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The Role of P_i, P_o, and P_f in Constitutive Equations and New Boundary Conditions in Cake Filtration

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Abstract−The constitutive equations proposed by Tiller and Shirato were analyzed and a new constitutive equation originating from the sediment thickness was proposed. A new boundary condition of the filter cake based on the solid compressive pressure of the first solid layer, p_f, was also proposed. Accurate average specific cake resistances at various pressures and the thickness of cake were calculated with the new constitutive equation and boundary conditions. The influence of p_f on the cake thickness and average porosity was studied theoretically. Using three constitutive equations, it was proved that the compressibility n obtained from filtration results instead of CPC (compression-permeability cell) of very compressible cake could not have an exact value.

Key words: Filtration, Cake Filtration, Compression-Permeability Cell, Porosity, Specific Resistance, Average Specific Cake Resistance, Compressibility, Solid Compressive Pressure

INTRODUCTION

Modern filtration theory originated from the Ruth's Compression-Permeability Cell (CPC) [Ruth, 1946]. He established a method of studying the internal structure of a cake by measuring the specific resistance of a cake having uniform porosity with CPC at various pressures. Grace [1953] performed CPC experiments for various particulate materials and measured the relation of specific resistance to the solid compressive pressure p_s. Tiller [1953] started studying the phenomena inside the cake by calculating the distribution of liquid pressure using a simple compression test and numerical integration. This calculated liquid pressure distribution was proved experimentally by Okamura and Shirato [1955].

Tiller [1955] analysed CPC results and proposed that the porosity and the specific cake resistance are related to the solid compressive pressure by a power function above a certain pressure p_i and suddenly become constant below the pressure. Shirato [1970] succeeded in representing the above conception with one equation; futhermore, this equation gives smooth change at the vicinity of p_i. Tiller and Crump [1977] accepted Shirato's equation with a small modification.

In this study, we want to analyze the equations proposed by Tiller and Shirato to elucidate the necessity of these conceptions. Then we shall prove that the porosity and specific resistance are not constant below a certain solid compressive pressure p_i, and they change until a very low solid compressive pressure by power function. Until now, almost all researchers except us have adopted the boundary condition of a cake as from zero to filtration pressure. We propose new boundary conditions as from a very low pressure exerted by the drag force and the weight of the first solid layer to filtration pressure.

The primary subject of this study is calculating the average specific cake resistance with the above two new conceptions and CPC experimental results. The cake thickness and the limit of compressibility by filtration experiments will be calculated by using constitutive equations and theoretically explicated with the above conceptions.

THEORETICAL ANALYSIS

1. Analysis of the Constitutive Equations of Tiller and Shirato

Tiller [1955] proposed constitutive equations as below based on CPC experiments as Fig. 1. Eqs. (1) and (2), which are a little different from the initial form [Tiller, 1955], have been established and used until now.

When p_s is greater than p_i in Fig. 1:

$$\alpha = ap_s^n \tag{1}$$

$$1-\varepsilon = Bp_s^\beta \tag{2}$$

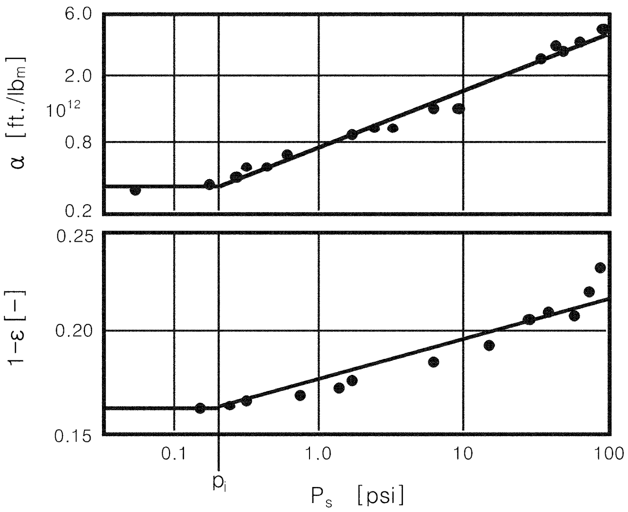

Fig. 1. Logarithmic plots of α, and $(1-\varepsilon)$ vs. p_s [Tiller, 1977].

