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FUZZY MODEL PREDICTIVE CONTROL OF NONLINEAR pH PROCESS

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Abstract–A new fuzzy model-based predictive control scheme was developed to control a nonlinear pH process. The control scheme is based on the Takagi-Sugeno type fuzzy model of the process being controlled. In the present fuzzy model predictive control method, the process model maintains a linear representation of the conclusion parts of fuzzy rules. Therefore, it has a significant advantage over other types of models in the sense that nonlinear processes can be handled effectively by embedding the linear characteristic. The fuzzy model of the pH process to be controlled was constructed and used in the predictive control algorithm. Results of computer simulations and experiments demonstrated the effectiveness of the present control method.

Key words : Fuzzy Model, pH Control, Predictive Control, Nonlinear Process, pH Process

INTRODUCTION

In many industrial areas, pH neutralization processes are widely used. The pH neutralization process is a typical nonlinear process, and satisfactory control performance can hardly be achieved by conventional controllers. Various control methods to control pH processes have been proposed including classical PID control schemes, adaptive control methods, gain-scheduling methods, genetic control algorithms and model based control strategies [Sing and Postlethwaite, 1997; Katarina et al., 1997; Charles and Edward, 1993; Lee et al., 1994; Loh et al., 1995; Henson and Seborg, 1994; Park et al., 1995]. Difficulties in the pH control problem arise mainly from its heavy nonlinearity and uncertainty. The increasing research efforts in recent years are due to the highly nonlinear character coupled with the rather simple mathematical model to make pH control suitable for illustrating new nonlinear control approaches.

As new control strategies, uses of black-box type models such as fuzzy or neural network models have attracted much attention for modelling and controlling highly nonlinear chemical processes. In the fuzzy control method, qualitative control algorithms are presented in the form of IF-THEN rules which are evaluated based on fuzzy inferences. Fuzzy control systems have some advantages over other control methods in the control of inherently nonlinear chemical processes. Two different approaches can be used in a fuzzy control scheme. The first approach is based on the utilization of fuzzy control rules obtained from the simulations of human control activities. This approach employs heuristic sets derived from operational knowledge of the operator. The output of the controller can be determined by the manufacturer of the controller according to the process conditions to be controlled. But, in this approach, a large amount of accurate operational knowledge is required to define perfect control rules for satisfactory control performance. In the second approach a fuzzy model is obtained first from the input and output plant data. The fuzzy controller is constructed by the implementation of a linear control scheme into the fuzzy model. This approach was originally proposed by Takagi and Sugeno [1985] and is characterized by the use of control rules derived from fuzzy model rules.

Recently, many pH control schemes based on the fuzzy model have been proposed. Katarina et al. [1997], obtained a fuzzy model for nonlinear pH processes and employed the DMC algorithm to control pH processes. Park [1995] also derived a fuzzy model of a pH process and developed an optimal trajectory control method for the pH process based on the fuzzy model. Sing et al. [1997] adopted fuzzy relational models (FRMs) to implement a predictive control scheme for the control of pH processes.

In this paper a new model predictive control scheme based on the fuzzy model is developed to control the pH neutralization process. It is well known that a moderate nonlinear process can be controlled satisfactorily by the linear model predictive controller. But, for the control of processes showing severe nonlinearity, such as the pH process, acceptable control performance cannot be obtained from the use of linear schemes. The validity of the fuzzy model developed is tested through computer simulations. A control algorithm based on fuzzy rules is designed and the effectiveness of the proposed control method is demonstrated both by simulations and by experiments.

DYNAMIC MODEL OF THE pH PROCESS

The pH process used in the present study, shown in Fig. 1, is based on the model developed by Loh et al. [1995]. The

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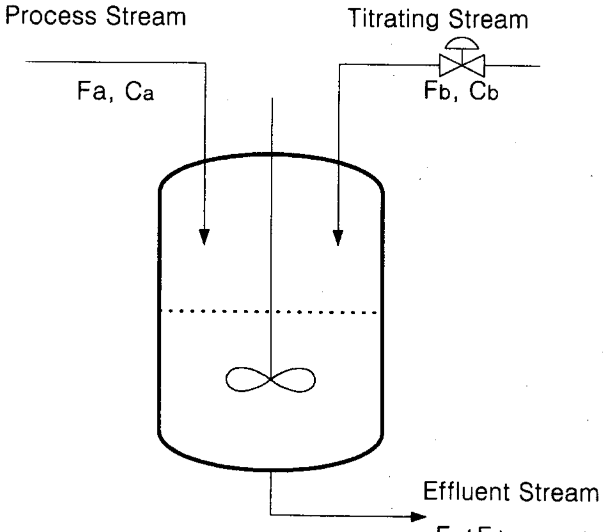

