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Statistical and Deterministic Chaos Analysis of Pressure Fluctuations in a Fluidized Bed of Polymer Powders

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Abstract−The absolute and differential pressure fluctuations in gas-solid fluidized beds have been analyzed by statistical and deterministic chaos methods. Linear low-density polyethylene (LLDPE) particles with a mean diameter of 1.23 mm were used as a fluidizing material. The statistical methods are composed of the mean, standard deviation, skewness and kurtosis, and the deterministic methods are composed of autocorrelation, mutual information function, pseudo-phase space and correlation dimension. The minimum slug velocity of LLDPE particles is found to be 0.34 m/s by using the statistical and deterministic methods. As slugs appear and grow with increasing gas velocity, pressure fluctuations in the fluidized bed of LLDPE are oscillated and more periodic.

Key words: LLDPE, Minimum Slugging Velocity, Pressure Fluctuation, Deterministic Chaos Analysis

INTRODUCTION

Fluidized beds have been used widely for combustion, FCC regenerators, and polymerization, to name but a few. Mostly, fluidized beds used for polymerization are operated in bubbling bed state at the given operation conditions. However, bubble size increases with increasing gas velocity (U_g); slugs are thereby formed that may increase friction among the particles and column walls [Lee et al., 2001, 2002].

To determine the fluidizing state, time-dependent variables such as histories of pressure or temperature have been used to characterize the fluidizing state. However, the nature of pressure fluctuation has not been fully understood due to the enormous complexity and lack of satisfactory mathematical tools [Karamavruc et al., 1995; Karamavruc and Clark, 1997]. For analyzing pressure fluctuation, the classical statistical analyses such as its mean, standard deviation, skewness and kurtosis, have been employed [Lee and Kim, 1987]. Also, the auto-correlation function and frequency spectrums of pressure fluctuations have been used to analyze fluidized bed behavior. Recently, to analyze pressure fluctuation, deterministic chaos analyses such as the mutual information, pseudo-phase space trajectory and correlation dimension have been employed [Karamavruc et al., 1995; Karamavruc and Clark, 1995, 1996; Bai et al., 1997]. Therefore, in this study, pressure fluctuations are analyzed by the classical statistical and deterministic chaos analyses in a fluidized bed of polymer powders.

THEORY

1. Mutual Information Function

The mutual information function is based on the concept of entropy [Shannon and Weaver, 1949]. Entropy is a good measure to determine uncertainty about outcome of a probabilistic experiment. If one is absolutely certain about the outcome of an event, then the entropy will be zero. However, if one is absolutely uncertain about the outcome of an event, then entropy will exhibit a maximum value. In other words, if one knows the outcome with certainty before it happens, the probability will be a maximum and, as a result, the entropy will have a minimum value.

The mutual information measures the dependence between the successive measurements [Fraser and Swinney, 1986]. When time delay is zero, the mutual information function has a maximum point and it decreases as time delay is increased. The mutual information function of a chaotic data will lie between absolutely random data and truly periodic data [Karamavruc and Clark, 1996].

A typical discrete data set can be obtained by an experimental observation such as $x(t)=\{x(t_1), x(t_2), \ldots, x(t_N)\}$. Values of x may be divided into bins, each with a range in x(t), and denoted by values $x_1, x_2, \ldots x_N$. For any data set, the probability of any value of x falling into a specific bin is $P(x_i)$. Hence, a set of probabilities $P(x_1)$, $P(x_2)$, …, $P(x_N)$ can be created from the original data set. If delayed time is τ, the data set becomes $x(t+\tau)=\{x(t_1+\tau), x(t_2+\tau), \ldots, x(t_N+\tau)\}$ and the set of probabilities becomes $P(x^*_1)$, $P(x^*_2)$, …, $P(x^*_N)$.

