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SHORT COMMUNICATION

CONVECTIVE INSTABILITY IN PACKED BEDS WITH INTERNAL HEAT SOURCES AND THROUGHFLOW

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Abstract – This article focuses on convective instabilities of throughflow in packed beds with internal heat sources. When a packed bed is heated with internal heat sources, the effects of throughflow on the onset conditions of convection have been examined numerically under the linear stability theory. The resulting conditions show that stationary instabilities occur at higher values of Darcy-Rayleigh number than the critical values as the amount of throughflow increases. The effects of free and rigid boundaries on the onset condition are also obtained for the Brinkman porous media with throughflow.

Key words : Convective Instabilities, Throughflow, Packed Bed, Internal Heat Source, Brinkman Porous Media

INTRODUCTION

As a Benard-Rayleigh problem, convection in a porous layer has a broad range of applications in connection with packed beds, nuclear reactors, solar ponds and geothermal energy collectors. In the case of a fluid-saturated porous layer heated from below, the critical Darcy-Rayleigh number at which natural convection occurs was calculated based on the linear stability theory by Horton and Rogers [1945] and by Lapwood [1948]. Unlike the case of a stationary state, when there is throughflow, the onset condition of natural convection is affected by fluid flow through a saturated porous medium.

The amount of flow can be represented by a Peclet number, and for a large Peclet number, Wooding [1960] presented the critical Darcy-Rayleigh number for the onset of natural convection. Homsy and Sherwood [1976] examined the effects of flow direction and of special boundary conditions of constant velocity and temperature at upper and lower boundaries. Recently, Jones and Persichetti [1986] solved the same problem by using the IMSL subprogram giving the estimated critical Darcy-Rayleigh number for arbitrary Peclet numbers, and Nield [1987] verified the validity of unusual results by Jones and Persichetti. In the case of porous media containing internal heat sources without throughflow, many studies have been performed analytically or numerically. Rudraiah et al. [1982] investigated the effects of various boundary conditions by considering the Brinkman model. Gasser and Kazimi [1976] also calculated the stability conditions for the case of internal heat generation with impermeable conducting boundaries.

In this note, the numerical results of various boundary conditions for a porous layer with throughflow and internal heat sources are examined by using the Brinkman model. It is of interest that when throughflow is present in a porous layer with internal heat sources, the critical Darcy-Rayleigh number with a Peclet number shows a rather unexpected trend.

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LINEAR STABILITY ANALYSIS

Here is considered a fluid-saturated porous layer. The horizontal porous layer consisting of spherical particles with porosity ε is bounded by rigid or free boundaries and z is the vertical coordinate with an origin at the lower boundary, as shown in Fig. 1. Fluid is injected through the lower boundary at a constant velocity and drawn off through the upper boundary at the same velocity. Therefore, there is a transverse throughflow within the porous layer at a uniform vertical velocity u_0. This problem is well described by Gershuni and Zhukhovitskii [1976]. Linear stability theory is applied to analyze this system and the Boussinesq approximation proper to the

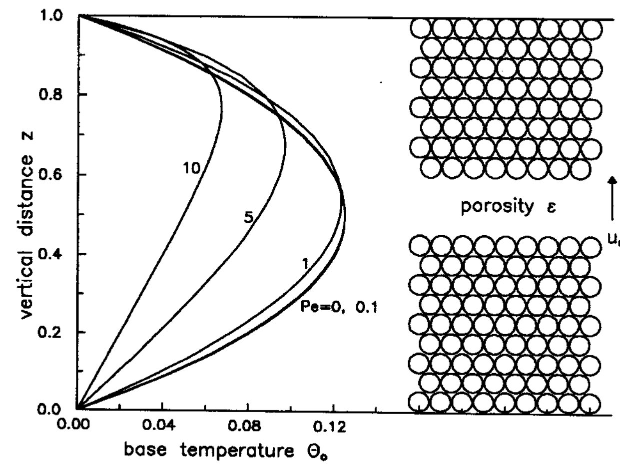

Fig. 1. Schematic diagram of packed beds with internal heat generation and throughflow.

