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Effects of Pore Size, Suspension Concentration, and Pre-Sedimentation on the Measurement of Filter Medium Resistance in Cake Filtration

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Abstract−Theoretical and experimental study was conducted on the factors influencing the measurement of filter medium resistance by Ruth's equation. It was determined that a filter medium having small pore size does not always give high filter medium resistance. The appropriate pore size of filter medium for filtration was analysed. The resistances of filter media measured with thick suspensions by Ruth's equation have negative values. This phenomenon can be analyzed as the effect of sedimentation during the long filtration time due to thick suspension. When sedimentation occurs before the start of filtration, the filter medium resistance measured by Ruth's equation gives a large value. It was determined that the result of the filtration of sediment was included in the filter medium resistance. A new method for measuring filter medium resistance by the filtration of the sediment is proposed. This method excludes the effects of suspension concentration and pre-sedimentation.

Key words: Filtration, Cake Filtration, Filter Medium, Resistance of Filter Medium, Pore Diameter, Filtration-Permeation, Filtration with Sedimentation

INTRODUCTION

The importance of filter medium in cake filtration was well understood before the beginning of filtration theory. In the history book "Chun Qiu' Confucius [BC 479] wrote that a feudal nation named 'chu' was invaded by other nations because the 'chu' did not deliver the filter media for ritual wine to the great kingdom 'zhou'. Without the filter medium, the wine could not be filtered clear enough to be fit for use at national ceremonies.

After establishment of the theory of cake filtration by Ruth [1946], it has been considered that cake filtration is performed by filter cake and the role of filter medium is supporting the filter cake only. Grace [1953] stated that the filter medium resistance could be neglected when it is less than 1% of final resistance of filter cake. But the filtration time changes up to several-fold according to filter media at the same operating conditions. Different values of filter medium resistances are frequently measured with the same material filtered, medium, and pressure by altering the concentration of suspension and other operating conditions.

In this study we want to analyse the method for measuring filter medium resistance proposed by Carman [1938], which has been used until now, and then to find the conditions for exact measurement of filter medium resistance. At first, the resistances of clean filter media were measured by permeating particle removing water through the media. The effects of suspension concentration and the sedimentation time before the start of filtration on the filter medium resistance were analysed.

THEORY

1. Measurement of Filter Medium Resistance in Cake Filtration

The so-called Ruth's Eq. is usually applied for analysing cake filtrations.

$$\frac{dV}{dt}=\frac{\Delta p}{\mu(\alpha_{av}CV+R_m)} \tag{1}$$

where t is filtration time (s), V is filtrate volume per unit filter area (m^3/m^2), Δp is applied pressure for filtration (Pa), μ is liquid viscosity (kg/ms), α_{av} is average specific cake resistance (m/kg), R_m is filter medium resistance (m^{-1}), and C is cake mass formed per unit volume of filtrate (kg/m^3).

By using the total and solid mass balance for filtration, C can be given by

$$C=\frac{\rho S}{1-S/S_c} \tag{2}$$

where ρ is liquid density (kg/m^3), S is mass fraction of solids in suspension (−), and S_c is mass fraction of solids in cake (−).

Actually, Carman [1938] proposed the above equations, and Ruth did not term it by his name. Tiller [1960] termed it as "Ruth's equation" and used this name for more than 50 years in his many papers.

The method determining the resistance of filter medium proposed by Carman [1938] is as follows. Combining Eq. (1) and Eq. (2), and rearranging leads to

$$\frac{dt}{dV}=\frac{\mu\alpha_{av}C}{\Delta p}V+\frac{\mu R_m}{\Delta p} \tag{3}$$

With precise data between filtrate volume and filtration time, the $\Delta t/\Delta V$ can be calculated. A typical plot of $\Delta t/\Delta V$ vs. V is shown in Fig. 1.

When there are no side effects such as sedimentation or pre-sed-

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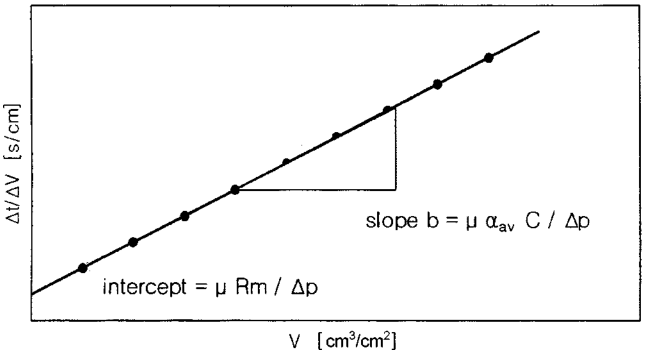

Fig. 1. A typical result of filtration experiment.

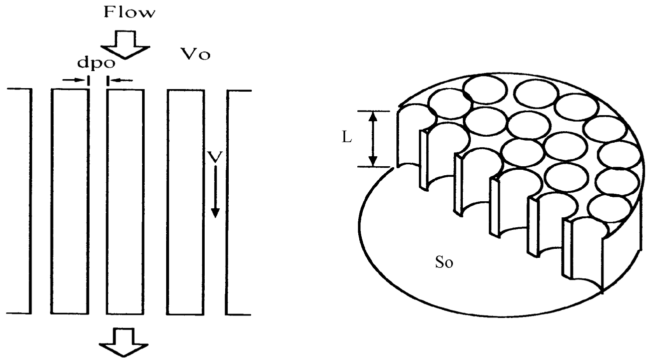

Fig. 2. Schematic diagram of a filter medium composed of homogeneous pores.

imentation during operation, the plot generally gives fairly good straight line. This relationship was discovered by Sperry [1917] and has been used until now.

