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Modeling of Three-Dimensional Groundwater Flow Using the Method to Calculate Fractal Dimension

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Abstract−A three-dimensional finite-difference groundwater flow model was developed by the use of fractal theory. The model developed in this study can simulate the groundwater flow in fractured aquifers as well as in porous aquifers. The model was designed to be able to use other parameters, such as permeability, hydraulic conductivity, porosity and fractal dimension besides hydraulic parameters which are used in MODFLOW. Methods which can calculate box fractal dimension used in the Baecher model and mass dimension used in Levy-Lee Fractal model were developed. The results of the model and MODFLOW agreed exactly in the case of the fractal dimension of 2.0 without regard to the use of a fractal equation. The fact that the drawdown along the distance from the well increases by increasing the fractal dimension shows the effect of fracture on groundwater flow.

Key words: Fractal, Fractal Dimension, Generalized Radial Flow, Groundwater Flow, Modflow

INTRODUCTION

Fractured aquifers consist of solid rock that contains primary porosity and a system of joints, cracks, microcracks, faults, and shear zones that create secondary porosity. Groundwater flow in fractured aquifers is dominated by secondary porosity; therefore, it is very important to analyze the groundwater flow in secondary porosity properly. If the groundwater flow in fractured aquifers is interpreted as the flow only through porous media, the interpretation could lead to the wrong results [Anderson and Woessner, 1992].

The following models are typically used to simulate the groundwater flow in fractured aquifers: equivalent porous medium model, discrete fractures model, and dual-porosity model [Ŝen, 1995]. But it is difficult to use those models to analyze field data since they have one or more of the following defects: the fractured system is too simplified to represent the real structure, it is practically hard to obtain the detailed data of the fractured aquifer, and operation processes are complex and difficult.

Various methods, including equivalent porous medium model, discrete fractures model, and dual-porosity model, have been developed to simulate the groundwater flow in fractured aquifers; however, none of the above methods could describe the complex and irregular geometry of fractured aquifers properly. Fractal theory can be applied to express complex and irregular geometry by using self-similarity, the property that any phenomenon shows the same aspect regardless of scale [Mandelbrot, 1985]. To represent the degree of self-similarity, fractal dimension, which is generalized to nonintegral value, instead of integral dimension, is used.

The distribution of fractures in rock is reported to show self-similarity. According to the water well test analyses conducted on 126 data from 54 fractured aquifers in Korea, about 76% of the data showed fractal flow dimension between 1.0 and 2.0, which shows that fractal model can be applied justifiably to the geological features in Korea [Hamm, 1995].

DEVELOPMENT OF MODEL

1. Theoretical Background of Fractal Model

The Theis method is used in well test analysis to determine hydraulic parameters; however, an assumption of two-dimensional radial flow cannot be applicable to fractured media. Therefore, it is very important to choose the appropriate flow dimension of the fractured system. If the fracture density is high and the system is isotropic, a three-dimensional spherical flow geometry is proper. If the fracture density is low and the system is anisotropic, a one- or two-dimensional flow geometry would be proper. However, it is difficult to choose the appropriate flow dimension [Barker, 1988].

Barker developed a generalized radial flow (GRF) model, a fractal groundwater flow model which generalizes the flow dimension to nonintegral values. The main assumptions made in developing the GRF model are as follows:

1. Flow is radial, n-dimensional flow from a single source.
2. Aquifer consists of homogeneous and isotropic fractured medium.
3. Darcy's law applies throughout the system.
4. The source is an n-dimensional sphere.

Considering groundwater flow toward a single well in a fractured aquifer, the surface area of flow is proportional to the real power of the distance from the well [Eq. (1)]. This is the fractal theory applied in the GRF model.

$$A=b^{3-n}\alpha_n r^{n-1} \tag{1}$$

$$\alpha_n=2\pi^{n/2}/\Gamma(n/2) \tag{2}$$

According to Eq. (1), the surface area of one-dimensional flow is

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proportional to r^0, which is constant regardless of the distance from the well. The surface area of two-dimensional flow is proportional to r^1 and that of three-dimensional flow is proportional to r^2.