linear stability theory is assumed to set the equation for density variation as follows :

$$\rho=\rho_0[1-\beta(T-T_0)] \tag{1}$$

where β is the volume expansivity of saturated fluid. The steady state temperature distribution is changed to non-uniformity inducing natural convection as the Peclet number increases. The Brinkman model representing viscous drag at the boundaries is used as an equation of motion for flow through a porous layer, and the energy and continuity equation can be written as follows :

$$\rho_0\vec{u}\cdot\nabla\cdot\vec{u}=-\nabla P-\frac{\mu}{K^*}\vec{u}+\mu\nabla^2\vec{u}+\rho\vec{g} \tag{2}$$

$$\vec{u}\cdot\nabla T=\alpha\nabla^2 T \tag{3}$$

$$\nabla\cdot\vec{u}=0 \tag{4}$$

where ρ, μ, K^*, g and α represent fluid density, fluid viscosity, permeability of porous media, gravitational acceleration and effective heat diffusivity, respectively and $\vec{u}$, P and T stand for velocity vector, pressure and temperature. To simplify the problem, non-dimensional variables are introduced by choosing L, SL^2/α and α/L as characteristic length, temperature and velocity scales, respectively, where S is a volumetric heat source. After manipulating Eqs. (1) through (4), the governing equations of disturbance under the linear stability theory can be expressed as follows :

$$\left(-\nabla^2+\frac{1}{Da}\right)\nabla^2 w_1=Ra\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\theta_1 \tag{5}$$

$$Pe\nabla\theta_1+w_1\frac{\partial\theta_0}{\partial z}=\nabla^2\theta_1 \tag{6}$$

where w_1, θ_1 and θ_0 are dimensionless vertical velocity perturbation, temperature perturbation and basic temperature, respectively. $Da=K^*/L^2$, $Pe=u_oL/\alpha$, and $Ra=g\beta SL^5/\alpha^2\nu$ refer to the Darcy number, Peclet number and Rayleigh number, respectively. A basic temperature distribution is needed to analyze the onset condition of convection, and in the case of the internal heat generation system, the governing equation for basic temperature distribution in the direction of z can be obtained as follows :

$$PeD\theta_0=D^2\theta_0+1 \tag{7}$$

where D represents d/dz. Eq. (7) with isothermal boundary condition yields a solution for θ_0 :

$$\theta_0=\frac{1}{Pe}z+\frac{1}{Pe[1-\exp(Pe)]}[\exp(Pez)-1] \tag{8}$$

For different values of Pe, Eq. (8) is plotted in Fig. 1, where we can see that with increasing Pe, a temperature boundary layer at the upper boundary decreases the effective thickness of the stratified layer of fluid and an adverse temperature gradient increases in the lower boundary. Infinitesimal perturbations, w_1 and θ_1, must have the z-directional forms of w and θ under the principle of the exchange of stabilities by utilizing a horizontal wave number $a=(a_x^2+a_y^2)^{1/2}$. Using the wave number, we can obtain the following stability equations from Eqs. (5) and (6).

$$(D^2-a^2)\left[(D^2-a^2)-\frac{1}{Da}\right]w=a^2Ra\theta \tag{9}$$

$$[PeD-(D^2-a^2)]\theta=-wD\theta_0 \tag{10}$$

It is assumed that the upper and lower boundaries are both rigid, or both free, or one free and one rigid, and in all cases both boundaries are isothermal. In other words, four boundary conditions of rigid-rigid, free-free, free-rigid and rigid-free denoted as upper-lower are given as