Fig. 1. Typical pH process.

rates of changes of acid and base compositions are given by

$$V\frac{dx_a}{dt}=F_aC_a-(F_a+F_b)x_a \tag{1}$$

$$V\frac{dx_b}{dt}=F_bC_b-(F_a+F_b)x_b \tag{2}$$

where F_a is the inlet flow rate, F_b is the flow rate of the base solution, C_a is the acid concentration in inlet flow, C_b is the base concentration of the titrating stream, and x_a and x_b are concentrations of acid and base solutions, respectively. The ionization reactions are given by

$$H_2O \leftrightarrow H^+ + OH^-$$
$$HAC \leftrightarrow H^+ + AC^-$$
$$NaOH \rightarrow Na^+ + OH^-$$

For the condition of electrical neutrality to be maintained the summation of electrical charges of each ion in the solutions should be zero, i.e.,

$$[Na^+]+[H^+]=[AC^-]+[OH^-] \tag{3}$$

where [X] denotes the concentration of ion X in the solution. The equilibrium can be represented by using equilibrium constants K_a and K_w such as

$$K_a=\frac{[AC^-][H^+]}{[HAC]}, \quad K_w=[H^+][OH^-] \tag{4}$$

Now we can define acid and base concentrations x_a and x_b as

$$x_a=[HAC]+[AC^-], \quad x_w=[Na^+]$$

Using these definitions we have from (3) and (4)

$$[H^+]+[H^+]^2\{K_a+x_b\}+[H^+]\{K_a(x_b-x_a)-K_w\}-K_wK_a=0 \tag{5}$$

From the definition of $pH=-\log_{10}[H^+]$, $pK_a=-\log_{10}K_a$, the titration curve is represented as

$$x_b+10^{-pH}-10^{pH-14}-\frac{x_a}{1+10^{pK_a-pH}}=0 \tag{6}$$

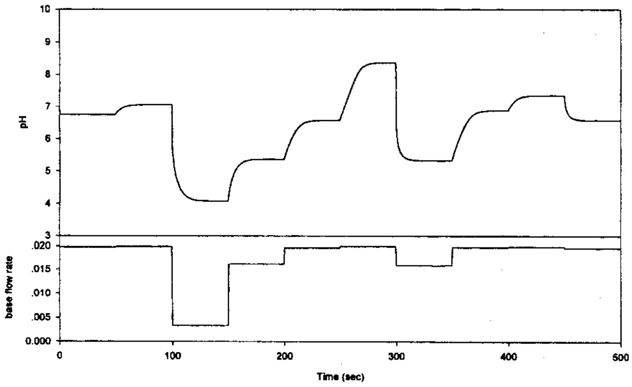

Fig. 2. Dynamic model response of pH process.

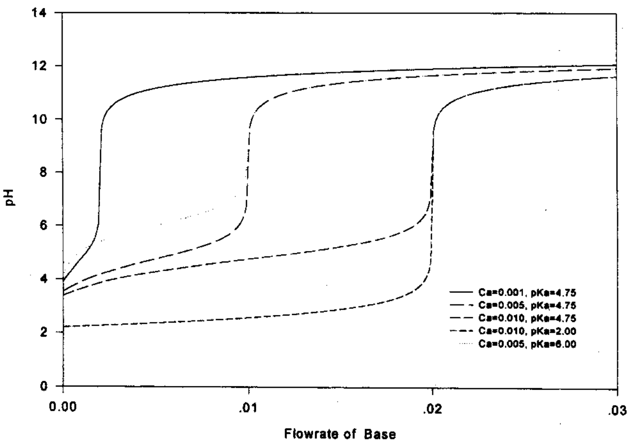

Fig. 3. Steady-state titration curves.