By Eq. (3), the slope of straight line b in Fig. 1 is $\mu\alpha_{av}$ C/Δp. Carman [1938] calculated α_{av} with the slope b obtained by experiment, the value C, liquid viscosity, and filtration pressure. This method has been used for more than 60 years as a prototype for obtaining average specific cake resistance. The last term $\mu R_m/\Delta p$ in Eq. (3) is the intercept in Fig. 1, and the resistance of filter medium R_m can be acquired.

This method has widely been employed for calculating filter medium resistance for a long time [McCabe et al., 1976], but has defects as follows. Filter media having larger size pores than particle diameter are frequently used in cake filtration. Considerable amount of particles passes through the filter medium at the initial period of filtration, then the resistance of filter medium cannot be measured by the above method. The intercept, in this case, usually shows negative value which means the negative filter medium resistance.

Occasionally, starting a filtration immediately after putting the suspension in a cell is not possible. The sealing time for a batch pressure filter requires over one minute. Sedimentation takes place during the period, and sediment is deposited upon filter medium. Bockstal et al. [1985] and Tiller [1990] assumed that the sediment is filter cake, but Yim [1999] explained the difference between sediment and cake. Anyhow, the existence of sediment upon a filter medium before filtration does not result in a correct medium resistance for filtration.

When sedimentation occurs during filtration, a false filter medium resistance could be measured by above method. Yim [1999] also suggested that accurate average specific cake resistance cannot be measured for a filtration with sedimentation. Without the phenomena mentioned above, Eq. (3) can be applied to filtration with sufficient reliability except for the initial period of filtration [Yim, 1999].

In this study, an experimental technique named filtration-permeation [Yim, 1986] is used together with conventional filtration. After filtration, the permeation of particle-removed water through a preformed cake follows in filtration-permeation. Average specific cake resistance is also measured with the permeation velocity through cake.

2. Calculation of Average Pore Diameter by Permeation through Filter Medium

A simple and fast method for measuring average pore diameter and the number of pores per unit area is proposed in this study.

The average pore diameter can be calculated by permeating particle-removed water through a filter medium, so measured pore diameter relates directly to the fluid flow. Matrix of a filter medium can be illustrated as Fig. 2, i.e. a filter medium is composed with cylindrical pores. Yim [2000b] also proposed another model of filter medium composed of long fibers, but in this study we use the pore model only.

When the particle-removed water passes through a filter medium with pressure Δp, the true flow rate $\overline{V}$ through the porous filter medium can be expressed by using empty tower velocity $\overline{V}_0$ and porosity ε by $\overline{V}_0/\varepsilon$. The pores in the filter medium are assumed to be composed of cylinders, and each cylinder has diameter d_{po} and height L. During permeation, the total area contacting water and pores $A_{s,po}$ is the product of the area of one pore ($\pi d_{po}\times L$) and the total number of pores N_{po} as

$$A_{s,po}=\pi d_{po}L\times N_{po} \tag{4}$$

The total number of pores can be calculated as Eq. (5).

$$N_{po}=\frac{\text{total void volume in filter medium}}{\text{volume of one pore}}=\frac{S_oL\varepsilon}{\pi d_{po}^2L/4} \tag{5}$$

where S_o is the filter medium area. The total area contacting filter medium and water $A_{s,po}$ is

$$A_{s,po}=4S_oL\varepsilon/d_{po} \tag{6}$$

The hydraulic radius r_H could be calculated as follows by definition.

$$\begin{aligned} r_H &\equiv \frac{\text{cross}-\text{sectional area}\times\Delta L}{\text{wetted perimeter}\times\Delta L} \\ &=\frac{\text{void volume}}{\text{surface area contacting solid and water}} \\ &=\frac{\text{void volume}}{A_{s,po}}=\frac{S_oL\varepsilon}{4S_oL\varepsilon/d_{po}}=\frac{d_{po}}{4} \end{aligned} \tag{7}$$

The drag force in fluid flow is usually expressed [McCabe et al., 1995] by

$$F_D=A_{s,po}\frac{k_1\mu\overline{V}}{g_cr_H} \tag{8}$$

When the expressions about $A_{s,po}$ in Eq. (6), r_H in Eq. (7), and $\overline{V}_0$ are introduced to Eq. (8), it becomes

$$F_D = 16\frac{k_1 L \mu \varepsilon \overline{V}_o S_o}{g_c d_{po}^2 \varepsilon} \tag{9}$$

This drag force is exerted on the void portion of filter medium εS_o; the pressure for flow Δp is related by

$$\Delta p = \frac{F_D}{\varepsilon S_o} = 16\frac{k_1 L \mu \overline{V}_o}{g_c d_{po}^2 \varepsilon} \tag{10}$$

The average pore diameter d_{po} is

$$d_{po}^2 = \frac{16 k_1 L \mu \overline{V}_o}{g_c \Delta p \ \varepsilon} \tag{11}$$

Thus the average pore diameter d_{po} is obtained by measuring flow rate $\overline{V}_0$ of particle-removed water through a filter medium. Naturally, the thickness of filter medium L, the porosity ε, and the viscosity of liquid μ are necessary for the calculation.

The number of pores per unit filter medium area can be obtained from Eq. (5).

$$\frac{N_{po}}{S_o} = \frac{4\varepsilon}{\pi d_{po}^2} \tag{12}$$

In conclusion, the average pore diameter and the number of pores of filter medium can be calculated with the flow rate and the porosity.