$$w=Dw=\theta=0 \quad \text{at rigid boundary} \tag{11}$$

$$w=D^2w=\theta=0 \quad \text{at free boundary} \tag{12}$$

NUMERICAL METHOD

An eigen group (Ra, Pe, a, Da) may exist in order for governing Eqs. (9) and (10) to satisfy any one among the four boundary conditions. Therefore, the outward shooting method is used to solve the eigenvalue problem by transforming a boundary condition problem to an initial value problem. For example, in the case of rigid-free boundaries, in order to solve Eqs. (9) and (10), the basic temperature profile solution must be obtained from Eq. (8), a priori. Since boundary conditions are all homogeneous, D^2w at $z=0$ can be assigned arbitrarily. If the proper values of Ra, $D^3w(0)$ and $D\theta(0)$ are assumed for a given Pe, a and Da, this can be considered an initial value problem. Integration is carried out from z=0 to z=1 using the 4-th order Runge-Kutta method with assumed values of Ra, $D^3w(0)$ and $D\theta(0)$. If the guessed values do not yield a satisfying result, that is $w=D^2w=\theta=0$ (below an error of 10^{-12}) at z=1, initial guesses are improved by Newton-Raphson iteration until convergence is achieved. Repeating this procedure, we can obtain a unique group (Ra, Pe, Da, a) with various Pe, a and Da values.

RESULTS AND DISCUSSION

The resulting critical Ra values for a special Da are plotted in Figs. 2 and 3 with Pe. The following points may be noted from these figures. As a limiting case of a stagnant system for Pe=0 and infinite Da, the critical Rayleigh numbers for different boundary combinations are calculated as Ra_c=37325.2 (rigid-rigid), 37949.9 (free-rigid), 16670.2 (rigid-free) and 16993.9 (free-free), respectively. The onset condition of natural convection in a homogeneous fluid layer without throughflow should have Rayleigh values larger than the critical Rayleigh value. It is found that Ra_c values are more dependent on upper boundary conditions and the Ra_c value for rigid-rigid or free-rigid boundaries is about twice as large as that for rigid-free or free-free boundaries. It is considered that the free boundary condition induces the onset condition of natural convection more easily since it is less restrictive to perturbation.

As we can see in the resulting figures, the Ra_c values on the characteristic curves are larger than that for Da=1. This result may be explained by the fact that the Ra_c values for a small Da must be increased since a larger buoyancy force is

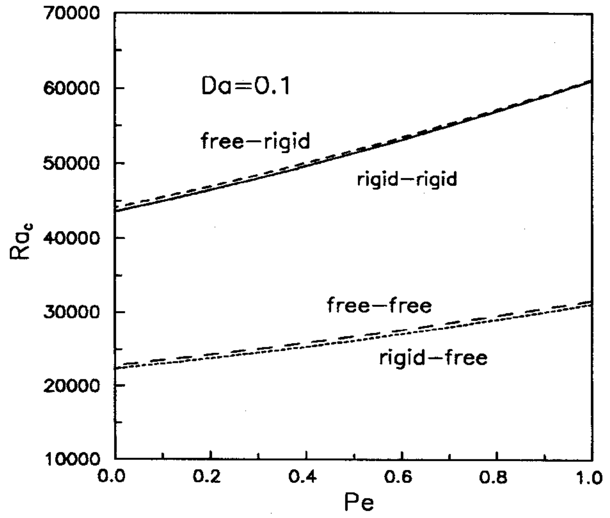

Fig. 2. Critical conditions of natural convection for Da=0.1 with the variations of Peclet number under internally heated basic temperature profile.

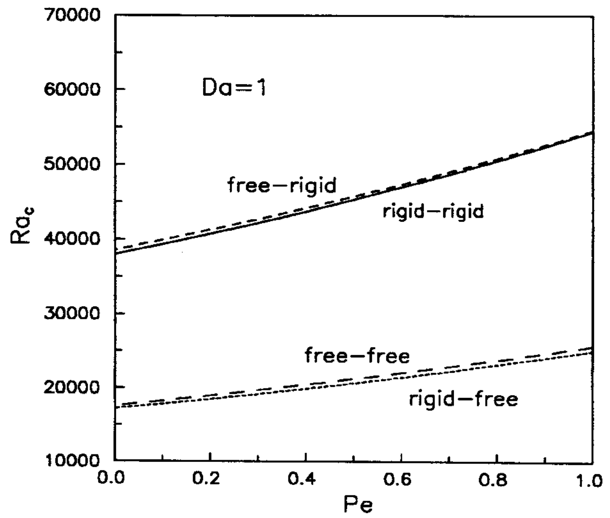

Fig. 3. Critical conditions of natural convection for Da=1 with the variations of Peclet number under internally heated basic temperature profile.