For the dynamic modeling, we assumed perfect mixing of acid and base solutions, constant density and instantaneous reaction. Figs. 2 and 3 show results of simulations based on the dynamic model of the pH process. Fig. 2 shows the response of the pH process to variations of the base flow rate, and Fig. 3 demonstrates steady-state titration curves for various operation situations.

FUZZY MODELING OF THE pH PROCESS

1. Takagi-Sugeno Fuzzy Model

In order to identify a fuzzy model of the pH process we adopted the Takagi-Sugeno type model [Takagi and Sugeno, 1985]. In this type of model the input space is divided into several fuzzy subspaces and the input-output relations of each of the subspaces are represented by linear equations. The relationship between inputs and outputs of the nonlinear system is given by the weighted summation of these linear equations.

A fuzzy system is a mathematical model which can realize nonlinear mapping to an arbitrary accuracy. Like neural network and universal function approximation, numerous approaches have been proposed for constructing fuzzy models from input-output data. Compared to other nonlinear approximation techniques, fuzzy models provide a more transparent representation of the identified model.

Suppose the rules of a fuzzy system are as follows :

R_l : If x_1 is A_l and x_2 is B_l Then $y_l = f_l(x_1, x_2)$

where x_1 and x_2 are input variables of the fuzzy system, y_l is an output variable and A_l and B_l are fuzzy sets characterized by their membership functions. The If-part of the rules describes fuzzy regions in the space of input variables and the Then-part is a function of the inputs, usually in the linear form of

$$f_l(x_1, x_2) = a_i x_1 + b_i x_2 + r_i$$

where a_i, b_i and r_l are consequent parameters. Such a simplified fuzzy model can be regarded as a collection of several linear models applied locally in the fuzzy regions defined by the rule premises.

2. Fuzzy Modeling of the pH Process

In terms of Takagi-Sugeno fuzzy rules, the fuzzy model has the form

$$R^n : \text{If } y(t) \text{ is } A_1^n, \ldots, y(t-k) \text{ is } A_k^n$$

$$\text{then } y(t+1) = \sum_{i=0}^{k} p_i^n y(t-i) + \sum_{j=0}^{l} q_j^n u(t-j) \tag{7}$$

where n is nth fuzzy rule (n=1, ···, M), k is the order of output, m is the order of input and A_i^n is the membership function of the fuzzy set.

The model output y(t+1) estimated by rule (7) can be represented as

$$\hat{y}(t+1) = \frac{\sum_{k=1}^{M} w_{t+1}^k y^k(t+1)}{\sum_{k=1}^{M} w_{t+1}^k} \tag{8}$$

where w is given by

$$w^k = \prod_{j=1}^{n} \mu_{A_j^i}(x_j) \tag{9}$$

As can be seen, the conclusion parts of the fuzzy model of pH process are of the form of the ARMA equation. The membership functions are constructed by dividing the output space within the operational range. The type, position, and number of the membership functions can be determined by

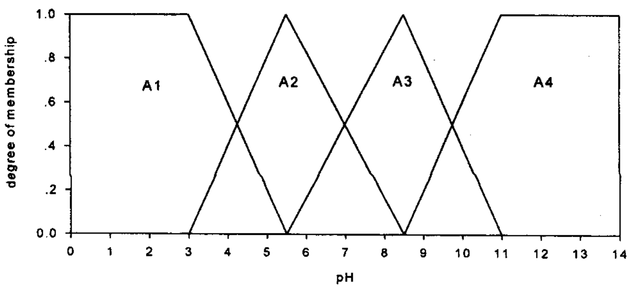

Fig. 4. Membership functions for pH.

Table 1. Membership function values

	Left point (pH)	Right point (pH)
A1	0.0	5.5
A2	3.0	8.5
A3	5.5	11.0
A4	8.5	14.0

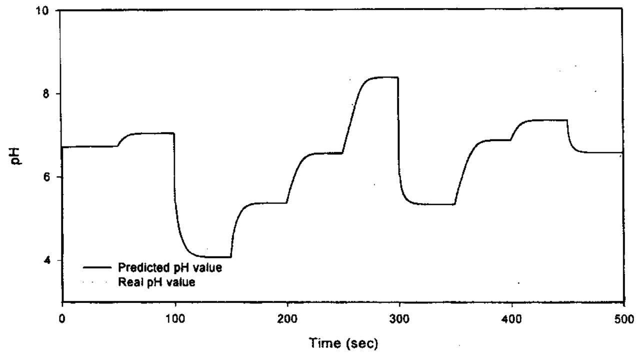

Fig. 5. Validation of the fuzzy model for simulation.