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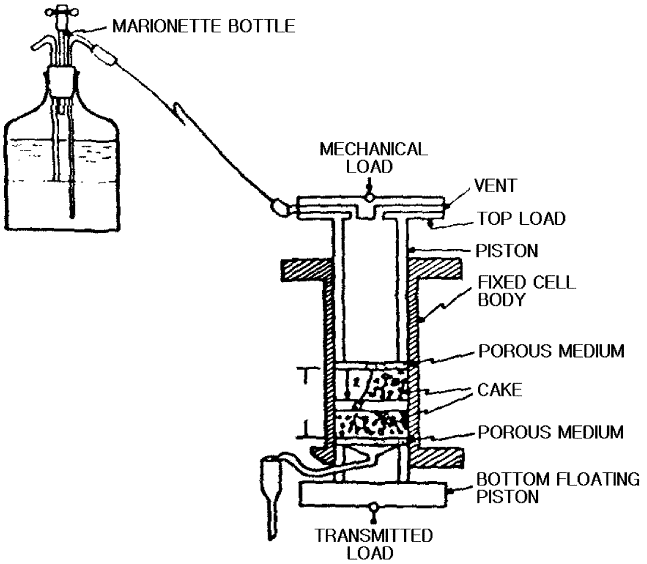

Fig. 2. Compression-permeability cell [Tiller, 1977].

Eqs. (1) and (2) show that the average specific resistance, α, and solidosity, $(1-\varepsilon)$, are solely functions of solid compressive pressure, p_s. The constants "a" and "n" in Eq. (1) and "B" and "β" in Eq. (2) can be determined with the slopes in Fig. 1. Among the four constants, the "n" alone has the name "compressibility". A cake is incompressible when n is zero, and extremely compressible when "n" is larger than 1.

The characteristics of a cake are determined by above four constants. The average specific cake resistance and the distribution of porosity through a cake can also be calculated with the constants.

When p_s is smaller than p_i:

$$\alpha=\alpha_i=ap_i^n=\text{constant} \tag{3}$$

$$1-\varepsilon=1-\varepsilon_i=Bp_i^\beta=\text{constant} \tag{4}$$

Eqs. (3) and (4) signify that cake porosity and specific resistance do not change under a certain value of solid compressive pressure, p_i. The point of contact between the straight line with slope and parallel line to x axis is p_i in Fig. 1. The p_i in the figure is 0.2 psi, namely 1.4 kPa. But we think that the existence of p_i could not be verified visually even in Fig. 1, which was presented as a proof of the pressure by the originator. And furthermore, we could not find any theoretical basis of the constancy of porosity and specific resistance at the range of pressure smaller than 1.4 kPa.

Actually, it is possible that the friction between the cake and the wall of CPC may prevent the reduction of the cake at small pressure. By the friction, the porosity and specific resistance may not change below p_i as shown in Eqs. (3) and (4). Therefore, we think that this phenomenon would be a special characteristic of a CPC, and could not be applicable to cake filtration.

To modify the sharp change at p_i, and simplify Eqs. (1) to (4), Tiller and Crump [1977] proposed Eqs. (5) and (6) which were the modifications of the equations originally proposed by Shirato et al. [1970]. Tiller call Eqs. (5) and (6) as "Shirato's equations", and we shall use the expression also in this paper.

$$\alpha/\alpha_o=(1+p_s/p_a)^n \tag{5}$$

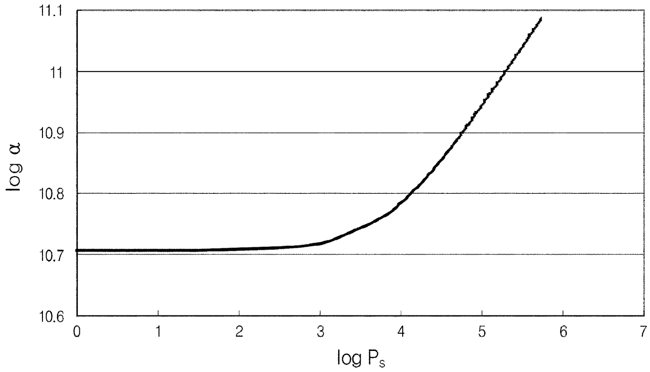

Fig. 3. Specific cake resistance vs. solid compressive pressure of calcium carbonate cake by Shirato's equation.

$$(1-\varepsilon)/(1-\varepsilon_o)=(1+p_s/p_a)^\beta \tag{6}$$

The new equations require determination of α_o, ε_o and p_a.

