dp_s}{\alpha}}{\int_{p_{sx}}^{\Delta P}\frac{dp_s}{\alpha(1-\varepsilon)}}=\frac{1-n-\beta}{1-n}B\frac{\Delta p^{1-n}-p_{sx}^{1-n}}{\Delta p^{1-n-\beta}-p_{sx}^{1-n-\beta}} \tag{6}$$

Combining above two Eqs.; one can get

$$L=L_0-x=\frac{W}{\rho_s}\frac{1-n}{1-n-\beta}\frac{1}{B}\frac{\Delta p^{1-n-\beta}-p_{sx}^{1-n-\beta}}{\Delta p^{1-n}-p_{sx}^{1-n}} \tag{7}$$

This equation enables us to calculate the p_{sx} at any cake thickness L. The expression velocity dx/dt could be calculated by Eq. (4) with this calculated p_{sx}. Thus the time needed to attain a certain degree of expression could be determined. In Fig. 3, two sets of experimental results at the same condition are shown in triangular and circular points. The line represents calculated results based on the proposed model. For above calculation, the particle density (ρ_s, 2,850 kg/m^3), filtrate viscosity (μ, 1.0 cP), CPC originated constants (a=2.87 $\times 10^7$, n=1.13, B=4.09$\times 10^{-3}$, β=0.32), expression conditions (W=3.18 kg/m^2, Δp=10^5 Pa), and p_{si} (36 Pa) determined by the filtration result were used. Not a constant originated from the experiment of expression was used. In conclusion, an expression process could be defined as the compression of the first solid layer from small p_{si} to expression pressure Δp. The solid compressive pressure of the other side of cake remains constant from the beginning to the end of expression at Δp.

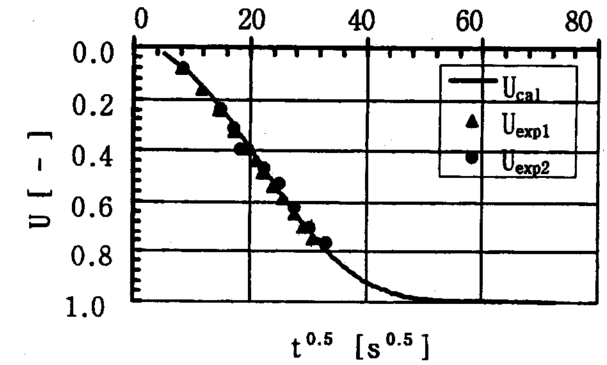

Fig. 3. Experimental and calculated expression results of bentonite floc at 1.0 bar.

NEW HINDERED SEDIMENTATION THEORY

In hindered sedimentation, the upper limit of suspension which usually called solid blanket is defined as the first solid layer. Contrary to the expression theory, the solid compressive pressure of the first solid layer p_{sx} remains constant during the course of the procedure and that of the bottom solid layer p_{sb} increases continuously until all of the solid exerts its own weight on the bottom of sedimentation vessel. The schematic diagram of the new sedimentation theory is shown in Fig. 4.

The Darcy's equation can be rearranged as follows:

$$\begin{aligned}\frac{dx}{dt}&=\frac{\text{driving factor}}{\text{resistance}}=\frac{(F_{gravitation}-F_{buoyancy}-F_{sb})/A}{\mu\alpha_{av}W}\\&=\frac{\{m_p(1-\rho/\rho_s)g-p_{sb}A\}/A}{\mu\alpha_{av}W}=\frac{W(1-\rho/\rho_s)g-p_{sb}}{\mu\alpha_{av}W}\end{aligned} \tag{8}$$

Here, dx/dt represents the velocity of solid blanket, i.e. the relative velocity between liquid and solid particles. The velocity is propelled by driving factor and hindered by resistance. At the start of sedimentation, the driving force for hindered sedimentation is the sum of total weight of solid being suspended $F_{gravitation}$ and the buoyant force $F_{buoyancy}$ which acts in the opposite direction. The force exerted on the bottom by the weight of piled particles F_{sb}, which does not contribute to sedimentation, is null at the start and increases up to the total weight of solid in suspension. F_{sb} can be described by the solid compressive pressure of the vessel bottom p_{sb} and sedimentation area A. The m_p is the total mass of suspended solid, ρ and ρ_s are density of liquid and solid respectively. To

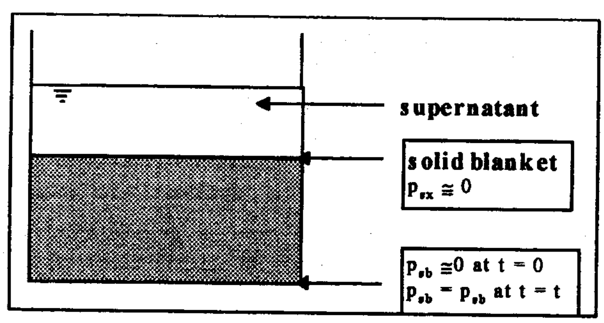

Fig. 4. Schematic diagram of hindered sedimentation.