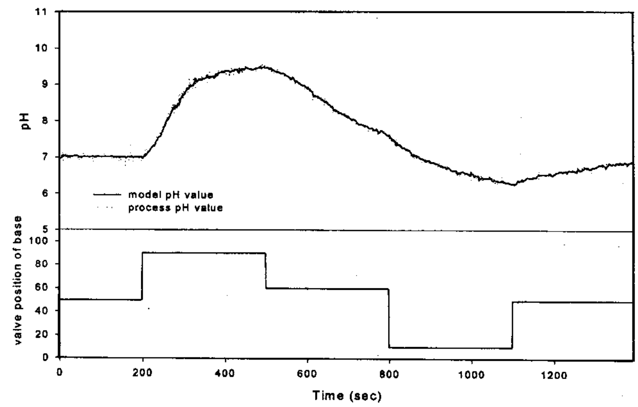

Fig. 6. Validation of the fuzzy model for experiments.

lustering, neural network, genetic algorithm and complex methods. Fig. 4 and Table 1 show membership functions for the pH process.

The well-known least squares method or recursive least squares method can be used to determine the parameters of the conclusion parts of the fuzzy model. For the on-line adjustment of parameters only the recursive least squares method is used in this study. In order to verify the fuzzy model obtained, we compared the behavior of the present fuzzy model with that of the dynamic model of the pH process and the actual pH process. Results are shown in Figs. 5 and 6. As can be seen, the present fuzzy model follows even the behavior of the experimental pH process very well. This fact demonstrates the effectiveness and usefulness of the present fuzzy model in the model-based control of the pH process.

FUZZY MODEL PREDICTIVE CONTROL

In the model-based predictive control future, the prediction of outputs is based on the dynamic process model to be controlled. Control commands to be applied at the present time are given by the minimization of the cost function composed of prediction errors and control inputs. Fig. 7 shows the basic structure of the general model predictive control method. In the predictive control method, the process output is forced to follow a desired output trajectory to be reached at the set point within a fixed time horizon in the future, as shown in Fig. 8.

The main idea of our approach is to combine the advantages

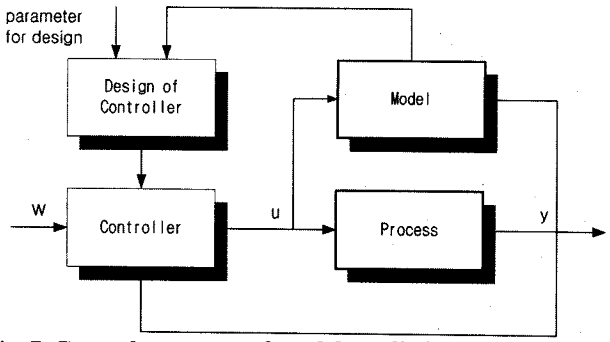

Fig. 7. General structure of model predictive control.

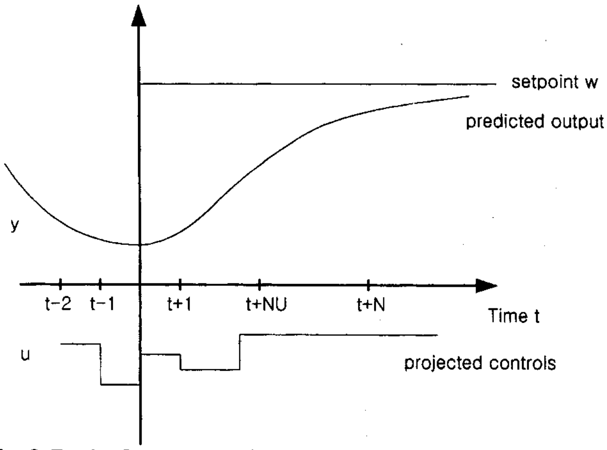

Fig. 8. Desired output trajectory.

of the fuzzy model and the general predictive control scheme in a way that is fast enough and suitable for real-time implementation. One of the authors proposed some predictive control strategies especially for bilinear processes [Lo et al., 1991; Oh et al., 1995; Yeo et al., 1989].