From Darcy's law, law of mass conservation and the fractal theory applied in the GRF model, the groundwater flow equation with generalized flow dimension can be derived as follows.

$$S_{sf}\frac{\partial h}{\partial t} = \frac{K_f}{r^{n-1}}\frac{\partial}{\partial r}\left(r^{n-1}\frac{\partial h}{\partial r}\right) \tag{3}$$

2. Calculation of Fractal Dimension

Generally, fractal dimension is calculated from the slope of a log-log plot of count against number. However, fractal dimension can be calculated in a wide variety of different ways. Nine geometric conceptual models for simulating fractured aquifers are the Enhanced Baecher model, the Nearest Neighbor model, the Levy-Lee Fractal model, the War Zone model, the Poisson Rectangle model, the Non-Planar zone model, the Fractal POCS (Projection Onto Convex Sets) model, the Fractal Box model and the Geostatistical model as shown in Fig. 1 [Dershowitz et al., 1998].

The model developed in this study was designed to calculate fractal dimension by using the Baecher analysis and Levy-Lee analysis among the above models. The Baecher analysis or Box Fractal analysis compares number of boxes necessary to contain all fracture trace center against box size to calculate box fractal dimension. Levy-Lee Fractal analysis or Point Density Fractal analysis compares circle radii to number of fracture trace centers within the circles to calculate mass dimension (or point density fractal dimension) (Fig. 2) [Dershowitz et al., 1998].

The Baecher analysis is one of the first well-characterized discrete fracture analysis methods. In this method, the fracture centers are located uniformly in space, using Poisson process, and the fractures are generated as disks with a given radius and orientation. The Levy-Lee Fractal analysis is a stochastic model which uses a Levy Flight fractal process to produce clusters of smaller fractures around widely scattered, larger fractures [Geier et al., 1988].

3. Development of Computer Model

The majority of the groundwater flow models used in Korea, including MODFLOW (Modular Three-Dimensional Finite-Difference Groundwater Flow Model), assume that aquifers consist of porous media only [McDonald and Harbaugh, 1994]. Therefore, these models are not appropriate to simulate the groundwater flow through fractured media which are extensively developed in Korea.