Fig. 3 is obtained with the constants proposed by Tiller and Crump [1977] for calcium carbonate. As shown in Fig. 3, there is only a small difference between Eqs. (1), (3) and Eq. (5). The specific resistance is constant below 1 kPa in the figure; this is identical to Eq. (3). After a smooth change between 1 to 10 kPa, a straight line with slope is followed for the larger solid compressive pressures; this phenomenon is the same as Eq. (1). In a recent paper Tiller et al. [2001] also analyzed the characteristics of a compressible cake with Eqs. (5) and (6). But they do not explain why the porosity and the specific resistance are constant at low solid compressive pressure.

2. Problem for Calculating Average Specific Resistance without p_i or p_a

In order to calculate the average specific cake resistance from above constitutive equations which are induced from the experimental results of CPC, Eq. (7) is usually applied. This equation can be derived from fundamental notion of cake filtration, but we omit the derivation process because it is a formula frequently used.

$$\alpha_{av}=\frac{\Delta p_c}{\int_0^{\Delta p_c}\frac{dp_s}{\alpha}} \tag{7}$$

Boundary conditions of Eq. (7) are established as follows. The solid compressive pressure, p_s, at the interface between cake and suspension is null, and that at the interface between cake and filter medium is the pressure drop in cake Δp_c. All the researchers and engineers who have been engaged in filtration use these boundary conditions; the appropriateness of these conditions will be discussed.

The integration of Eq. (7) using Eq. (1), i.e., without using Tiller's p_i or Shirato's p_a, gives

$$\alpha_{av}=\frac{\Delta p_c}{\int_0^{\Delta p_c}\frac{dp_s}{ap_s^n}}=\frac{a(1-n)\Delta p_c}{\Delta p_c^{1-n}}=a(1-n)\Delta p_c^n \tag{8}$$

The values of α_{av} at different filtration pressures also can be obtained from filtration experiments. When the relationship between α_{av} and Δp_c is drawn in logarithmic scales, the slope represents the cake compressibility, n, according to Eq. (8). This method of determining the compressibility is commonly used by engineers and researchers, and is recognized as reasonable for cakes having comparatively small compressibility.

But Eq. (8) has a critical defect. When compressibility, n, is greater than 1, the average specific cake resistance is negative according to Eq. (8). A negative value of average specific cake resistance is impossible in cake filtration.

Carman [1938] expressed compressibility which is measured by filtration experiments as Eq. (9).

$$\alpha_{av}=\alpha_o \Delta p_c^s \tag{9}$$

For cakes with small compressibility, Eq. (9) is identical with Eq. (8).

Ruth's filtration equation is usually expressed as

$$q=\frac{dV}{dt}=\frac{\Delta p}{\mu(\alpha_{av}W+R_m)} \tag{10}$$

Filtration pressure, Δp [Pa], and viscosity, μ [kg/m·s], have positive values; dry cake mass per unit area of filtration, W [kg/m^2], is zero or positive; and filter medium resistance, R_m [m^{-1}], has large positive value. Therefore, the negative average specific cake resistance means that the liquid must flow toward the direction of applied pressure. This phenomenon is actually not possible, so Eq. (8) cannot be applied to the cake which has compressibility greater than 1. It is also doubtful applying the equation to a cake having compressibility close to 1, e.g. 0.8 or 0.9.

3. Role of p_i or p_a Calculating Average Specific Cake Resistance

3-1. Average Specific Cake Resistance by Tiller's p_i

Substituting Eq. (1) and Tiller's Eq. (3) into Eq. (7) leads to

$$\alpha_{av}=\frac{\Delta p_c}{\int_0^{\Delta p_c}\frac{dp_s}{ap_s^n}}=\frac{\Delta p_c}{\int_0^{p_i}\frac{dp_s}{ap_i^n}+\int_{p_i}^{\Delta p_c}\frac{dp_s}{ap_s^n}}=\frac{a(1-n)\Delta p_c}{\Delta p_c^{1-n}-np_i^{1-n}} \tag{11}$$

Different from Eq. (8), Eq. (11) cannot be represented in a simplified form. Tiller and Leu [1982] proposed that the value of p_i is small enough to be eliminated for a cake having moderate compressibility. In this case Eq. (11) is identical with Eq. (8).