The cost function for the predictive control strategy considered here can be represented as

$$J(N_1,N_2)=E\left\{\sum_{j=N_1}^{N_2}[y(t+j)-w(t+j)]^2+\sum_{j=1}^{N_2}\lambda(j)[\Delta u(t+j-1)]^2\right\} \tag{10}$$

where N_1 is the minimum cost interval, N_2 is the maximum cost interval and $\lambda(j)$ is the weighting vector on the control inputs. As the desired output trajectory a first-order delay model given by (11) is widely used.

$$\begin{aligned}&w(t)=y(t),\\&w(t+j)=\alpha w(t+j-1)+(1-\alpha)w\\&J=1,2,\ldots,0\le\alpha\le1\end{aligned} \tag{11}$$

In the fuzzy model predictive control method, the future outputs are estimated by using the fuzzy model of the process to be controlled, and control commands are computed by a similar procedure as in the general predictive control method. Fig. 9 shows the basic structure of the fuzzy model predictive control scheme.

In the fuzzy model predictive controller the model parameters vary according to sampling times because of the varia-

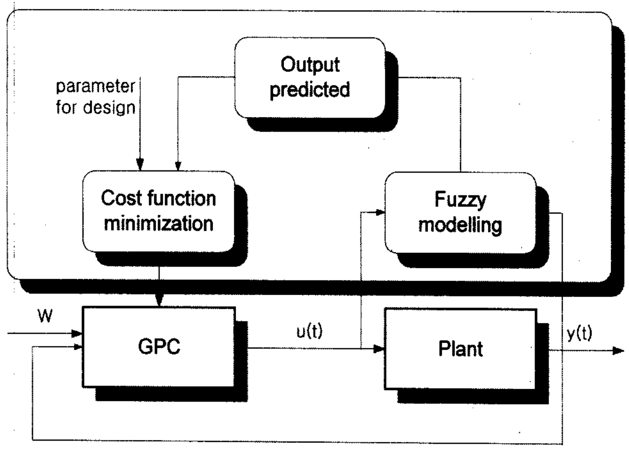

Fig. 9. Basic structure of fuzzy model predictive control method.

tions in the suitability of fuzzy rules. For example, if the conclusion part of a fuzzy rule is given by (12)

$$y^k(t+1)=p_0^k y(t)+p_1^k y(t-1)+q_0^k u(t)+q_1^k u(t-1) \tag{12}$$

and the prediction period N is 3. Predicted outputs are given by (13)-(15).

$$\hat{y}(t+1)=\hat{p}_0^1 y(t)+\hat{p}_1^1 y(t-1)+\hat{q}_0^1 u(t)+\hat{q}_1^1 u(t-1) \tag{13}$$

$$\hat{y}(t+2)=\hat{p}_0^2 y(t+1)+\hat{p}_1^2 y(t)+\hat{q}_0^2 u(t+1)+\hat{q}_1^2 u(t) \tag{14}$$

$$\hat{y}(t+3)=\hat{p}_0^3 y(t+2)+\hat{p}_1^3 y(t+1)+\hat{q}_0^3 u(t+2)+\hat{q}_1^3 u(t+1) \tag{15}$$

where

$$\hat{w}_{t+1}^k=\frac{w_{t+1}^k}{\sum_{k=1}^{M} w_{t+1}^k}$$

$$\hat{p}_i^1=\sum_{k=1}^{M}\hat{w}_{t+1}^k p_i^k \qquad (i=0,\ldots,m)$$

$$\hat{q}_j^1=\sum_{k=1}^{M}\hat{w}_{t+1}^k q_j^k \qquad (j=0,\ldots,n)$$

The general output predictions can be obtained from a generalization of the above relations and is given by

$$Y_F=GU_F+HU_p+FY_P \tag{16}$$

where Y_F is the future output vector, U_F is the future input vector, Y_P is the past output vector and U_P is the past input vector.

