

## A Tuning of the Nonlinear PI Controller and Its Experimental Application

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**Abstract**—Many industrial chemical process control systems consist of conventional PID and nonlinear controllers, even though many advanced control strategies have been proposed. In addition, nonlinear control methods are widely used even for linear processes to achieve better control performance compared with linear PID controllers. However, there are few tuning methods for these nonlinear controllers. In this work, we suggest new controller tuning methods for the error square type of nonlinear PI controller. These control methods can be applied to a large number of linear and nonlinear processes without changing control structures. We also propose new tuning rules for integrating processes. In addition, we suggest application guidelines for performing the proposed tuning rules at the pilot scale multi-stage level control system. Finally, in this work we confirmed good control performances of the proposed tuning methods through both simulation studies and experimental studies.

Key words: Nonlinear PI Controller, PID Controller, Error Square Controller Tuning, Experimental Application, ITAE Tuning Criteria

### INTRODUCTION

Many industrial chemical process control systems generally consist of conventional PID controllers [Cohen and Coon, 1953] even though many advanced control strategies have been proposed. A number of the processes, in which nonlinear PID controllers are introduced, have been rapidly increasing industrially in order to obtain good control performance, even though these processes are just linear processes.

However, the tuning and analysis of nonlinear PID controllers is much more difficult than that of conventional linear PID controllers [Sung and Lee, 1996, 1999; Lee, 1989], because the optimal tuning parameters depend on the magnitude of the set point or disturbances as well as the controller type [Jutan, 1989]. Also, very few guidelines for the application of the controller are available. In addition, to apply the nonlinear controller to integrating processes, it is necessary to find a new control structure or more efficient tuning methods [Auinn Jr. and Sanathanan, 1989].

In this work, we suggest application guidelines and tuning rules for the error square type of nonlinear PI controllers that are most frequently used in the process industry. We also propose tuning rules for integrating processes. The proposed tuning rules for integrating processes can be directly applied to an open-loop stable process without a change of its own strategy. In addition, we perform a real experimentation of the pilot scale level control system with the servo problem situation [Park et al., 1997]. We also obtain good control performance through this experimental application. It demonstrates that the new controller tuning methods can be applied to real chemical processes.

### THEORETICAL APPROACH

#### 1. PID Controller and Nonlinear PI Controller

The ideal PID controller algorithm is described as the following equation.

$$m(t) = K_c \left[ e + \frac{1}{\tau_i} \int e dt + \tau_d \frac{de}{dt} \right] \quad (1)$$

In a practical implementation, it is difficult to realize the derivative term in the PID controller. Also the inherent noise in the process measurement is amplified by the derivative term. One popular method of dealing with this drawback is to remove the derivative term and increase its proportional action to compensate the removed derivative action.

Here, we consider two kinds of error square type of nonlinear PI controllers as shown in following equations.

$$K_c = \tilde{K}_c (1 + \beta |e|) \quad (2)$$

Structure 1

$$m(t) = \tilde{K}_c (1 + \beta |e|) \left[ e + \frac{1}{\tau_i} \int e dt \right] \quad (3)$$

Structure 2

$$m(t) = \tilde{K}_c (1 + \beta |e|) e + \frac{\tilde{K}_c}{\tau_i} \int e dt \quad (4)$$

In the case of structure 1, this type of controller would permit us to use a low value of gain so that the system is stable near the set point over a broad range of operating levels with changing process gains. Another advantage of this nonlinear controller is that error square at the set point reduces the effects of noises. These error square controllers are suitable for average level control because the error de-

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viation never comes at zero. However, it might not be reasonable that the integral time constant of error-square controller changes with deviation. Therefore, the controller can be modified as structure 2. The controller of structure 2 can be intended to maintain a constant damping factor.

**2. Error Square Type of Controller Tuning**

In order to develop tuning formulae for these two algorithms, we use a similar approach of Lopez et al.'s [1967] using the first order plus time delay model. The disturbance  $D(s)$  enters directly into the process  $G(s)$ . In this research, we seek correlation among  $K\tilde{K}_c$ ,  $\tau_i/\tau$ , and  $\beta y_{sp}$  for various values of  $\theta/\tau$  to develop an optimal tuning method. Lopez et al. [1967]'s recommendation of using ITAE optimal tuning criteria was followed by Eq. (5)

$$\min(\text{ITAE}) = \int |e| dt \tag{5}$$

Unit step changes for the input load and the set point were introduced and corresponding ITAE values were calculated. The simulation was combined with a simplex search optimization algorithm with random initial guesses. Also, we can overcome the local minimum problem by using a wide range of random initial guesses.