EXPERIMENTAL

1. Filtration Apparatus

The filtration cell of a conventional Büchner funnel has enlarged cross sectional area at the upper part of filter medium to contain more volume of suspension. The particles settle upon the edge of enlarged portion. To prevent this, we employed a 200 mL straight cylinder with constant diameter as the filtration cell.

A 400 mL graduated cylinder was used for measuring the volume of filtrate. This cylinder was connected to an aspirator and 40 L air tank simultaneously. Before starting filtration, the connection to the filtrate cylinder was locked. The air tank was depressurized to a desired pressure by using the aspirator. Then the aspirator was stopped, and filtration began with the vacuum in air tank.

By this experimental technique, pressure fluctuation by aspirator could be prevented. Normally 200 mL filtration and 200 mL permeation were performed in this study, i.e., the volume change during experiment was 400 mL. The volume of the air tank was 40 L, so there was 1% decrease in pressure from the beginning to the end of experiment. This technique was developed by the author [1986, 1990].

2. Filter Medium and Particles

Eight filter media Advantec Toyo 5A, Advantec Toyo 5C, Whatman No. 4, Whatman No. 5, Whatman GF/C, polysulfon and cellulose acetate membrane of Micro filtration system Co., and a coffee filter of Bonmac Co., were used in this study.

Calcium carbonate manufactured in Jin chemical Co. (A) was chosen to know the effect of average pore diameter of filter medium to the resistance of filter medium.

Particle size distribution of the calcium carbonate was analysed by Malvern Mastersizer, and the particle diameters are from 0.3 to 50 μm. The average volumetric particle size is 8.54 μm, and about 10 vol% of the particles are smaller than 1 μm.

The calcium carbonate manufactured by Yakuri Pure Co. (B) was used for the experiment to know the influence of suspension concentration and pre-sedimentation on filter medium resistance. Particle size analysis by the same method gives a distribution between 0.2 to 60 μm, the average particle size of 10.6 μm, and also 10 vol% of the particles were below 1 μm.

The difference between two kinds of calcium carbonate by particle size analysis is not prominent. The calcium carbonate A showed large value of average specific cake resistance and filter medium resistance for the filter medium Toyo 5C, but calcium carbonate B did not show this phenomenon.

Calcium carbonate had been dried in a drying oven for 5 hours at 105 °C, then stored in a desiccator. Fast measurement of weight was performed to prevent the adsorption of humidity, then the particle was put into particle-removed water and agitated with magnetic stirrer during 30 minutes.

The water passing through the membrane of reverse osmosis was refiltered with 0.2 μm membrane filter just before the application to eliminate the possibility of contamination by particles.

RESULTS AND DISCUSSION

1. Determination of Average Pore Size by Permeation of Particle-Removed Water

To know the property of filter media, the volume of permeate as a function of time was measured by permeating particle-removed water through the filter medium at the pressure of 1.47 kPa. The permeation time t and V, which is the volume of permeate divided by filter medium area, are presented in Fig. 3.

The filter media Toyo 5A and Whatman No. 4 show almost the same experimental results. The results with Toyo 5C and Whatman No. 5 are quite different from above, i.e., the time needed for the permeation of certain volume requires several-fold. Whatman GF/C shows similar results as Toyo 5A and the coffee filter gives a little faster permeating velocity, as these results are not shown in Fig. 3. At 1.47 kPa, permeation did not take place through the micro filtration (MF) membranes, so the pressure was elevated to 10 kPa.

All of the experimental results in Fig. 3 show very good linearity. This means that the rate of permeation preserved from the beginning to the end. The reciprocal of the slope of the straight line is the velocity of permeation of particle-removed water, and is termed

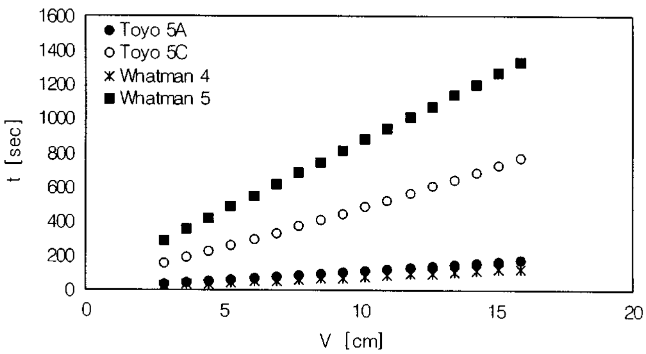

Fig. 3. Permeation of particle removed water through various filter media at 1.47 kPa.

Table 1. Average pore diameters and retained diameters of various filter media

	Pore diameter (μm)	Retained diameter (μm)
Toyo 5A	2.8	7
Toyo 5C	0.95	1
Whatman GF/C	2.2	1.2
Whatman No. 4	3.6	20-25
Whatman No. 5	1.2	2.5
Coffee filter	4.2	Not available
MF (CA)	0.43	0.2
MF (PS)	0.51	0.2

$\overline{V}_0$ in Eq. (9). Thus, the average pore diameter of filter medium d_{po} can be obtained with the thickness of filter medium L, porosity ε, and the viscosity μ.

The average pore diameters of filter media by this method and the retained particle diameters of filter media given by the manufacturer are shown in Table 1. These two kinds of diameters do not coincide well. It is not so strange because the two kinds of measurements are totally different in method and purpose. The authors think that there exists the tendency of correspondence between the two kinds of diameters.