The mutual information function is defined as [Mansuripur, 1987]

$$I(X, X+\tau) = H(X) + H(X+\tau) - H(X, X+\tau) \quad (1)$$

where

$$H(X) = -\sum_{i=1}^{N} P(x_i)\log_2 P(x_i) \quad (2)$$

$$H(X+\tau) = -\sum_{j=1}^{N} P(x_j)\log_2 P(x_j) \quad (3)$$

$$H(X, X+\tau) = -\sum_{i}^{N}\sum_{j}^{M} P(x_i, x_j)\log_2 P(x_i, x_j) \quad (4)$$

where X denotes the whole system that consists of all the meas-

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‡This paper is dedicated to Professor Dong Sup Doh on the occasion of his retirement from Korea University.

ured data and N and M are the number of possible outcomes. H(X) and H(X+τ) are the average entropy corresponding to x(t) and x(t+τ). H(X, X+τ) is the joint entropy for any measurement of x(t) will fall into bin x_i and its delayed version will fall into x_j [Karamavruc and Clark, 1996].

2. Pseudo Phase Space Method

When only one variable is measured, the time delayed pseudo phase space trajectories method or the embedding space method is used for analyzing the signals. In the method, a plot of signal x(t) vs. signal from the same source, but shifted in time by an increment τ, is used. The pseudo phase space trajectories method examines the relationship between x(t+τ) and dx(t)/dt, x(t+2τ) and $d^2x(t)/dx^2$, and so on [Packard et al., 1980]. The choice of time constant τ is not obvious and according to the size of τ, the trajectories can be stretched or compressed around a diagonal. According to Takens [1981], almost any choice of delay time (τ) may be acceptable, whereas Roux et al. [1983] claimed that quality of the phase portraits depends on the choice of τ. According to Fraser and Swinney [1986], the first minimum of mutual information, I(X, X+τ) as described below, is the best choice of τ. However, this does not work for all the cases, because τ exhibits any minimum value instead of following a decaying value with increasing τ value.

3. Correlation Dimension

The measured signals can be characterized by dimension of the attractor if one exists. An attractor is described as a stable structure of long-term trajectories in a bounded region of a multi-dimensional pseudo-phase-space plane [Thompson and Stewart, 1987]. The correlation dimension of an attractor characterizes the spatial correlation between the measured points on the attractor [Karamavruc and Clark, 1997]. For time series x_i (I=1 to N), we can reconstruct a d-dimensional space with M points $\{X_i\}$ in the phase space where M is the length of vector, X_i and N are the number of points. The distances between the pairs of points $\|X_i - X_j\|$ can be calculated (i and j are integers).

A correlation integral is defined [Grassberger and Procaccia, 1983] as

$$C(r) = \lim_{M\to\infty} \frac{1}{M^2} \{\text{number of pairs of points (i, j) with distance } \|X_i - X_j\| < r\}$$

where X_i is the d-dimensional reconstructive vector, defined as

$$X_i = \{x_i, x_{i+\tau}, \ldots, x_{i+(d-1)\tau}\} \quad (i=1, 2, \ldots, M) \tag{5}$$

$$D_c = \lim_{r\to 0} \frac{\ln C(r)}{\ln r} \tag{6}$$

It can be shown that C(r) is proportional to r^{D_c} when r, the radius of the hyper-sphere, is small compared with the size of attractor. The correlation dimension is then defined as the usual practice to select random pairs of (X_i, X_j). A plot of logC(r) vs. log(r) is then produced for a given embedding dimension, and the correlation dimension is calculated from slope of line in the linear part. The embedding dimension of the attractor then increases and another correlation dimension can be calculated. The slope or the correlation dimension becomes independent of the embedding dimension when the latter is sufficiently large [Bai et al., 1997].