$$G=\begin{bmatrix} g_0^1 & 0 & 0 & \cdots & 0\\ g_0^2 & g_1^2 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ g_0^N & g_1^N & \cdots & \cdots & g_{N-1}^N \end{bmatrix}$$

$$H=\begin{bmatrix} h_1^1 & h_2^1 & \cdots & h_m^1\\ \vdots & \vdots & \ddots & \vdots\\ h_1^N & h_2^N & \cdots & h_m^N \end{bmatrix}$$

$$F=\begin{bmatrix} f_0^1 & f_1^1 & \cdots & f_n^1\\ \vdots & \vdots & \ddots & \vdots\\ f_0^N & f_1^N & \cdots & f_n^N \end{bmatrix}$$

The cost function used in the present fuzzy model predictive control scheme is

$$J=\sum_{j=N_1}^{N_2}[\hat{y}(t+j)-w(t+j)]^2+\sum_{j=1}^{N_2}\lambda(t)[u(t+j-1)]^2 \tag{17}$$

From the substitution of (16) into (17) we have

$$\begin{aligned}J&=\{(Y_F-W)^T(Y_F-W)+\lambda U_F^T U_F\}\\&=\{(GU_F+HU_P+FY_P-W)^T(GU_F+HU_P+FY_P-W)+\lambda U_F^T U_F\}\end{aligned} \tag{18}$$

Without suppression on future inputs the input vector which minimizes the cost function (18) is given by

$$U_F=(G^TG+\lambda I)^{-1}G^T(W-HU_P-FY_P) \tag{19}$$

Only the first element of the input vector (19) is applied to the process at the present time t

$$u(t)=g^T(W-HU_P-FY_P) \tag{20}$$

where

$$g^T=[1\ 0\ 0\ \cdots\ 0](G^TG+\lambda I)^{-1}G^T \tag{21}$$

Table 2. Experimental conditions

	Case (A)	Case (B)
H_3PO_4 (M)	0.01	0.005
NaOH (M)	0.05	0.05
Volume of reactor (*l*)	2	2
Sampling time (sec)	2	2
Limit time (sec)	6000	4500
CPU (MHz)	Pentium 166 (MMX)	Pentium 166 (MMX)

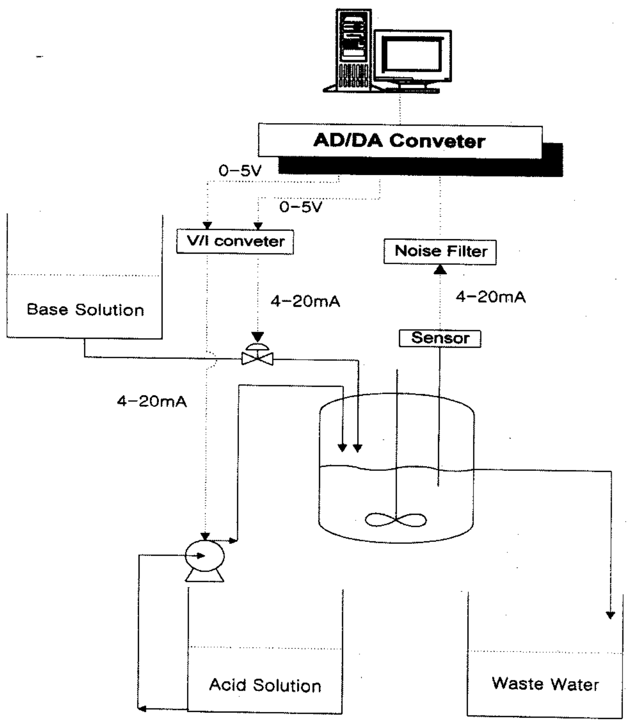

Fig. 10. Schematics of experiments.

EXPERIMENTS

Experimental conditions are summarized in Table 2. Fig. 10 shows the experimental apparatus used in the present study. The flow rate of acid solution is maintained at a constant value by the acid pump while the flow rate of the base solution is adjusted by the pneumatic valve to achieve the desired pH value. The pH values measured by the pH sensor are sent to the noise filter to eliminate measurement noises. The control commands computed by the fuzzy model predictive control scheme described before are sent to the control valve via AD/DA converter.