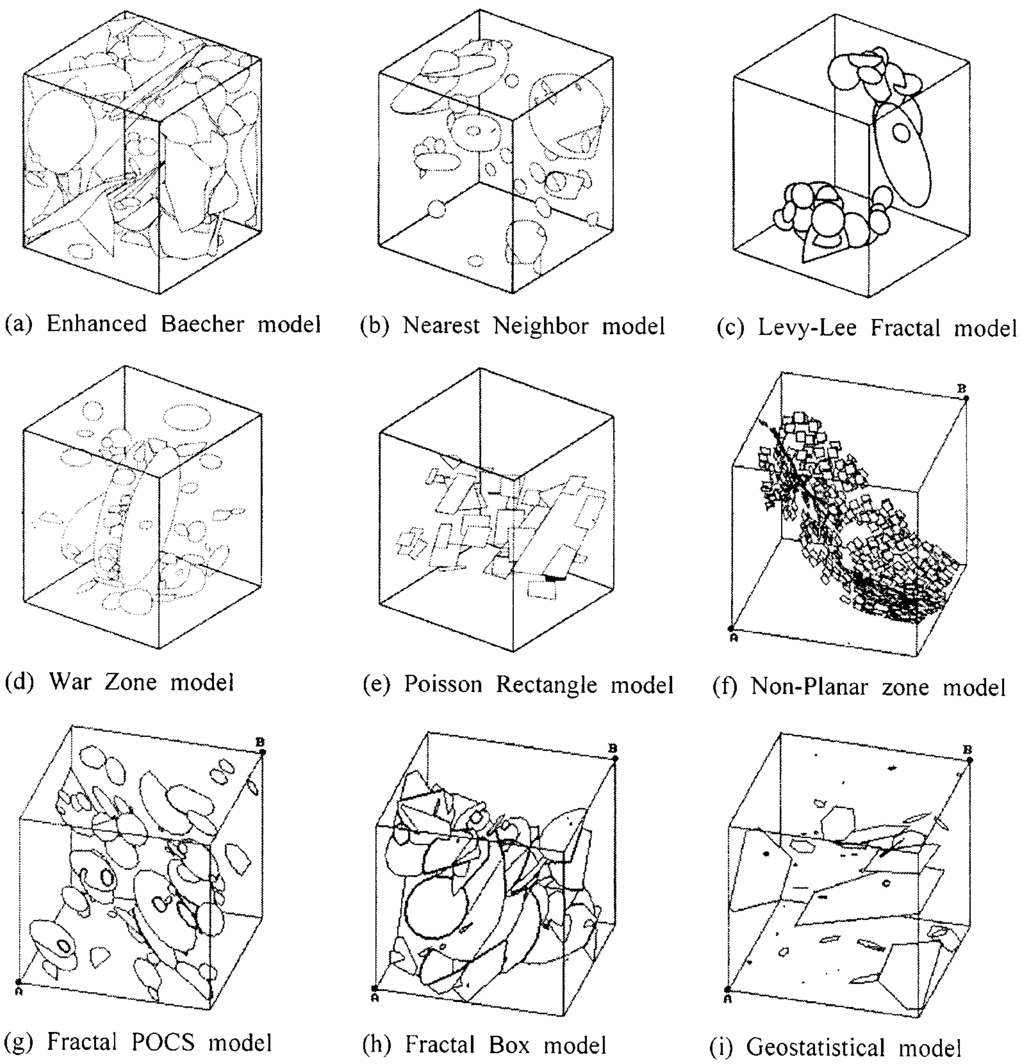
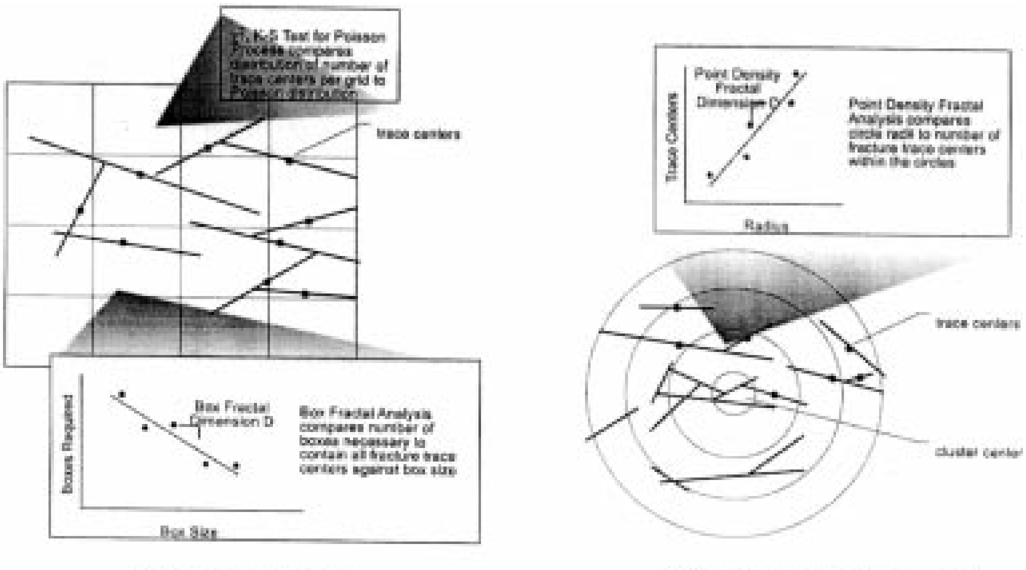


Fig. 1. Geometric conceptual models [Dershowitz et al., 1998].