The eight filter media are divided into three groups according to the above results. We named "large pore filter medium" to the coffee filter of 4.2 μm, Whatman No. 4 of 3.6 μm, Toyo 5A of 2.8 μm, and Whatman GF/C of 2.2 μm pore diameter. The Whatman No. 5 of 1.2 μm and Toyo 5C of 0.95 μm are named as "middle size pore filter medium". The MF membrane by polysulfon of 0.51 μm and cellulose acetate of 0.43 μm are classified as "small pore filter medium". The classification is totally optional for analyzing the experimental results obtained in this study.

2. The Average Pore Sizes and Resistances of Filter Medium Measured by Filtration

The resistance of the filter medium is obtained with the experimental filtration results and Eq. (3). In principle, the medium composed of large pores has smaller value of resistance than that composed of small pores when the same number of pores per unit area is postulated.

2-1. Resistance of Filter Medium Composed of Large Pores

The experimental result of "filtration-permeation" of 1% calcium carbonate suspension with filter medium Whatman GF/C is presented in Fig. 4.

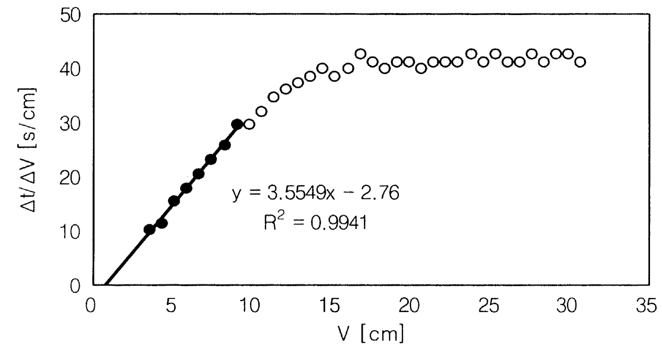

Fig. 4. Filtration-permeation of 1% $CaCO_3$ suspension at 0.5 atm with the filter medium Whatman GF/C.

Filtration takes place in the region of straight line with slope, and thereafter a permeation period follows. During the permeation period, the particle-removed water permeates through the preformed cake. The average specific cake resistance obtained from the filtration period is 1.9×10^{11} m/kg, and that from the permeation period is 1.4×10^{11} m/kg. The difference between two values is caused by the sedimentation during filtration which does not exist during permeation [Yim and Ben Aim, 1986].

In Fig. 4, the intercept value with y axis of the extrapolation line for the filtration data is −2.76 s/cm, i.e., the negative value. According to Eq. (3), this value must be $\mu R_m/\Delta p$. As the viscosity and pressure drop have positive values, the filter medium resistance R_m is negative. In an equation, this negative value means that the existence of the filter medium makes faster flow than without a filter medium. But it is not possible.

We explained this phenomenon as follows. When the small particles pass through the large pores, the mass of cake actually formed upon the filter medium is smaller than that calculated by Eq. (2). So the resistance of cake does not increase in proportion to V, the value of $\Delta t/\Delta V$ is smaller than that expected by the equation.

After the above phenomenon progresses for a short period, normal cake begins to be formed. From the moment of normal cake formation, the relationship of $\Delta t/\Delta V$ vs V shows a straight line. The first point in Fig. 3 is measured after the normal cake formation. By the particles passed through the large pores, the intercept of the extrapolation line with y axis gives negative value.

This could be proved experimentally if it were possible to have earlier data than that we have in Fig. 3. As the author analysed [Yim and Kim, 2000a], to obtain the filtration data of the initial period is very difficult because of the fast rate of flow, large flux and the sudden change of flux. It could be expected that the actual result begins

Table 2. Resistances of filter media measured by filtration (Whatman GF/C, Toyo 5A, Whatman No. 4 and coffee filter)

	Pore diameter (μm)	R_m ($\times10^{10}$ m^{-1})
Whatman GF/C	2.2	0.21
	2.2	−1.12
	2.2	−1.5
	2.2	−0.91
	2.2	−0.91
Toyo 5A	2.8	−1.7
	2.8	−0.82
	2.8	0.864
	2.8	−1.4
	2.8	−1.3
Whatman No. 4	3.6	0.82
	3.6	0.19
	3.6	0.07
	3.6	−1.5
	3.6	−0.54
Coffee filter	4.2	0.89
	4.2	0.14
	4.2	1.47

from the origin and is connected to the first experimental result in Fig. 3 with a concave curve.

Table 2 shows the R_m values obtained from filtration at the same condition with filter media of large pores. Various values of R_m were measured with the same kind of filter medium at the same condition. The reason for data dispersion will be discussed later.

Whatman GF/C and Toyo 5A have four negative values of R_m for 5 times experiments. Whatman No. 4 has two negative values over five experiments. Whatman No. 4 and Toyo 5A have larger pores than Whatman No. 5. It is natural that the larger pores permit passing through of more particles before the formation of cake. Thus the filter media having larger pores could have negative values of R_m.

But the coffee filter which has the largest pores shows three positive values of R_m. It was suggested that the fiber of the coffee filter has many branches to give clear coffee from the beginning by forming a cake.

All of the experiments except one (17 over 18) in Table 2 have a value of R_m below 1×10^{10} m^{-1}.

2-2. Resistance of Filter Medium Composed of Small Pores

The experimental result of "filtration-permeation" of 1% calcium carbonate suspension with polysulfon MF membrane is shown in Fig. 5. It resembles Fig. 4, but the intercept of the extrapolated line for the filtration data with y axis is positive, i.e., +6.2 s/cm. The filter medium resistance calculated by this intercept is 2.9×10^{10} m^{-1}. The large resistance is natural because the filter medium is composed of 0.51 μm pores. The mean pore diameter in Fig. 4 is 2.2 μm.