EXPERIMENTAL

Experiments were carried out in a fluidized bed (0.38 m-I.D.× 4.3 m-high) and the apparatus consisted of three sections: blower, fluidized bed and cyclone [Lee et al., 2001, 2002]. Airflow rates were measured by a flow meter (Tri-Sense Model No. 37000-00, Cole-Parmer Co.) in the range of 0-0.56 m/s. The solid particles used in this study were 1.23 mm linear low-density-poly-ethylene (LLDPE) particles having density of 919 kg/m^3. The distributor used in this study was a perforated plate having 76 holes in 5 mm diameter to provide uniform air distribution in the bed. Pressure taps were mounted flush with the wall of the column at 0.1 m height intervals from 0.55 m above the distributor [Lee et al., 2001, 2002].

The opening of pressure tap was covered with a screen (400 mesh) to prevent leakage of LLDPE particles from the bed. The other side of the tap was connected to one of the input channels of differential pressure transducer (Validyne P306D) from which an output voltage proportional to pressure difference between two channels was obtained. The remaining channel was exposed to the atmosphere so that the absolute pressure drop fluctuations across the entire bed were measured. Differential pressure drop fluctuations were

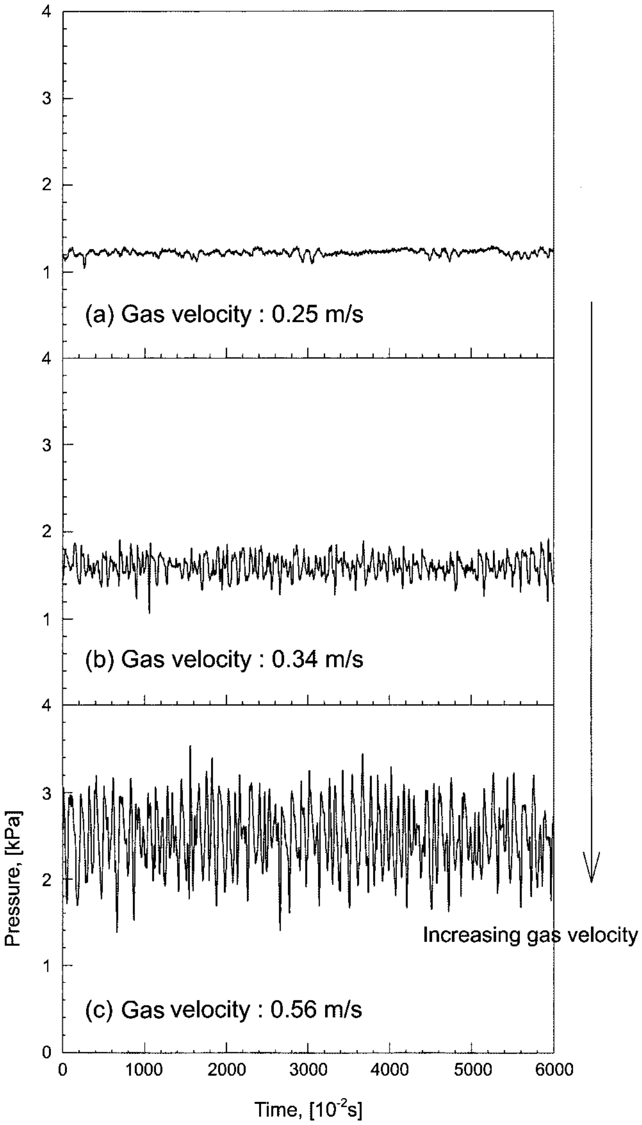

Fig. 1. Typical response signals of absolute pressure fluctuations.

measured between two pressure taps. The output signal from the pressure transducer was monitored by an oscilloscope, and sent it via A/D converter to a microcomputer for digital recording. The sampling interval of the signals was selected at 10 ms and 6,000 samples were collected during a sampling period of 60 s for each experimental condition.

RESULTS AND DISCUSSION

Typical pressure fluctuations in a fluidized bed of polymer particles are shown in Fig. 1 where oscillations of pressure fluctuations increase with increasing gas velocity (U_g) due to the formation of bubbles and slugs in the bed [Lee et al., 2001; Cho et al., 2001].