(a) Baecher analysis (b) Levy-Lee fractal analysis

Fig. 2. Analysis of fracture spatial structure [Dershowitz et al., 1998].

In this study, the fractal groundwater flow equation on the basis of Barker's generalized radial flow model was used to offset this limitation and simulate the groundwater flow in fractured aquifers. General groundwater flow equation and the fractal groundwater flow equation which is expressed as three-dimensional Cartesian coordinates can be represented as Eqs. (4) and (5), respectively. Comparing these equations with each other, we can see that the fractal dimension, n, which represents the degree of fractures in fractured aquifers is included in the fractal groundwater flow equations.

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}=\frac{\phi\mu c}{0.00633k}\frac{\partial p}{\partial t} \quad (4)$$

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}+\frac{(n-2)}{x^2+y^2}\left(x\frac{\partial p}{\partial x}+y\frac{\partial p}{\partial y}\right)=\frac{\phi\mu c}{0.00633k}\frac{\partial p}{\partial t} \quad (5)$$

On the basis of the fractal groundwater flow equation [Eq. (5)], a three-dimensional fractal groundwater flow model was developed. The model can simulate the groundwater flow not only in porous aquifers but also in fractured aquifers. The model was designed to be compatible with MODFLOW; however, other parameters such as permeability, hydraulic conductivity, and porosity can be used besides the hydraulic parameters used in MODFLOW, such as transmissivity, horizontal anisotropy factor (HAF), vertical conductance (VCONT), and specific storage. In the earlier version of the model developed in the previous study, the fractal dimension can only be input by a user with a specific value. The model developed in this study was designed to calculate fractal dimension directly from the Baecher analysis and the Levy-Lee analysis as well as to be input by a user with a specific value.

The Baecher analysis or the Levy-Lee analysis is one of the most popular methods to calculate the fractal dimension. The data to calculate the fractal dimension are a reciprocal of box size and the number of boxes required in the Baecher analysis, and the radius and the number of trace centers in the Levy-Lee analysis. To improve the accuracy of the calculation, the number of data is assigned to be more than 5. The program calculates a slope, that is a fractal dimension, from the logarithm of data using the least-squares method automatically. The correlation factors were applied to verify the reliability of the calculated values.

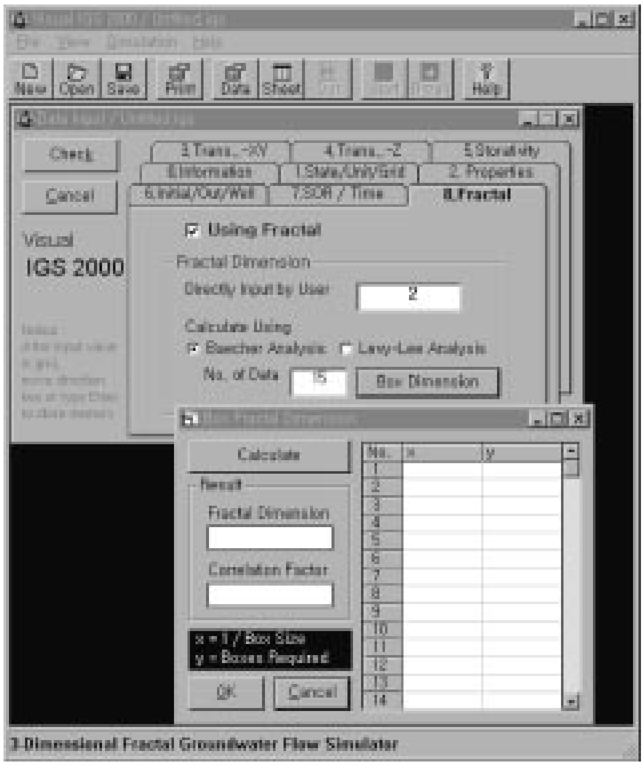


Fig. 3. VIGS (Visual Inha Groundwater Simulator) 2000.