Table 3 shows the R_m values obtained from filtration at the same condition with filter media composed of small pores. In spite of the same condition, there is large dispersion in values, but no results having negative values.

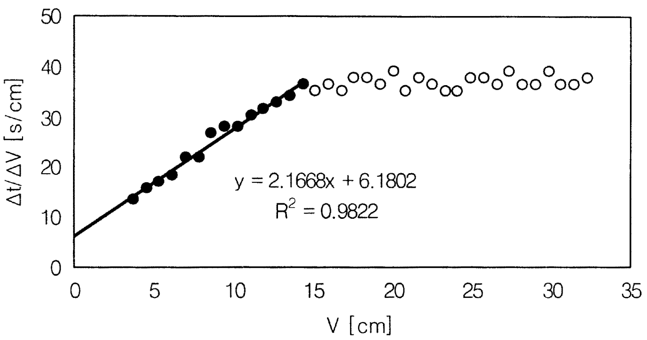

Fig. 5. Filtration-permeation of 1% $CaCO_3$ suspension at 0.5 atm with polysulfon MF membrane.

Table 3. Resistance of filter medium (cellulose acetate and polysulfon MF membrane)

	Pore diameter (μm)	R_m ($\times10^{10}$ m^{-1})
MF (CA)	0.43	1.38
	0.43	3
	0.43	1.4
MF (PS)	0.51	0.12
	0.51	2.85
	0.51	0.3

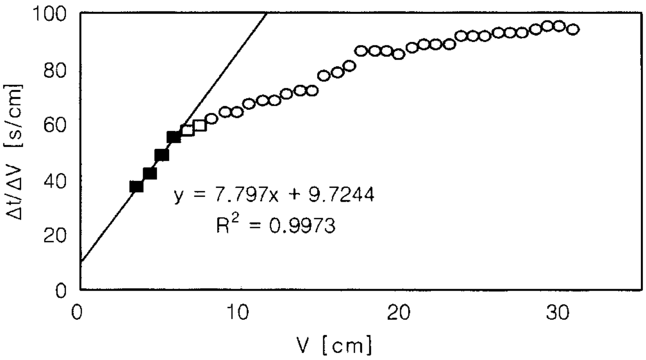

Fig. 6. Filtration-permeation of 1% $CaCO_3$ suspension at 0.5 atm with Whatman No. 5.

2-3. Resistance of Filter Medium Composed of Middle Size Pores

Fig. 6 shows the result of the same filtration-permeation except for the filter medium, in this case Whatman No. 5. The tendency is different from former results, and the differentiae can be summarized as follows.

First, the period of filtration is short compared with the large and small pore media. Second, the value of Δt/ΔV increases continuously even in the permeation period, and does not attain to an equilibrium value. Third, the slope of the straight line during filtration is large, and the average specific cake resistance calculated by this slope is 3.21×10^{11} m/kg. This average specific cake resistance is 1.7 times greater than that measured with Whatman GF/C. The value of Δt/ΔV in this experiment is about two times larger than the former experiments. Contrary to the conventional conception, the filter medium has an influence on the experimental value of average specific cake resistance.

The filtration-permeation with Toyo 5C shows the same trend. This phenomenon is obtained from the middle size pore media, not from the large nor small pore media. The average pore diameter of Whatman No. 5 by the method proposed in this paper is 1.2 μm, and that of Toyo 5C is 0.95 μm.

The resistances of Toyo 5C and Whatman No. 5 measured at the same condition are shown in Table 4. On the whole, the values are large compared with former results and there is no negative filter medium resistance.

The average particle diameter of calcium carbonate is 8.54 μm, but about 10 vol% is smaller than 1 μm and larger than 0.5 μm.

Table 4. Resistance of filter medium (Toyo 5C, Whatman No. 5)

	Pore diameter (μm)	R_m ($\times10^{10}$ m^{-1})
Toyo 5C	0.95	11
	0.95	4.2
	0.95	11
	0.95	9.3
	0.95	6.1
Whatman No. 5	1.2	20
	1.2	13
	1.2	22
	1.2	12
	1.2	9.7

The average pore diameter of filter media mentioned here is about 1 μm. The small particles can block up the pores. It was determined that the blocking up the pores with small particles was measured as the increase of filter medium resistance and average specific cake resistance.

And it is also possible that the small particles migrate through cake and block up the small pores in the cake and in the filter medium. This phenomenon can increase the value of $\Delta t/\Delta V$ during permeation period. With these concepts, the three differentiae for the filtration with middle size pore medium could be explained.

The amount of particles about 0.5 μm diameter is very small, so the filtration with MF membranes which have the pore sizes of 0.43 and 0.51 μm is not influenced by the blocking up.

For the filtration with large pore medium, Whatman GF/C of 2.2 μm pore size, in Fig. 4 the blocking up is totally possible by particles about the pore size. In fact, the passing through of small particles through pores is the dominant phenomenon at the initial period of cake formation, and the large particles remain over the filter medium. With these large particles, normal cake begins to form. The author thought that the conversion was very rapid. Thus the R_m is calculated as a negative value by experimental data due to above phenomena.

In conclusion, the filtrations of the suspension of calcium carbonate with Whatman No. 5 and Toyo 5A show that the filter medium has a great influence on the filtration time. The time needed for the filtration of 200 mL with Whatman GF/C was 386 s, and with Whatman No. 5 was 842 s, that is 2.2 times longer.