In this equation the "a" and Δp_c are positive. When n is greater than one, (1−n) is negative. Thus the numerator of Eq. (8) is negative. The denominator has negative value, because the pressure drop across the cake, Δp_c, is much larger than, p_i, and (1−n) is negative. The numerator and denominator are negative altogether, so the average specific cake resistance has positive value. This means that Eq. (11) is valid when "n" is greater than 1. We assumed that this is the main role of p_i.

3-2. Average Specific Cake Resistance by Shirato's p_a

Substituting Eq. (5) to Eq. (7), average specific resistance is calculated as follows.

$$\alpha_{av}=\frac{\Delta p_c}{\int_0^{\Delta p_c}\frac{dp_s}{\alpha_o(1+p_s/p_a)^n}}=\frac{\alpha_o(1-n)\Delta p_c}{p_a\left[\left(1+\frac{\Delta p_c}{p_a}\right)^{1-n}-1\right]} \tag{12}$$

Eq. (12) also cannot be represented in a simple form. In most cases Δp_c is greater than p_a; the average specific cake resistance in the equation is positive because both the numerator and denominator are negative when the compressibility, n, is greater than 1. So Eq. (12) is valid for all values of "n". As shown in Fig. 3, Tiller and Crump [1977] accepted the merits of Shirato's equation, i.e., the positive average specific cake resistance and the smooth variation of specific resistance in the vicinity of p_i, and they gave up the conception p_i.

3-3. Physical Significance of p_i and p_a

The apparent meaning of p_i and p_a is that the porosity and specific resistance do not change below the solid compressive pressures. As mentioned before, we cannot find an acceptable physical meaning of these pressures. At this stage it is important that the calculated average specific cake resistance of a very compressible cake (n>1) is negative without the conception p_i or p_a.

4. Porosity Variation at Extremely Low Pressure by Sedimentation

Shirato et al. [1983] sedimented a suspension completely and measured the final height of the sediment. This procedure has no direct connection with the operation of filtration. But the particulates in the lower part of sediment support the weight of particles in the upper part. The solid compressive pressure increases with the depth of solid. Experimental results with compressible material demonstrated that an increase of mass of two times does not give double sediment height. The measured height is smaller than expected. Shirato et al. [1983] proved by sedimentation experiment that the porosity of the sediments of ferric oxide, Mitsukuri-Gairome clay, and zine oxide decrease even at very low solid compressive pressures, i.e., as low as 100 Pa. The β in Eq. (2) of Mitsukuri-Gairome clay thus obtained is 0.101, and that of other two materials are 0.096 and 0.094. According to the equations of Tiller and Shirato, the β must be zero at such a very low solid compressive pressure.

They also calculated the average specific resistance by measuring hindered sedimentation velocity at very low solid compressible pressure. The experimental results are a little dispersed, but in general they coincide with the extrapolation line of CPC results. It means that the Eqs. (1) and (2) are valid until a very low solid compressive pressure. Tiller and Leu [1983] mentioned that Eqs. (1) and (2) can be applied down to very small solid compressive pressure for cakes of small compressibility which have small p_i enough to be ignored. Shirato did not apply the above experimental results to cake filtration theory.

The first author [1986] of this paper proved by experimental means that the porosity of a very compressible cake having n=1.13 also decreases as Eq. (2) until very low solid compressive pressure of 1 Pa (not 1 kPa) using bentonite floc. With these facts, the whole expression procedure of a cake formed by bentonite floc is calculated and verified by experiments [Yim and Kwon, 1997]. Tiller and Leu [1982] thought that p_i is greater than 1 kPa for such extremely compressible cakes based on CPC results.

5. Proposed Constitutive Equation

With the above complete sedimentation results, it is not possible to expound on Tiller's Eqs. (3), (4), and Shirato's Eqs. (5), (6). We think that the constancy of porosity less than 0.2 psi in Fig. (1) originates from the friction generated between cake and cell wall. According to complete sedimentation results of a very compressible material, we propose that Eqs. (1) and (2) are valid until very low solid compressive pressure.