As can be seen in Fig. 2, the mean values of absolute pressure and its fluctuations increase due to the slug formation and the bed expansion with increasing U_g [Lee et al., 2001, 2002]. However, the mean values of absolute pressure fluctuations decreased with increasing bed height due to lack of particles.

The standard deviations of absolute pressure fluctuation with variation of U_g are shown in Fig. 3. As can be seen, the standard deviations increase with square root of U_g due to the presence of bubbles and slugs in the bed. The increase of standard deviation in pressure fluctuations might be attributed to increase in bubble size with increasing U_g [Lee and Kim, 1988]. However, the determination of minimum slug or onset slug velocity is difficult from the analysis of pressure fluctuation, mean values and standard deviations. Therefore, the other analyses of pressure fluctuations are applied in this study.

Variation of skewness in pressure fluctuations with gas velocity in the bed of LLDPE particles is shown in Fig. 4. As can be seen, skewness of the absolute pressure fluctuations and differential pressure fluctuations decreases with an increase in U_g in the fixed and bubbling fluidized beds, but those increase with a further increase in U_g in the slugging fluidized bed. The size and shape of bubbles vary and skewness of both the absolute and differential pressure fluctuations decrease with U_g in bubbling fluidized beds as found previously [Lee and Kim, 1988]. However, the distribution of pressure fluctuations lies around the mean of pressure fluctuations in the slugging fluidized beds. Consequently, skewness of both the absolute and differential pressure fluctuations increases with increasing U_g. Therefore, the minimum points of skewness with gas velocity might represent the transition gas velocity from bubbling to slugging fluidized beds.

The effect of gas velocity on kurtosis of pressure fluctuations in the bed of LLDPE particles is shown in Fig. 5. As can be seen, kur-

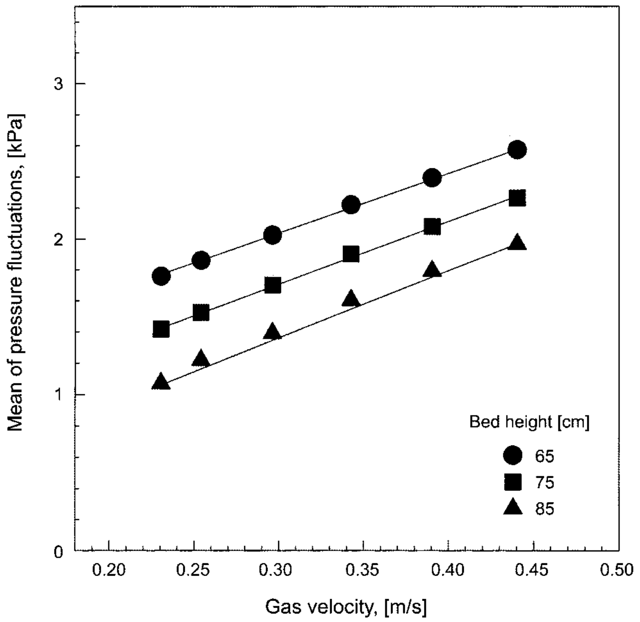

Fig. 2. Effect of gas velocity on the mean pressure drop of absolute pressure fluctuations.