2-4. The Overall Comparison Between the Size and the Resistance of Filter Media

The whole experimental results mentioned above are shown in Fig. 7. The three points which look like two points at the right end side are the experimental resistances of the coffee filter. The R_m values are almost the same, very small, and all positive. By analysis already mentioned, positive values of R_m mean that there is no passing through of particles at the initial period of filtration.

The five points (looks like four points) at the position of 3.6 μm are the results with Whatman No. 4. There is a little dispersion in R_m, and two points have negative values. At 2.8 μm, five points, which are shown as four points, are the results with Toyo 5A, four points are negative. The number of points at 2.2 μm are five, and four points have negative values. These points are obtained from Whatman GF/C.

Above results are obtained from the large pore filter media. On the whole, passing of the particles through filter medium is observed at the initial period of filtration, and the phenomenon gives the negative values of R_m. But after a little while, typical cake filtration follows, and reliable average specific cake resistances are measured. Until now, almost all cake filtrations have been performed with these kinds of filter media. These large pore filter media are adequate for measuring average specific cake resistance, but inadequate for measuring the resistances of filter media. Because a negative filter medium resistance, which is theoretically not possible, is frequently obtained.

Next, the five results with Whatman No. 5, pore diameter of 1.2 μm, are shown. The Rm is situated from 9.7×10^{10} m^{-1} to 22×10^{10} m^{-1}, and this is a broad distribution. The large values of R_m mean that the pores are blocked with particles. The broad distribution of data signifies that the blocking does not happen each time at the same degree. The five times experimental results with Toyo 5C are shown at 0.95 μm. The resistances are smaller than previous results, but the extent of dispersion is also large.

The results for 0.51 and 0.42 μm are performed with MF membranes, three times experiments with each membranes. The value of R_m is a little larger than that with large pore medium. It is natural that the medium of small pore size has large resistance. The distribution of experimental results is not broad.

The pore sizes above are obtained through the permeation of particle-removed water.

3. Resistances of Filter Media and Concentration of Suspension

The mass fraction of solid S in Eq. (2) changes when filtration is carried out with the various concentrations of suspension. The solid mass in cake per unit volume of filtrate C changes with S by Eq. (2). The new value of dt/dV in the experiment is measured corresponding to the new C for different suspension concentration based on Eq. (3). According to traditional theory the average specific cake resistance and the filter medium resistance do not change with the concentration of suspension at the same filtration pressure.

The resistances of filter medium Toyo 5C are shown in Fig. 8 obtained from the filtration of calcium carbonate fabricated by Yakuri Pure Co. for suspension concentrations of 1, 2, 3, 4 and 8%. The filtrations with this calcium carbonate for Toyo 5C do not exhibit the blocking of filter medium, so to speak, giving normal filtration-permeation data.

Contrary to the widely approved theory above mentioned, the resistance of the filter medium decreases as the suspension concentration increases. When the concentration exceeds 3%, the values of resistance are negative and arrive at -1.3×10^{11} m^{-1} for 8% sus-

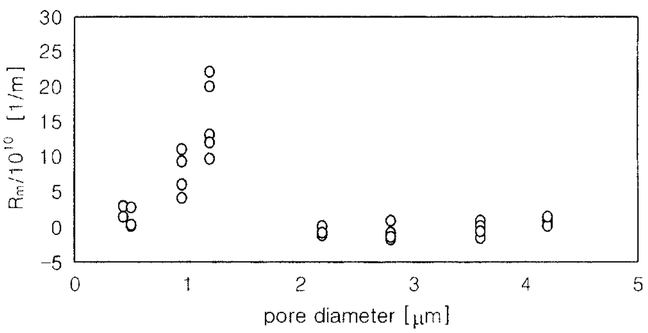

Fig. 7. Resistances of filter medium of various pore size.

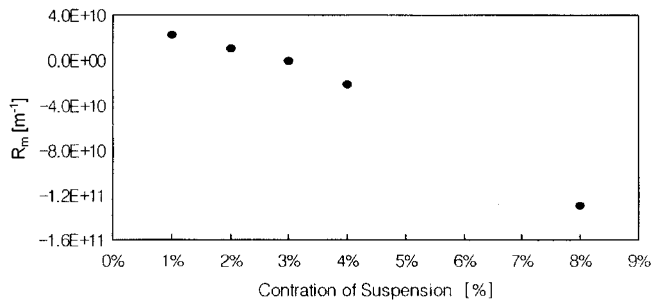

Fig. 8. Resistances of filter medium for various suspension concentration at 0.5 atm with Toyo 5C.

Table 5. Time required for 90 mL filtrate

Conc. of susp.	Time (s)
1%	74
2%	113
3%	194
4%	244
8%	493

pension. Explanation of this phenomenon with previous concepts is impossible.

The time for filtration depends deeply on the suspension concentration when other conditions are the same. Table 5 shows the time needed for 90 cm^3 filtrate according to suspension concentration at 0.5 atm and 12.3 cm^2 filter area. The time for the 1% suspension was 74 s and that of 4% was 244 s, i.e., about three-fold increase. For 8% it was 494 s, and this is about seven-fold increase in filtration time. Sedimentation in suspension could take place for such a long filtration time.

In the early stage of filtration, the progress of filtration is faster than that of sedimentation, so the influence of sedimentation on filtration is negligible. The filtration time for dilute suspension such as 1% is comparatively short and the effect of sedimentation is not significant. For filtration of a thick suspension, the time for filtration is sufficiently long enough for the development of sedimentation in suspension.