For p_s>0, i.e., for all of the solid compressive pressures

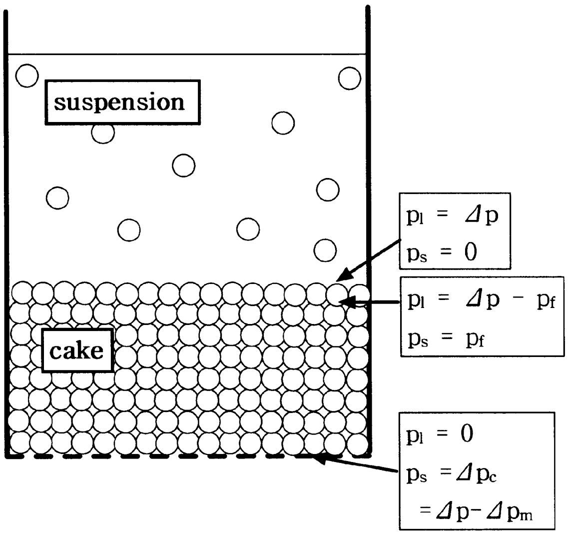

Fig. 4. Boundary conditions of a filter cake.

$$\alpha = ap_s^n \tag{13}$$

$$1-\varepsilon = Bp_s^\beta \tag{14}$$

The calculated average specific resistance is negative when compressibility is greater than one, as mentioned earlier at the analysis of Eq. (8). This defect can be corrected with the following new concept.

6. New Boundary Condition

A schematic diagram of a filter cake is shown in Fig. 4. Almost all engineers and researchers, except ourselves, use boundary conditions of a filter cake as follows.

The pressures at the cake surface closest to the suspension are

$$p_l = \Delta p \tag{15}$$

$$p_s = 0 \tag{16}$$

Here, p_l is the liquid pressure and Δp is filtration pressure. Eq. (15) means that the liquid pressure at the starting point of a cake is filtration pressure. With this notion, the solid compressive pressure p_s at the starting surface of a cake was assumed zero as Eq. (16), and has been used very widely. Shirato et al. [1967] applied this notion to the expression procedure. In this paper, we protest against Eq. (16) for applying to filtration.

The boundary condition of solids closest to the filter medium is

$$p_l = \Delta p_m \tag{17}$$

$$p_s = \Delta p_c = \Delta p - \Delta p_m \tag{18}$$

Liquid pressure transforms into drag force during the flow through the interstices of cake, and the drag force pushes the solid particles toward the filter medium. The solid compressive pressure, p_s, is generated by the drag force; thus the liquid pressure, p_l, decreases. The liquid pressure at the end of the cake has very small value Δp_m, which is the pressure drop across the filter medium. The solid compressive pressure at the point is $\Delta p - \Delta p_m$. This means that all of the pressure drop across the cake, Δp_c, has been transferred to the cake particles at point of contact between the cake and the filter medium. The above concepts are the ordinary boundary conditions that have been adopted in filtration for a long time.

In this study, we want to examine the first solid layer where the suspension enters the cake. A precise definition of the first solid layer is not easy, but as a matter of convenience a solid layer distinguished from the second solid layer is assumed as shown in Fig. 4.

The weight and drag force of the first solid layer pushes the second solid layer. The particles in the second layer rearrange by the forces. The porosity and, in case of floc, specific surface also change according to the forces.

But the particles of the first solid layer do not change in porosity nor any other characteristics by their own small weight or drag force. This concept is the important point of our theory.

In this study, the solid compressive pressure of the first solid layer, p_f, is defined as the sum of drag force and the weight of the first solid layer divided by filtration area. We think that the average porosity and average specific cake resistance is largely affected by the p_f.

At the first solid layer, the solid compressive pressure changes from zero to p_f. The change of solid compressive pressure at the first solid layer does not influence the porosity and the specific resistance. So we propose a new boundary condition of a filter cake close to the suspension as

$$p_l = \Delta p - p_f \tag{19}$$

$$p_s = p_f \tag{20}$$

The value of p_f is very small during the procedure of cake filtration. The operation of squeezing water out of previously filtered cake with a piston is called expression. In the operation, the solid compressive pressure which directly pushes the particles touching the piston can be termed as p_f. The p_f at the start of expression is very small, becomes larger during operation, and finally reaches the expression pressure. Yim and Kwon [1997] calculated the whole expression procedure with the above conception, and verified the calculation with experimental results.