**3. Load and Set Point Change**

The optimal tuning formulae were obtained by using linear regression of the dimensionless groups  $K\tilde{K}_c$ ,  $\tau_i/\tau$ ,  $\beta D_{max}$  and  $\beta y_{sp}$ . The nonlinear PI tuning parameters for the load changes are well described by the following equations. The range of the  $\theta/\tau$  value is described as the following equation.

$$0.1 \leq \theta/\tau \leq 1.5 \tag{6}$$

A large disturbance could make a deviation large enough to cause the loop gain to exceed 1. And these tuning methods have controlled values such that it is feasible that these types of controllers have several near optimal points.

A similar approach was used for the step set point change. The range of the  $\theta/\tau$  value is shown as the following equation.

**Table 1. Parameters for load changes**

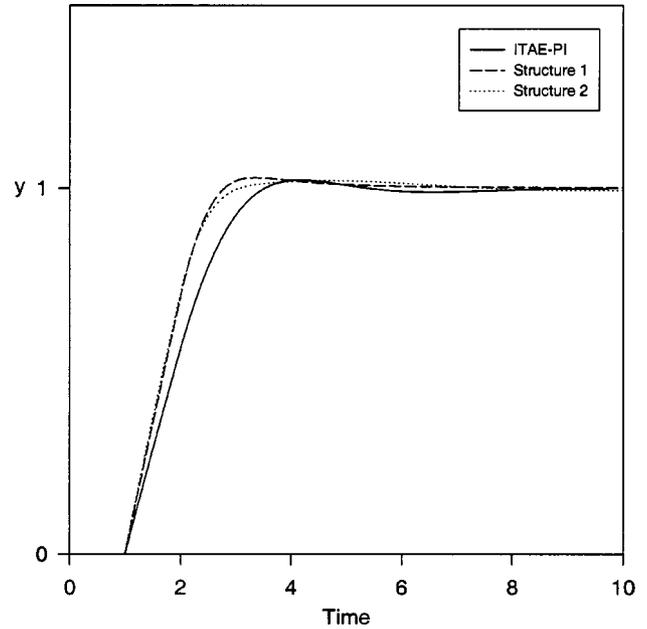
Structure 1	Structure 2
$K\tilde{K}_c = 0.152(\theta/\tau)^{-1.11}$	$K\tilde{K}_c = 0.1682(\theta/\tau)^{-1.164}$
$\tau_i/\tau = 1.096(\theta/\tau)^{-0.4833}$	$\tau_i/\tau = 1.49(\theta/\tau)^{1.308}$
$\beta \times D_{max} = 5.335(\theta/\tau)^{-0.4013}$	$\beta \times D_{max} = 10.61 - 6.164(\theta/\tau)$

**Table 2. Parameters for set point changes**

Structure 1	Structure 2
$K\tilde{K}_c = 0.02586 + 0.0982(\theta/\tau)^{-1.049}$	$K\tilde{K}_c = 0.1245(\theta/\tau)^{-0.965}$
$\tau_i/\tau = 0.997 + 0.08(\theta/\tau)^{-2.351}$	$\tau_i/\tau = 0.09 + 1.57(\theta/\tau)$
$\beta \times \Delta y_{sp} = 4.259 + 0.5362(\theta/\tau)^{0.667}$	$\beta \times \Delta y_{sp} = 4.12 + 1.05(\theta/\tau)^{0.763}$

**Table 3. Parameters for integrating processes in the case of load changes**

ITAE Criteria	IAE Criteria
$K\tilde{K}_c = 0.665(\theta/\tau)^{-1.004}$	$K\tilde{K}_c = 2.818(\theta/\tau)^{-0.396} - 2.104$
$\tau_i/\tau = 3.682(\theta/\tau)^{1.004}$	$\tau_i/\tau = 3.859(\theta/\tau)^{1.017} + 0.044$
$\beta D_{max} = 0.1588(\theta/\tau)^{-0.9806}$	$\beta D_{max} = 0.165(\theta/\tau)^{-0.864}$



**Fig. 1. Comparison of control performance between nonlinear PI controller and ITAE-PI controller for the servo problem ( $\theta/\tau=1.0$ ).**

$$0.1 \leq \theta/\tau \leq 1.5 \tag{7}$$

**4. Integrating Processes**

In this section, we propose an optimal tuning method that is obtained by the same method for the open loop stable process. We could find a new controller tuning method for the set point change and load change. However, in the step change we have the same control performance with the conventional ITAE-PI tuning method. Also in the ISE criteria, it is better controller tuning to eliminate the nonlinear characteristic term. In the load change, we could obtain a good tuning rule for the nonlinear PI controller.