Tiller et al. [1995] attempted to analyze the phenomenon of filtration with sedimentation, but the analysis did not include the variation of suspension concentration by sedimentation. The author [Yim, 1999] explained the phenomena as follows. At the start of filtration, the concentration of suspension is the initial concentration. Then the rate of filtrate becomes slow when filtration progresses and cake forms to some extent. The thick suspension yields more quantity of cake with relatively small amount of suspension, and the procedure becomes slow. By the slow rate of filtration, sedimentation takes place upon the horizontal cake, so the concentration of suspension entering cake becomes larger than the initial concentration.

The existence of clear supernatant observed at the upper side of suspension during filtration proves that the initial concentration was not supplied to the cake during the period. It is reasonable to suppose that the sedimentation during filtration thickened the concentration of suspension entering to the cake, i.e., the mass fraction of solids of suspension S in Eq. (2) should have larger value than initial value. The larger S enlarges the value of C by Eq. (2). Therefore, filtration is performed with the larger value of C for the thick suspension except for the initial period. According to Eq. (3), the slope of the graph $\Delta t/\Delta V$ vs. V is $\mu\alpha_{av}C/\Delta p$. The large value of C yields the large slope. The large slope does not begin with the start of filtration. It comes out after formation of cake to a certain thickness.

In Fig. 9, the experimental filtration result of calcium carbonate suspension of 8% is presented. The intercept of extrapolated line with y axis from 8% data has a negative value.

Black points represent the filtration period, and permeation begins at the white points. The intercept is −25.8 s/cm and the resistance from this value is -1.3×10^{11} m^{-1}. From the experiment of 1% suspension for which the influence of sedimentation is relatively small,

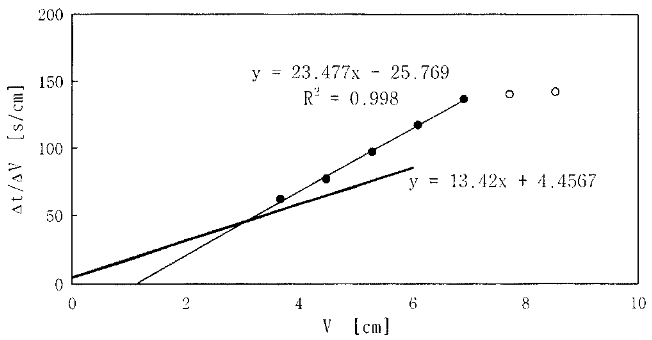

Fig. 9. Filtration-permeation of 8% $CaCo_3$ suspension at 0.5 atm through Toyo 5C with the calculated line from 1% data.

the average specific cake resistance and the resistance of filter media, 2.2×10^{10} m^{-1}, is obtained. The thick line in Fig. 9 is calculated from above values of 1% and the initial 8% concentration with Eq. (3).

As sedimentation does not influence the filtration at the initial period, it can be postulated that the filtration begins as the thick line. With the progress of filtration, the sedimentation effect enlarges the S, C, and the slope in Eq. (3). This is shown in the experimental result in Fig. 9.

The slope becomes larger after a little while, so the extrapolation of experimental data could result in an intercept smaller than expected. The small value intercept gives small filter medium resistance. We concluded that the thicker suspension yields the small filter medium resistance by this reason as shown in Fig. 8. So it is meaningless to measure filter medium resistance by these experimental data.

If it were possible to have accurate data of the initial period of filtration, our concept could be verified. As mentioned above, it is not possible due to the rapid rate of flow and fast change of flux [Yim and Kim, 2000a]. Tiller [1990] analyzed the initial period of filtration with the experimental result of Hosseini [1977], but the experimental result has only two data points at the position corresponding to the thick line in Fig. 9.

Tiller et al. [1995] postulated that the value of S does not change by sedimentation; it is different from ours. And they also assumed that the sediment is cake; this will be discussed soon.

4. Filter Medium Resistance and the Influence of Pre-sedimentation Time on Filtration

Filtration can be started immediately after pouring suspension into a filter cell when vacuum filtration apparatus is used. For pressurized filtration, a time for sealing the filter cell is usually necessary before starting filtration. The sealing time is about one to two minutes. The sedimentation before the start of filtration is termed pre-sedimentation. To know the effect of pre-sedimentation to the filter medium resistance, the filtration-permeations of 4% suspension are performed with pre-sedimentation times of zero, 1.1, 2.3, 4.4 minutes. The experimental results are shown in Fig. 10.

The small black circles represent the experimental results without pre-sedimentation. The data for filtration period give good linearity, and the filter medium resistance from the intercept is -2.2×10^{10} m^{-1}; the negative value is due to the effect of pre-sedimentation.

The small triangles are the data from 2.3 minutes pre-sedimen-

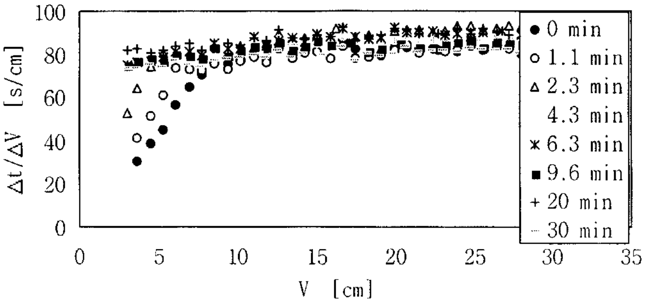

Fig. 10. Filtration-permeation of 4% $CaCo_3$ suspension at 0.5 atm through Toyo 5C for various pre-sedimentation times.

tation. Even though the extrapolation is performed with only three experimental points, the intercept with y axis is positive and the filter medium resistance by this intercept is 8.4×10^{10} m^{-1}.