The output space was divided into 4 subspaces to give the fuzzy subspace and the total prediction interval N was set to 4. The linear relation which makes the conclusion part of the fuzzy rules consists of 6 elements : y(t), y(t–1), y(t–2), u(t), u (t–1) and u(t–2). These elements are updated by the recursive least squares algorithm. The sampling period was two seconds.

RESULTS AND DISCUSSIONS

1. Simulations

In order to validate the proposed fuzzy model predictive control method, computer simulations were performed for various set point changes. The dynamic model developed before was used as the pH plant and the Takagi-Sugeno type fuzzy model was used in the prediction of future outputs. The out-

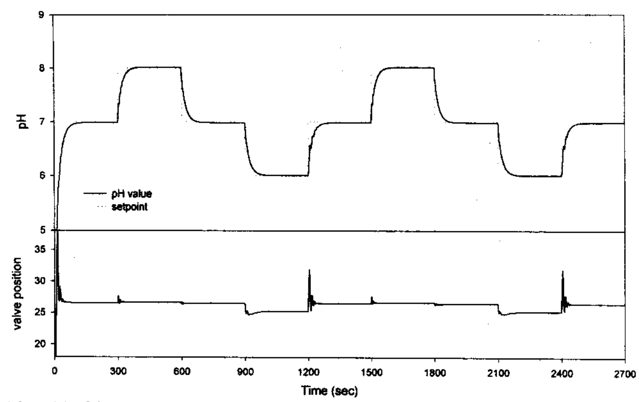

Fig. 11. Simulation results : without noise.

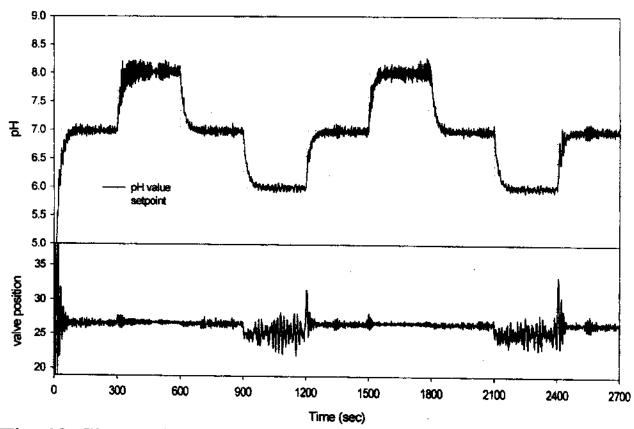

Fig. 12. Simulation results : with noise (±0.05).

put space was divided into 4 subspaces to be used as fuzzy subspaces. The linear relationships of the conclusion parts of the fuzzy rules consist of second order output and second order input. Parameters were updated by the recursive least squares method. The set point was forced to vary between pH 6 and 8 intentionally.

Results of simulations are shown in Fig. 11 and 12. Fig. 11 shows the results of control simulations without introduction of any noises. Even with frequent changes in set point, very good control performance could be achieved. Effects of random noises can be seen in Fig. 12. Random noises of magnitude 0.05 were assumed in the process with sampling period of one second. The control results seem to be contaminated a little bit by the noises, which can easily be smoothed out by the use of an additional output filter, although not shown here. It is obvious that the control performance is not affected by random noises confined within a certain magnitude. From the two figures, one can conclude that for this pH process, the fuzzy model has good dynamic performance and is also able to capture the steady-state relationship of the process.

2. Experimental Results

As in the simulations, the output space was divided into 4 subspaces to be used as fuzzy subspaces. The linear relations of the conclusion parts of the fuzzy rules consist of 2nd order outputs and inputs which are updated by the recursive least squares method. Sampling period was set to 2 seconds. Figs. 13 and 14 show the results of on-line control experiments based on the proposed fuzzy model-based predictive control method.

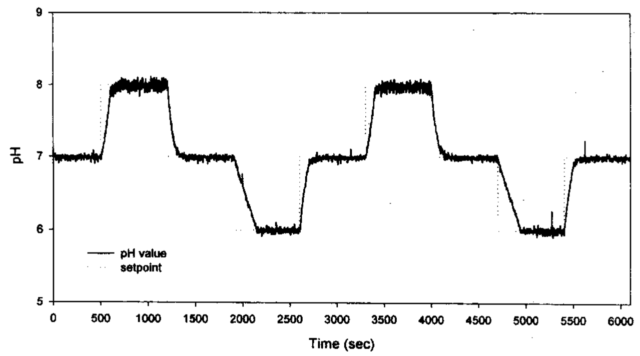

Fig. 13. Experimental results (0.01 M H_3PO_4, 0.05 M NaOH) (A).