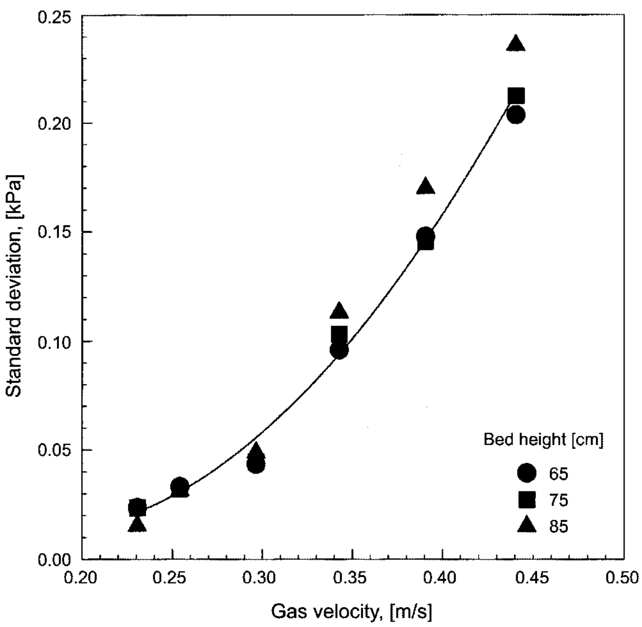

Fig. 3. Effect of gas velocity on the standard deviation of absolute pressure fluctuations.

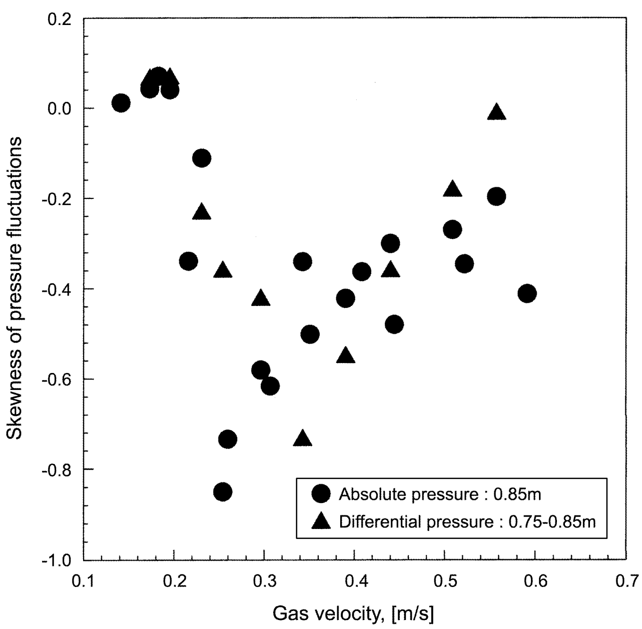

Fig. 4. Effect of gas velocity on the skewness values.

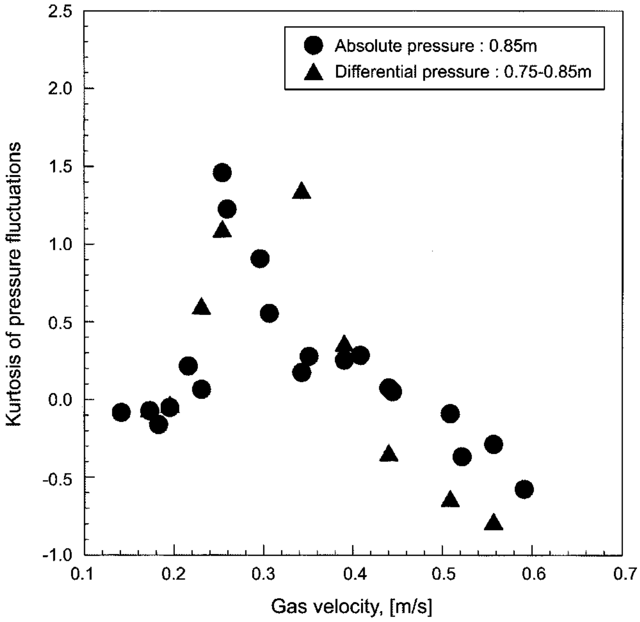

Fig. 5. Effect of gas velocity on the kurtosis values.

tosis of the absolute pressure fluctuations and that of differential pressure fluctuations exhibit a maximum value with increasing U_g. The variation of kurtosis is found to be opposite to that of skewness with increasing U_g. The increase of kurtosis with increasing U_g in bubbling fluidized beds can be attributed to deviation from the mean value in the major distribution of probability densities due to the increase of bubble size in bubbling fluidized beds [Lee and Kim, 1988]. However, the major probability densities of pressure fluctuations appeared to be located at its mean value and kurtosis decreases in the slugging fluidized beds. In contrast to skewness, the maximum point of kurtosis with gas velocity might be utilized to determine the transition velocity from bubbling to slugging fluidized beds.