know the p_{sb}, the same notion used for expression is adopted here. Eq. (7) reduces to Eq. (9) by the replacement of Δp with p_{sb}.

$$L = L_0 - x = \frac{W}{\rho_s}\left(\frac{1-n}{1-n-\beta}\right)\frac{1}{B}\left(\frac{p_{sb}^{1-n-\beta} - p_{sx}^{1-n-\beta}}{p_{sb}^{1-n} - p_{sx}^{1-n}}\right) \qquad (9)$$

For hindered sedimentation, p_{sx} is very small compared to p_{sb} during the course of sedimentation process. Omitting p_{sx}, the equation leads to;

$$L = L_0 - X = \frac{W}{\rho_s}\left(\frac{1-n}{1-n-\beta}\right)\frac{1}{B}\,p_{sb}^{-\beta} \qquad (10)$$

So the solid compressive pressure at the bottom p_{sb} can be calculated by the above equation with the constants (i.e. n, B, β) by CPC, total solid mass per unit sedimentation area (W), solid density (ρ_s), and the height of suspension from bottom to solid blanket (L).

$$p_{sb} = \left\{LB\frac{\rho_s}{W}\,\frac{(1-n-\beta)}{(1-\beta)}\right\}^{-\frac{1}{\beta}} \qquad (11)$$

With this p_{sb}, the sedimentation velocity can be calculated by Eq. (8). Experimental and calculated sedimentation results are shown in Fig. 5. The whole sedimentation procedure is represented by new theory. It is noted that theoretically predicted values are in good agreement with experimental data. The bridging effect by vessel wall is not considered in new theory. The details of calculation are presented by the author [1995] (n=0.3, a=3.11×10^8, B=0.080, b=0.123, W=53.9 kg/m^2). To apply this new theory for sedimentation, it is necessary that the concentration of suspension must be high enough to contact each particle for transmitting solid compressive pressure, and must not be too high to cause the bridging effect. This theory enables us to calculate such variables as the solid compressive pressure variation at a certain position [Yim, 1995], which may give us the information regarding the internal phenomena during sedimentation.

NEW CONCEPTION TO CENTRIFUGAL FILTRATION

For a filtration in centrifugal field, it is obvious that the

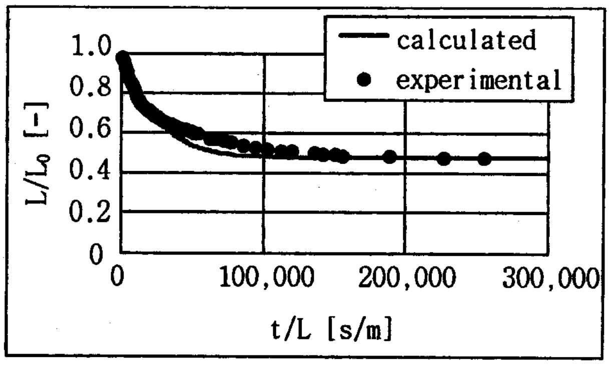

Fig. 5. Experimental and theoretical results of hindered sedimentation for the 16.8 wt% $CaCO_3$ suspension.

first solid layer of a cake receives centrifugal force which is not small. So the solid compressive pressure of the first solid layer p_{sx} could not be considered as zero. The average specific resistance during centrifugal filtration can be written as Eq. (12).

$$\alpha_{av} = \frac{\Delta p - p_{sx}}{\int_{p_{sx}}^{\Delta p}\frac{dp_s}{\alpha}} = \frac{\Delta p - p_{sx}}{\int_{p_{sx}}^{\Delta p}\frac{dp_s}{a p_s^n}} = \frac{a(1-n)(\Delta p - p_{sx})}{\Delta p^{1-n} - p_{sx}^{1-n}} \qquad (12)$$

Almost of all the p_{sx} here is originated from the centrifugal force and the drag force by liquid flow. To measure the exact value of p_{sx} by experimental mean is difficult. As the p_{sx} in this case is much greater than normal filtration, the average specific resistance by above equation is higher than that calculated by traditional boundary conditions 0 to Δp.