**SIMULATION RESULTS**

Several different sets of responses were simulated for both disturbance and set point changes. As shown in Fig. 1, the proposed method shows better control performance compared with the ITAE-PI tuning method. Nonlinear PI controllers use larger proportional action and relatively less integral action than ITAE-PI. Also, we can obtain better control performance by using the nonlinear PI than that of the ITAE-PI controller and the IMC-PID controller [Rivera et al., 1986]. In addition, we confirmed through extensive simulation studies that the nonlinear PI controller also gives good control performance for the long time delay processes.

Consider the following process transfer function:

$$G_p(s) = \frac{e^{-3s}}{(s+1)^2(2s+1)} \tag{8}$$

Chen [1989] obtained the process model as the following equation with the on-line identification method.

$$G_m(s) = \frac{e^{-4.48s}}{2.95s+1} \tag{9}$$

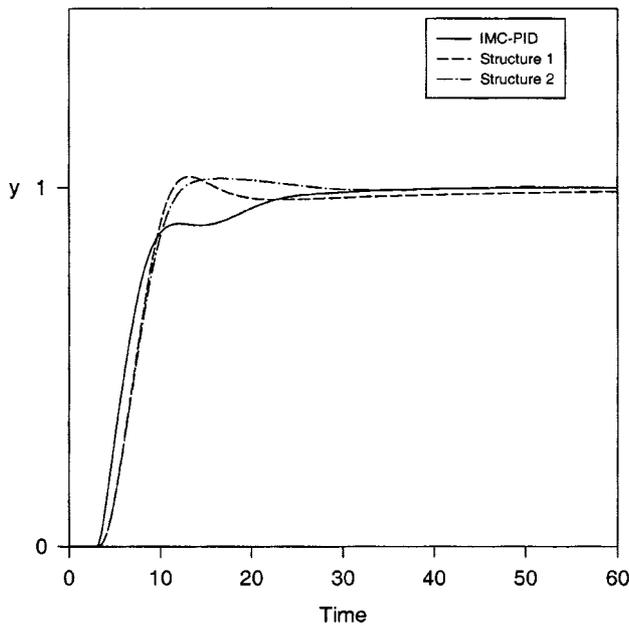


Fig. 2. Auto-tuning results of nonlinear PI controller.

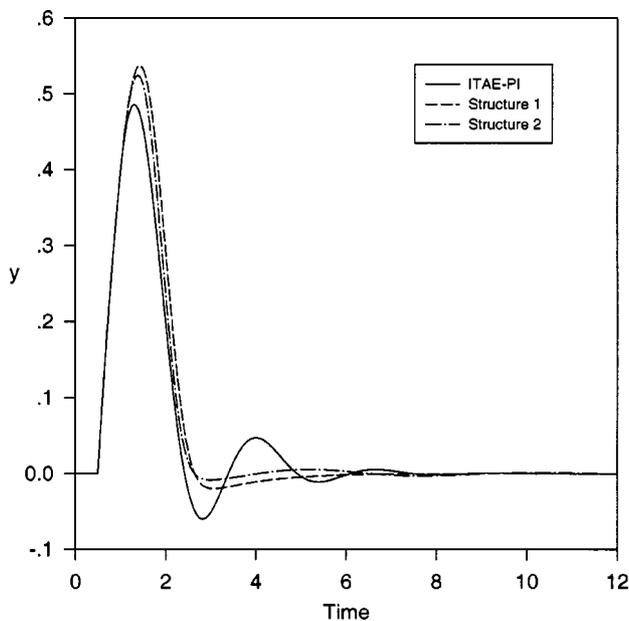


Fig. 3. Comparison of control performance between nonlinear PI controller and ITAE-PI controller for the regulatory problem ( $\Delta L=0.8D_{max}$ ).

In Fig. 2, we compare the response of the IMC-PID controller and the nonlinear PI controller tuned by the proposed tuning rule [Rivera et al., 1986].