Various autocorrelation coefficients with time lag at different gas velocities (0.17, 0.25, 0.34 and 0.56 m/s) are shown in Fig. 6. As can be seen, the autocorrelation coefficients of the absolute pressure fluctuations are similar with those of differential pressure fluctuations. When the gas velocity is at 0.17 m/s, the autocorrelation coefficients damp to zero value almost immediately, which may indicate a low degree of correlation between the neighboring values of pressure fluctuations. But the autocorrelation coefficients pronounce slow oscillation, which indicates a high degree of correlation with increasing U_g. Because of formation and coalescence of bubbles and slugs, pressure fluctuations produce periodic signals [Karamavruc and Clark, 1997]. Therefore, the autocorrelation coefficients exhibit slow oscillation.

The mutual information function of pressure fluctuations with gas velocity is shown in Fig. 7. The number of bins used is under 1% of total data because the choice of the number of bins was based on the evaluation of the most suitable time shifting constant τ [Karamavruc and Clark, 1997]. The mutual information functions exhibit stronger gradient with U_g that may represent chaotic nature of pressure fluctuations with increasing U_g. The final steady levels of the mutual information function of pressure fluctuations increase with an increase in U_g which may indicate the pressure fluctuations are more periodic [Karamavruc et al., 1995; Karamavruc and Clark, 1997]. In this figure, the final states of mutual information functions increase sharply when the gas velocity at 0.34 m/s. It might indicate that the pressure fluctuation at 0.34 m/s is more periodic than those at lower gas velocities. It might imply that the state of fluidized beds changes from bubbling to slugging in the bed. With a further increase in U_g, pressure fluctuations became more periodic due to the coalescence of slugs with increasing slug size [Ka-

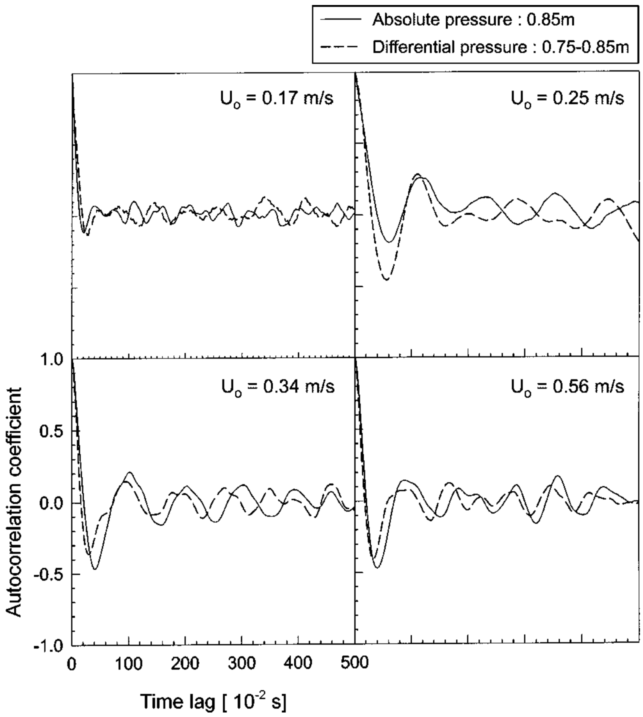

Fig. 6. The autocorrelation coefficient of pressure fluctuations.