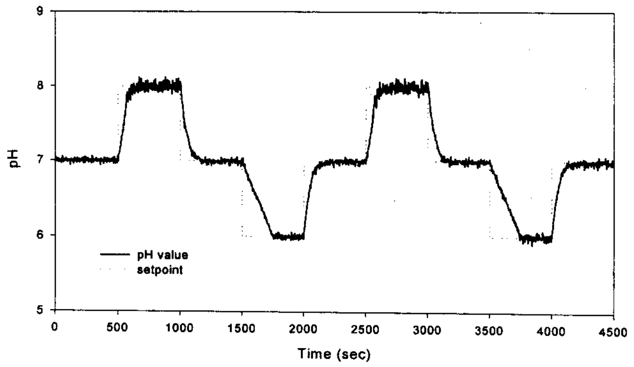

Fig. 14. Experimental results (0.005 M H_3PO_4, 0.05 M NaOH) (b).

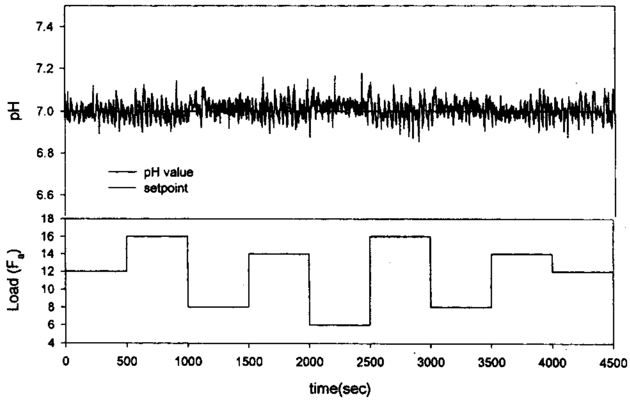

Fig. 15. Experimental results (disturbance change).

Phosphoric acid solution and NaOH solution were used as acid and base solutions, respectively. With constant concentration of the base solution (0.05 M NaOH), the concentration of acid solution was varied from 0.005 M to 0.01 M. Even with fluctuations in the acid concentrations, which are not usual in actual operations, very satisfactory control results are achieved.

In order to demonstrate the performance of the present control strategy even with the presence of external disturbances, we introduced changes into the flow rate of the acid solution with the set point being kept constant (=7.0). Results of the experiment are shown in Fig. 15. We can see that pH values of the outlet stream are confined within a permissible range, which means that effects of disturbances are well rejected.

Within the whole range of operations, the pH values of the output stream follow the reference trajectory fairly well. The good control results indicate that the predictive controller receives adequate information about the changes in process behavior from the fuzzy model.

CONCLUSION

In order to control a pH process, a new fuzzy model predictive control method was proposed and tested through simulations and experiments. As the fuzzy model for the pH process to be controlled, the Takagi-Sugeno type fuzzy model was employed and combined with predictive control algorithm. The fuzzy model was tested and shown to give accurate predictions on process outputs. Because of the linear relations in the conclusion parts of the fuzzy rules, well-known linear prediction control algorithms can be easily utilized. From both computer simulations and on-line control experiments, we could confirm the effectiveness of the present fuzzy model-based predictive control method. The fuzzy control scheme appears to be one of the most promising control strategies for nonlinear chemical processes, and the combination of the fuzzy techniques with neural networks is yet to be investigated.

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NOMENCLATURE

R : fuzzy rule
F_a : flow rate of the influent stream
F_b : flow rate of the titrating stream
x_a : concentration of the acid solution
x_b : concentration of the base solution
C_a : concentration of the influent stream
C_b : concentration of the titrating stream
Y_F : future output vector
U_F : future input vector
Y_P : past output vector
U_P : past input vector
y(t) : current process output
u(t) : current control input
w : weight or membership grade
μ : member function
N_1 : minimum output horizon
N_2 : maximum output horizon
N : control horizon
$\lambda(j)$: control-weighting sequence

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