In comparing the output responses to step disturbance of the ITAE-PI controller and the nonlinear PI controller, the maximum peak of the nonlinear PI controller is higher than that of the ITAE-PI controller. This means that the closed loop system with the nonlinear PI controller is less stable than the closed loop system with the conventional ITAE-PI controller at that point. However, the nonlinear PI controller has smaller settling time than the conventional ITAE-PI controller.

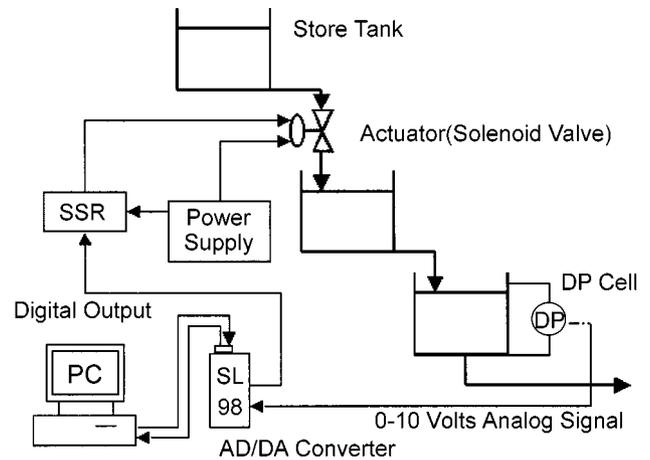


Fig. 4. Schematic diagram of level control system.

Table 4. Experimental conditions for level control system

Process identification	Control experiment
Sampling time; 3 sec	Sampling time; 3 sec
Time; 0-6,000 sec	Time; 0-6,000 sec
Sampled data; 2000	Sampled data; 2000
Step change; $\pm 0.1, \pm 0.15$	Set point change; $\pm 3.0$ cm

## EXPERIMENTAL STUDY

### 1. Introduction and Control Structure

The experimental system consists of two non-interactive sequential liquid-level tanks, and the schematic diagram is well described in Fig. 4. The control objective is to keep the level of the last tank at the desired value by adjusting the opening time of the solenoid valve attached below the store tank. Therefore, the manipulated and controlled variables are the opening time of the actuator and the average flow rate of the valve. We used the PWM (Pulse Wide Modulation) logic to operate the solenoid valve and determine the sampling time of the control system as 3 seconds, and also used the Microsoft Visual Basic language to hold the value of opening time obtained by the delay function and to calculate the controller output determined by the new controller tuning method. Experimental conditions are described in Table 4.

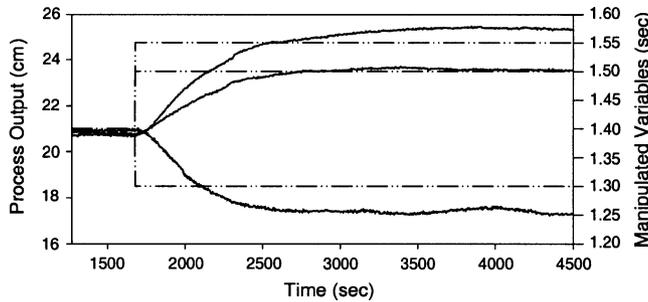
### 2. Process Identification

To identify the unknown process, there are many identification methods. To obtain the information of the process, we used the open loop step change of the controller output or manipulated variable as the test signal; that is the simplest and the accurate method for the first order plus time delay model. We also performed a set point change of the manipulated variable several times at a wide range of manipulated variables to obtain accurate process information and non-linearity. As a result, the process information had similar parameters at each process identification test. In particular, the values of  $\theta/\tau$  are obtained around 0.24 and the deviation of gain is not over 10% of average of the process gain. So we can obtain suitable process information and decide our reference process is a linear process.

The identification step starts with a step change of controller output or manipulated variable from 1.4 to 1.5 sec under the open loop