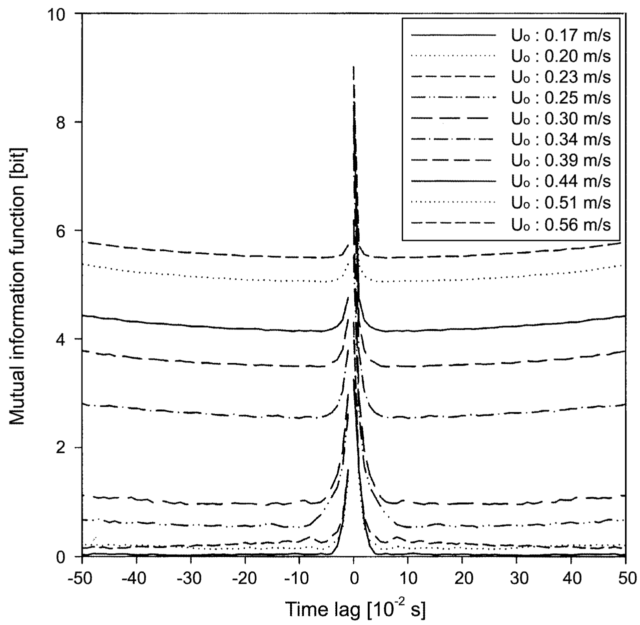

Fig. 7. The mutual information function of differential pressure fluctuations.

ramavruc et al., 1995; Karamavruc and Clark, 1997].

It is known that the first minimum value of mutual information functions provide a better criterion to determine the appropriate time delay for reconstruction of pseudo phase space portrait than choosing the first zero of the auto-correlation function [Fraser and Swinney, 1986; Karamavruc et al., 1995]. As can be seen in Fig. 7, the mutual information function reaches the first minimum time delay that is found to be 6. By using this time delay, the pseudo phase space trajectories of differential pressure fluctuations in various gas

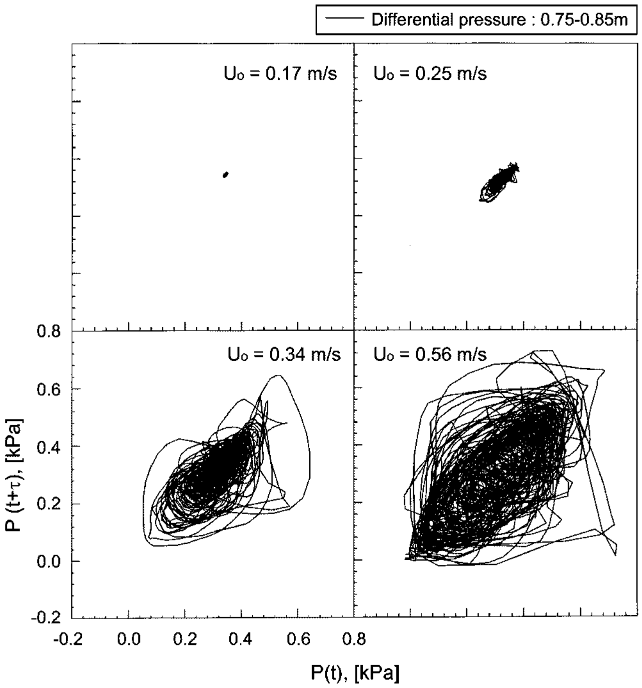

Fig. 8. Phase space construction on pressure fluctuations.