As the pre-sedimentation time is prolonged, the amount of suspended particles decreases and the filtration period is shortened. The resistance of filter medium seems to be increased in Fig. 10 because the resistance of sediment is also measured as that of filter medium. In fact, the pre-sediment is situated above the filter medium, and is filtered at first. After the filtration of sediment, the filtration of suspension is followed. Normally the value of V for the filtration of sediment is too small to be exhibited in Fig. 10. The filtration-permeation results with more than 4 minutes pre-sedimentation times do not yield filtration period, and give only the data of the permeation period.

Summing up above analyses, the accurate filter medium resistance cannot be measured when pre-sedimentation is included in the filtration experiment.

The authors think that the variation in the experimental results in Tables 2 and 3 may originate from the small difference in starting time after pouring suspension, i.e., the small difference of pre-sedimentation time. The intensity of pouring suspension to the filtering cell would cause the different moment of beginning sedimentation that can yield different value of filter medium resistance.

5. Filtration of Sediment - Exclusion of the Effect of Sedimentation and Pre-sedimentation

Considerable amount of sediment is necessary to have the data for the filtration of sediment. A 200 mL suspension with 32 g calcium carbonate was prepared. The suspension is poured into the filter cell, and completely sedimented for 40 minutes. The height of sediment was 3.9 cm. Filtration-permeation was performed with this sediment and supernatant.

The material to be filtered, which traditionally is called a suspension, is the sediment. The mass fraction of solid in suspension, or sediment, S is 0.465. The height of sediment is reduced to 1.87 cm was filtration finished, and the height remained constant during permeation period. During permeation, all of the particles were changed into cake. The mass fraction of solid in cake calculated with the final height is 0.737. With this and Eq. (2), the value of C was 1,260 kg/m^3. The filtration-permeation result is shown in Fig. 11.

The eight data points from the origin represent filtration of sediment. Thus the experimental result of filtration of sediment is obtained. Tiller et al. [1995] thought that sediment is cake. In case of the filtration of a cake, the eight points from the beginning in Fig. 11 cannot be obtained.

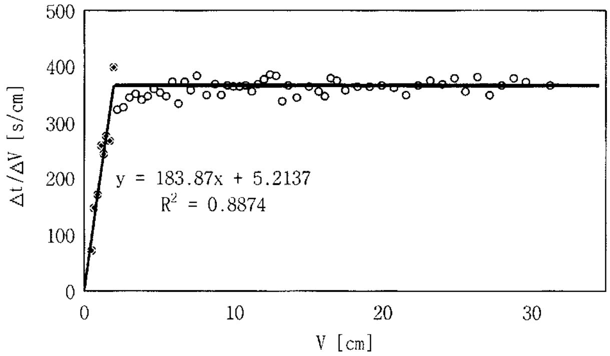

Fig. 11. Filtration-permeation of sediment (46.5% $CaCO_3$) at 0.5 atm.

Here, definition of a "cake for filtration" is necessary. We define a cake as "the matrix of particles redistributed by filtration operation".

The experimental results with more than 4.3 minutes pre-sedimentation in Fig. 10 also have the periods of filtration of sediment which have a very steep slope. But with such low concentration, the amount of sediment is very small and the filtration is finished at very small values of V.

The straight line obtained by eight filtration data is shown in Fig. 11. The resistance of filter medium by the intercept of y axis is 2.61×10^{8} m^{-1}. The value is smallest of all filter media resistances mentioned until now except for the negative resistances.

In this experiment, the effect of sedimentation was excluded by using previously sedimented material, and the influence of pre-sedimentation was also eliminated. Therefore, we concluded that the filter medium resistance thus measured is reliable.

Considerably large amount of sediment is required to perform the experiment. When large amount of thinner suspension is sedimented to get the sediment, the large and dense particles settle earlier than the small and light particles. This uneven sediment cannot represent the characteristics of suspension.

The effect is not prominent in hindered sedimentation with thick suspension used in this study. The average specific cake resistance by filtration of sediment and that of 1% suspension is identical within experimental error limits.

CONCLUSION

The experimental and theoretical study about the factors influencing the measurement of the filter medium resistance by Ruth's equation yields the following results.

1. An equation for measuring the average pore diameter of filter medium with the permeation velocity of particle removed water, porosity and thickness of filter medium was developed. The average pore diameter of seven kinds of filter media was measured by the method.

2. The resistance of 7 filter media was measured by filtration experiment and Ruth's equation. The resistances of filter media having pore diameters of about 0.5 μm and over 2.2 to 4.2 μm are smaller than 2.85×10^{10} m^{-1}. The resistances having pore diameters of 0.95 and 1.2 μm are from 4.2×10^{10} m^{-1} up to 2.2×10^{11} m^{-1}. This

means that the small filter medium resistance is not found in the media having large or small pores.

3. The filter medium resistance was measured as negative when thick suspension was filtered. It is suggested that the sedimentation during the long filtration time by thick suspension is the cause of negative filter medium resistance. In this case, filter medium resistance cannot be measured exactly.

4. The pre-sedimentation before the start of filtration results in the increase of the filter medium resistance. The sediment formed by pre-sedimentation is measured as filter medium resistance. The existence of pre-sedimentation gives false filter medium resistance.

5. To exclude the effects of suspension concentration and pre-sedimentation time, a new experimental method to measure filter medium resistance using the filtration of sediment is proposed.

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