**Table 5. Results of the process identification**

Process gain	31.54
Process time constant	358.85
Process time delay	86.81



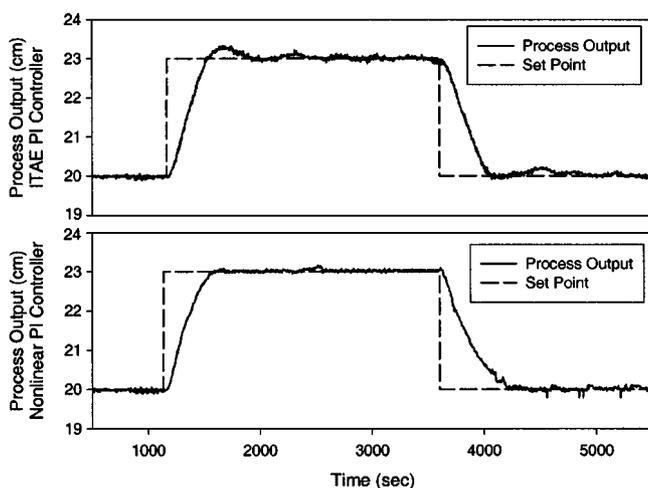
**Fig. 5. Process identification using upper and lower direction manipulated variable step change.**

**Table 6. The estimated controller parameters from the linear ITAE-PI tuning rule and the proposed tuning rule of nonlinear PI controller**

	ITAE-PI	Nonlinear PI
$\tilde{K}_c$	0.0682	0.01462
$\tau_i$	362.44	358.7943
$\beta$	-	0.8934

process system. After reaching another steady state, we performed another step change of the manipulated variable from 1.4 to 1.55 and 1.4 to 1.3 to obtain information on the non-linearity. From the sampled data during the intervals from 0 to 5,000 sec, we can identify the process and we can obtain the process information. Using numerical analysis, we can get the process information, as shown in the following Table 5 and Fig. 5. We can obtain these parameters by averaging the obtained parameters from each identification test.

We used these model parameters to calculate the tuning param-



**Fig. 6. Control performance of the nonlinear PI controllers compared with linear ITAE-PI controller for the set point change.**

ters of the conventional PI controllers as well as the nonlinear PI controllers. The optimum parameters for the linear ITAE-PI tuning methods and non-linear PI controller tuning methods are described in Table 6.

### 3. Real Application and Experiment

In this article, the experiment included process identification, parameter tuning of nonlinear controller, and set point tracking. Also, we determined the time interval as 3 sec. At the first steady state, the differential pressure transmitter indicates the process output value as about 20.0 cm.

We performed the set point tracking several times. From the above process information, we can use the proposed tuning methods for the nonlinear controller structure 1. In this experimental study, we obtained reasonable results that show the proposed tuning method of the nonlinear PI controller has good performance.

The results of the nonlinear PI controller tuning method are shown in Fig. 6. Using the proposed controller tuning method, we can obtain good control performance compared with ITAE PI controller. It shows that the nonlinear PI controller has the same rise time, less oscillation response and less controller output deviation compared with the conventional ITAE-PI controller. The nonlinear controller is more robust at the error deviation after rise time. Fig. 6 shows that the nonlinear PI controller is more suitable than any other linear controller at the highly noisy level control system. So we guess that this new controller tuning method can be applied to conventional and industrial processes effectively.

### CONCLUSIONS

We propose the application guideline and tuning formulae based on ITAE criterion for an error square type nonlinear PI controller. The nonlinear PI controller with tuning rule provides superior control performance for set point change in several processes. However, ITAE tuning criteria for the nonlinear PI controller seem to have the same control performance in disturbance rejection. In addition, from an experimental study of our new tuning method for level control system, it can be noted that the proposed method provides reliability and efficiency of the application to industrial processes. Moreover, we can obtain good control performance. Our pilot scale control system has a great deal of non-linearity, a long time delay and a large time constant. It also means that the proposed controller tuning method is suitable for highly nonlinear, noisy and industrial systems. Furthermore, in the case of commercial distillation columns, it is hard to get the process model exactly and we can calculate only the process gain. In these cases, the nonlinear PI controller is more suitable than linear controller because the nonlinear PI controller would permit us to use a low value of gain so that the system is stable near the set point over a broad range of operating levels with changing process gains. Another advantage of this kind of nonlinear controller is that error square at a set point reduces the effects of noise and the nonlinear PI controllers use larger proportional action and relatively less integral action than ITAE-PI.

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### NOMENCLATURE

$K$	: process gain
$K_c$	: controller gain (Conventional PID Controller)
$\tilde{K}_c$	: controller gain (Nonlinear PI Controller)
$\tau$	: process time constant
$\theta$	: process time delay
$e$	: amount of the process error
$L$	: step load disturbance
$D_{max}$	: maximum load disturbance
ITAE	: integral of time weighted absolute error
$\tau_I$	: controller integral time constant
$\beta$	: non-linearity parameter
$y_{sp}$	: process set point

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