We will consider two examples of (1) a moderately compressible (n=0.4) and (2) a highly compressible (n=1.7) cake as were chosen by Tiller [1982]. The pressure exerted by centrifugation is assumed 2.47×10^6 Pa (for the centrifugation of 4,000 rpm, the radius to the wall and to the surface of suspension is 0.3 m and 0.2 m respectively). For the first example, the values of a and n are assumed as 6.29×10^8 and 0.4, respectively [Tiller, 1982]. The several values calculated for various p_{sx} are listed as follows:

p_{sx} (Pascal)	1	10^3	10^5	2.46×10^6
α_{av} (m/kg)	1.36×10^{11}	1.37×10^{11}	1.53×10^{11}	2.27×10^{11}

The average specific resistance calculated by the traditional boundary condition is 1.36×10^{11} m/kg. For a moderately compressible cake, the increase in p_{sx} does not give a notable influence on average specific resistance except for great p_{sx}. The fast rotation and large vessel diameter can result in great p_{sx} which increases the average specific resistance. This means that a little dense cake could be formed by the centrifugal force on the cake surface.

For the highly compressible cake, the value of a is assumed 50 and n is 1.7 [Tiller, 1982]. The average specific resistances are calculated at the same operational conditions.

p_{sx} (Pascal)	1	10^3	10^5	2.46×10^6
α_{av} (m/kg)	8.65×10^7	1.09×10^{10}	2.93×10^{11}	3.67×10^{11}

Applying the Tiller's P_i concept with the value of 10^4 Pa [1982], the average specific resistance by Eq. (4) is 3.25×10^{10} m/kg and this value can not be varied at the centrifugal field. The average specific resistance calculated by new conception grows large as p_{sx} increases. It means that the cakes formed by centrifugal filtration have denser cake than that formed by normal filtration. This phenomenon has been mentioned for a long time in the text book for undergraduate student [McCabe et al., 1995].

NEW CONCEPTION TO CROSS-FLOW FILTRATION

For cross-flow filtration, a thin cake is formed and sustained at a constant thickness for a relatively long time. At the beginning of cross-flow filtration the flow through a filter which we define downward flow is fast because there is no cake. Particles are captured on the filter surface by downward drag force by this fast flow. Cake is continuously formed until the drag force caused by parallel flow (parallel to filter medium) is greater than that caused by downward flow.

To know the exact value of solid compressive pressure p_{sx} is not easy, but it could be calculated by Eq. (1) with the measured average specific resistance. The value of p_{sx} may not be small because the thin cake which cause fast flow. The cake thickness can be represented by Eq. (7) with new boundary conditions.

$$L = L_0 - x = \frac{W}{\rho_s}\,\frac{1-n}{1-n-\beta}\,\frac{1}{B}\,\frac{\Delta p^{1-n-\beta} - p_{sx}^{1-n-\beta}}{\Delta p^{1-n} - p_{sx}^{1-n}} \tag{7}$$

When the mass of cake per unit filter area W is measured after filtration, the cake thickness can be calculated using particle density, CPC data, Δp, and p_{sx}.

CONCLUSION

It was concluded that there is a possibility of unifying the theories of several solid-liquid separation processes by using Darcy's equation and the new boundary condition. New expression and hindered sedimentation theory using Darcy's equation are proposed, and the calculated results are presented and compared with experimental data. The expression procedure is redefined as a phenomenon of increasing solid compressive pressure of the first solid layer, and hindered sedimentation process is also defined as a procedure of increasing the solid compressive pressure of the bottom solid layer. The phenomena of filtration, expression, hindered sedimentation have been explained by the notion of first solid layer, and the difference between operations is only the boundary condition. Theoretical analyses of centrifugal and cross-flow filtration by the new conception are presented.

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NOMENCLATURE

a : empirical constant defined by the Eq. (2) [-]
A : sedimentation area [m^2]
B : empirical constant by CPC [-]
$F_{buoyancy}$: buoyant force of solid [N]
$F_{gravitation}$: weight of solid being suspended [N]
F_{sb} : force exerted on the bottom by the weight of sediment [N]
L : thickness of the expressed cake or height of the compression zone in hindered sedimentation [m]
L_0 : initial thickness [m]
n : compressibility of a cake defined by the Eq. (2) [-]
p_l : liquid pressure [Pa]
p_s : solid compressive pressure [Pa]
p_{sb} : solid compressive pressure at the bottom [Pa]
p_{si} : solid compressive pressure of the first solid layer at the start of expression [Pa]
p_{sx} : solid compressive pressure of the first solid layer [Pa]
Δp : filtration or expression pressure [Pa]
t : time [s]
v : filtrate volume per unit filter area [m^3/m^2]
W : mass of dry cake per unit filter or expression area [kg/m^2]
x : piston displacement in expression [m]

Greek Letters

α : local specific resistance of a cake [m/kg]
α_{av} : average specific resistance of a cake [m/kg]
β : empirical constant by CPC [-]
μ : viscosity of liquid [kg/ms]
ρ : density of liquid [kg/m^3]
ρ_s : true solid density [kg/m^3]

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