An example of input screen to calculate the fractal dimension is shown in Fig. 3. The model was developed as a WINDOWS application by using Visual Basic language for convenience of data input, output, simulation process.

RESULTS AND DISCUSSION

1. Calculation of Fractal Dimension

A method to calculate the fractal dimension by the use of both the Baecher analysis and Levy-Lee analysis was developed to be used in the fractal groundwater flow model. To verify the usefulness of the calculated fractal dimension from the both methods, it is necessary to determine the fractal dimension from the field data by using both analyses and compare the results from the model with those from the field; however, it is difficult to calculate the fractal dimension exactly of the field from existing models because a great deal of data are needed to describe the complex and irregular geometry of the geologic features by using fractal dimensions. Moreover, the fractal dimension to be assigned in the model is only one to one domain, while spacial irregular geometry of fractures is too complex to be represented by only one fractal dimension.

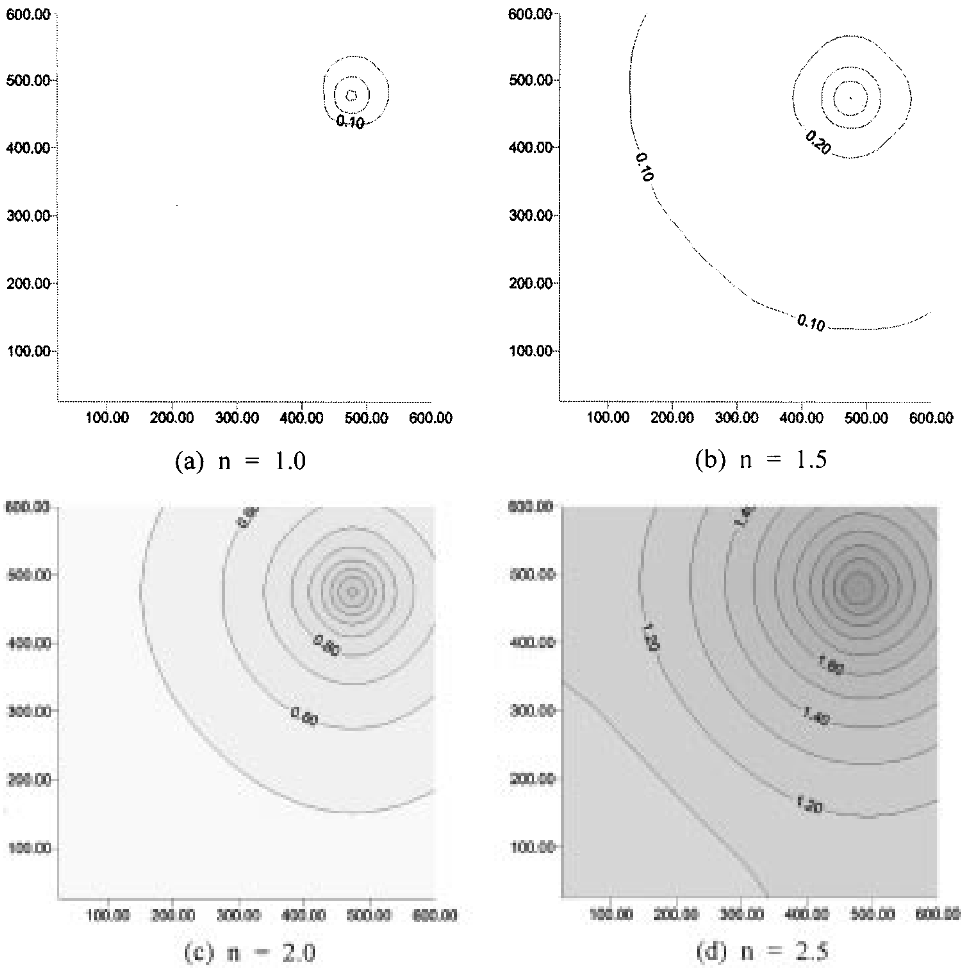
There are a number of subtleties to calculate the fractal dimension by using the above methods. They are all subject to sampling biases and the limitations of available data. As a result, we rely on the "preponderance of evidence" to decide the appropriate model and its parameters [Geier et al., 1988]. In this study, a method is attempted to determine the fractal dimension and apply it to the groundwater flow model. Further study in applying fractal dimensions to groundwater flow in the field would eliminate the limitations in application and improve the results of the model.