necessary for the onset of convection when small permeability restrains fluid mobility. The linear relationship of Ra_c vs. Pe shows that throughflow with internal heat sources in a porous medium may have stabilizing effects. In order to compare the results of Jones and Persichetti, the critical Ra values for rigid-free boundaries are shown in Fig. 4. In this research, there appears no minimum point such as the one Jones and Persichetti showed in their study of convective stability in a porous medium with throughflow. Thus, the analysis of Nield may not be applied to the case of throughflow with internal heat sources. It is interesting that Ra_c of rigid-free boundaries is smaller than that of free-free boundaries and Ra_c of free-rigid boundaries is larger than that of rigid-rigid boundaries for the entire range in Figs. 2 and 3. Considering the parabolic temperature profile of internal heat sources in Fig. 1, this result may be explained by the fact that the unstable upper boundary layer has a strong ef-

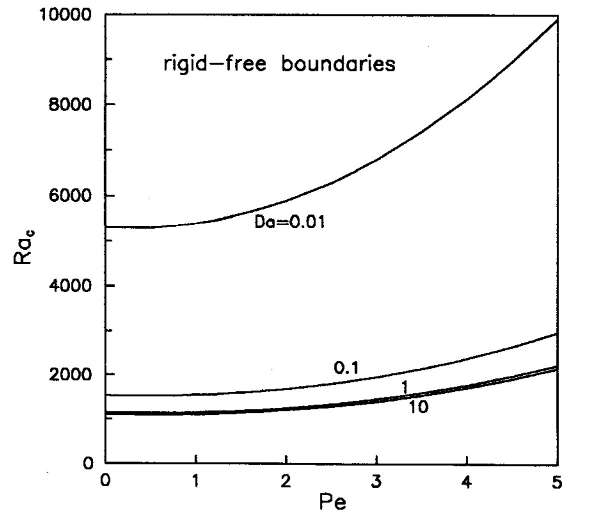

Fig. 4. Critical conditions of natural convection for rigid-free boundaries in internally heated packed beds with throughflow.

fect on the lower temperature profile when the lower boundary is free than when it is rigid. In other words, the unstable effect of the upper boundary can be distributed more readily to the lower layer that is bounded by a free boundary than to that bounded by a rigid boundary. As an unstable temperature effect is propagated to a whole layer, the instability of the upper layer may be alleviated.

CONCLUSION

The onset of convective instabilities for porous media heated with uniform internal sources has been analyzed. For a Brinkman porous system with throughflow, stability criteria have been obtained numerically under the linear stability theory, and the effects of flow boundary conditions on the onset condition have been examined quantitatively. In comparison with other theoretical results the present stability criteria look very promising. It seems apparent that the internally heated system becomes more stable with the increase of throughflows independent of flowing boundary conditions.

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NOMENCLATURE

a	: dimensionless horizontal wave number, $(a_x^2+a_y^2)^{1/2}$
D	: dimensionless operator, d/dz
Da	: Darcy number, K^*/L^2
Pe	: Peclet number, u_0L/α
Ra	: Rayleigh number, $g\beta SL^5/\alpha^2\nu$
K^*	: permeability of porous media [m^2]
L	: depth of a system [m]
$\vec{u}$	: velocity vector [m/s]
g	: gravitational acceleration [m/s^2]

S : volumetric heat source [K/ms^2]
T : temperature [K]
x, y, z : Cartesian coordinates
w : dimensionless vertical velocity

Greek Letters

α : effective thermal diffusivity [m^2/s]
β : volume expansivity [K^{-1}]
μ : dynamic viscosity [kg/ms]
ν : kinematic viscosity [m^2/s]
θ : dimensionless temperature
ρ : density of fluid [kg/m^3]

Subscripts

0 : undisturbed base quantity
1 : disturbed quantity
c : critical condition

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