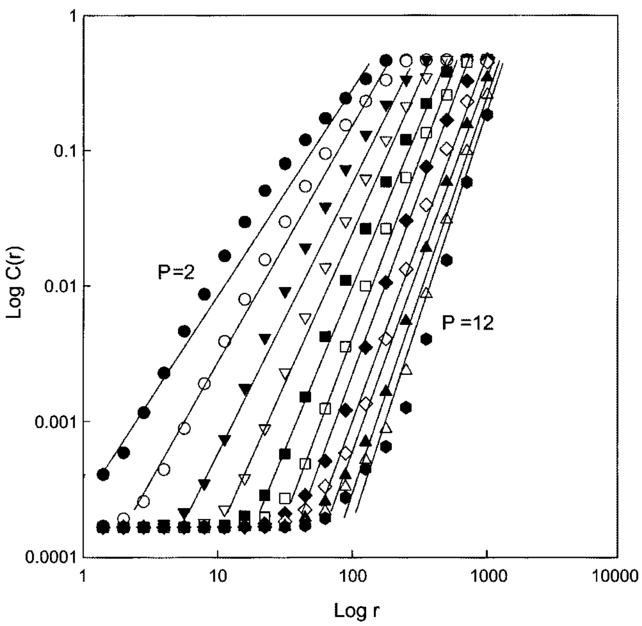

Fig. 9. Log-log plot of C(r) of absolute pressure fluctuation (U_g= 0.56 m/s, bed height=65 cm).

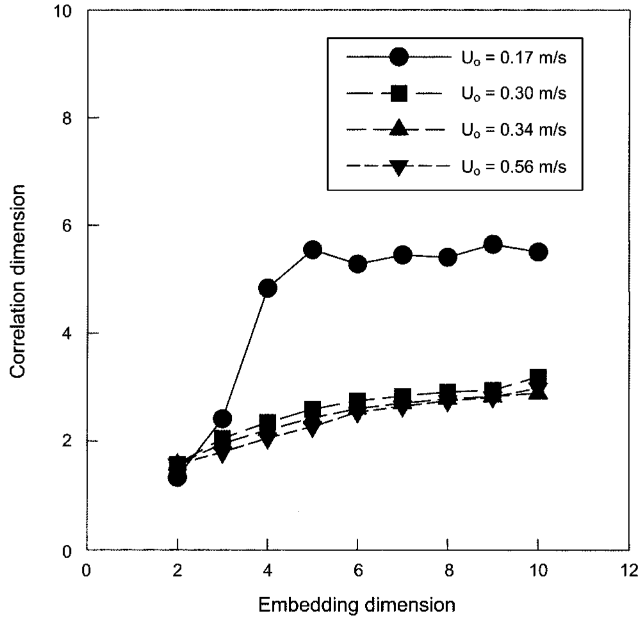

Fig. 10. Correlation dimension of absolute pressure fluctuations.

velocities are reconstructed as shown in Fig. 8. As can be seen, the phase space trajectories become larger with increasing U_g. Amplitude of the pressure fluctuations is short at gas velocity of 0.17 m/s. As the gas velocity is increased, amplitudes of pressure fluctuations become higher due to bubble coalescence. At the gas velocity of 0.34 m/s, the phase space trajectories reach lower-pressure regions due to slug formation. Compared with the phase space trajectories at U_g=0.34 m/s, the phase space trajectories at U_g=0.56 m/s became larger owing to an increase of slug size.

The correlation dimension as a function of the embedding dimension for the absolute pressure fluctuations is shown in Fig. 9 and Fig. 10. As can be seen, at the given gas velocities, the correlation dimensions reach a limiting value with increasing the embedding dimension. It might indicate that the present fluidized bed system in the given gas velocity range is the deterministic chaos system. Also, the correlation dimensions decrease with increasing U_g. Compared with the correlation dimension at U_g=0.17 m/s, the correlation dimensions at U_g=0.30 m/s decrease sharply. It might indicate that the state of fluidization changes due to the formation of large bubbles and slugs.

CONCLUSIONS

The absolute and differential pressure fluctuations in a slugging fluidized bed resulted in the following conclusions.

The transition velocity from bubbling to slugging fluidized regimes has been determined by the statistical and deterministic chaos methods. With these methods, the minimum slugging velocity of LLDPE particle is found to be 0.34 m/s. From the auto-correlation and mutual information functions, it has been found that the pressure fluctuations become periodic with increasing gas velocity. By using the phase space trajectories, it is found that the slug properties have changed with gas velocity. It has been found that the deterministic chaos analysis is the additional method to quantify the hydrodynamics of fluidized beds.

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