2. Modeling for the Case of Not Applying Fractal Equation

To prove the validity of the model developed in this study for the case of not applying fractal groundwater flow equation, the results of the model and MODFLOW were compared with each other. From the comparison of the results, it was found that both results were agreed exactly in the cases of isotropic and anisotropic single layered systems. However, a little difference within the tolerable error range existed between the two results in the cases of multilayered systems. Both the model results using the petroleum engineering data and the hydrogeology data agreed fairly [Chon and Choi, 1997].

3. Modeling for the Case of Applying Fractal Equation

The groundwater flow in fractured aquifers was simulated by applying a fractal groundwater flow equation. In applying a fractal equation, it should be considered that the region which the fractal dimension could affect is estimated to be several hundred meters [Hamm, 1995]. The modeling was performed to the fractured flow system by applying several dimensions, designated to be 1.0, 1.5, 2.0, and 2.5, respectively. The drawdowns at 24 hours after pumping were plotted in XY plane as shown in Fig. 4. When the fractal dimension was 2.0, both the results of using the fractal equation and not using the fractal equation and the result of MODFLOW were in exact agreement. And as the fractal dimension increased,



(a) n = 1.0 (b) n = 1.5 (c) n = 2.0 (d) n = 2.5

Fig. 4. Drawdown in XY plane at 24 hours after pumping with respect to each fractal dimension.

the drawdown along the distance from a well also increased. This shows the fact, that the fractal dimension represents the fracture density: as the fractal dimension increases, the fractal density and the influenced range along the radial distance from the well also increase.

CONCLUSION

1. Three-dimensional fractal groundwater flow model which can simulate the groundwater flow not only in porous aquifers but also in fractured aquifers using fractal groundwater flow equation was developed by finite-difference method.

2. A method which can calculate box fractal dimension used in the Baecher model and mass dimension used in Levy-Lee fractal model was also developed. The calculated fractal dimension can be used in the fractal groundwater flow model to simulate the groundwater flow in fractured aquifers as well as porous aquifers.

3. In the case of not applying fractal equation, the results of the model and MODFLOW agreed well in each case and the two models showed good compatibility with each other.

4. Both the results of the model using the fractal equation and not using the fractal equation, and the results of MODFLOW, agreed exactly for a fractal dimension of 2.0. Since the fractal dimension represents the fracture density, the drawdown along the distance from a well increases by the increase of the fractal dimension.

5. The model can be used with both parameters used in hydrogeology and petroleum engineering, while MODFLOW can only be used with parameters used in hydrogeology.

NOMENCLATURE

A : surface area of groundwater flow [L^2]
b : extent of the flow region [L]
h : hydraulic head [L]
k : absolute permeability [L^2]
K_f : hydraulic conductivity of the fractured system [LT^{-1}]
n : dimension of the fracture flow system [-]
p : pressure [M/LT^2]
r : radial distance from the center of the source (measured along the flow paths) [L]
S_{sf} : specific storage of the fracture system [L^{-1}]
t : time [T]

Greek Letters

α_n : area of a unit sphere in n dimension [-]
$\Gamma(x)$: gamma function [-]
μ : viscosity of water, $ML^{-1}T^{-1}$
ϕ : porosity [-]

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