7. Calculation of Average Specific Cake Resistance with New Conceptions

Substituting the new constitutive equation, Eq. (13), and new boundary condition, Eq. (20), to Eq. (7) gives average specific cake resistance as

$$\alpha_{av} = \frac{\Delta p_c - p_f}{\int_{p_f}^{\Delta p_c} \frac{dp_s}{\alpha}} = \frac{\Delta p_c - p_f}{\int_{p_f}^{\Delta p_c} \frac{dp_s}{ap_s^n}} = \frac{a(1-n)(\Delta p_c - p_f)}{(\Delta p_c)^{1-n} - p_f^{1-n}} \tag{21}$$

Compared to Δp_c, the p_f in $(\Delta p_c - p_f)$ is sufficiently small to be neglected, and the result can be expressed as

$$\alpha_{av} = \frac{a(1-n)\Delta p_c}{\Delta p_c^{1-n} - p_f^{1-n}} \tag{22}$$

The numerator is negative when n is greater than 1. And the denominator is also negative because (1–n) is negative and Δp_c is much greater than p_f. It means that the average specific cake resistance of a very compressible cake thus calculated is always positive. So Eq. (1) can be applied to very compressible cake as we proposed. The equations proposed for calculating average specific cake resistance are presented in Table 1.

The average specific cake resistances by Tiller and Shirato are

Table 1. Average specific cake resistances by Tiller, Shirato and Yim

	Average specific cake resistances	Boundary conditions	Equation no.
Tiller	$\alpha_{av}=\dfrac{a(1-n)\Delta p_c}{\Delta p_c^{1-n}-np_i^{1-n}}$	$p_s=0$ $p_s=\Delta p_c$	(11)
Shirato	$\alpha_{av}=\dfrac{a_o(1-n)\Delta p_c}{p_a\left[\left(1+\dfrac{\Delta p_c}{p_a}\right)^{1-n}-1\right]}$	$p_s=0$ $p_s=\Delta p_c$	(12)
Yim	$\alpha_{av}=\dfrac{a(1-n)\Delta p_c}{\Delta p_c^{1-n}-p_f^{1-n}}$	$p_s=p_f$ $p_s=\Delta p_c$	(22)

based on the hypothesis that the porosity and the specific resistance do not change under a certain solid compressive pressure. But we propose that the porosity and specific resistance changes until very low solid compressive pressure, and also propose that the boundary condition of a cake begins with the solid compressive pressure of the first solid layer.

EXPERIMENTAL

A pressure filter with 4.0 cm diameter was used for ordinary filtration. Compression-permeability cell of the same diameter was adopted for establishing constitutive equations. Sedimentation tests were performed in a 8.5 cm cylindrical cell.

After drying the bentonite particle at 105 °C, sieving was performed with 100 mesh sieve, then suspension was made with the fine particles. Filtration, CPC, and sedimentation experiments were performed with the flocs flocculated by cationic polymer flocculant having molecular weight of 10^7.

RESULTS AND DISCUSSION

1. Experimental Results of CPC

Bentonite suspension was flocculated with the anionic flocculant. The floc was filtered at 1,000 Pa. Experimental results of CPC with preformed cake are shown in Fig. 5.

We carried out two sets of CPC experiments for the bentonite floc, and have the constitutive equations as

$$\alpha=2.87\times10^7\ p_s^{1.125} \tag{23}$$

$$1-\varepsilon=4.09\times10^{-3}\ p_s^{0.317} \tag{24}$$

That is, a of Eq. (1) is 2.87×10^7, and n is 1.125. This cake is extremely compressible by the definition of Tiller and Horng [1983]. With our experimental apparatus, it was not possible to measure below 1.45×10^4 Pa.

2. Determination of p_i of Tiller, and p_a of Shirato

Tiller determined the value of p_i in Eq. (3) with the experimental result of CPC, as illustrated in Fig. 1. But the existence of p_i is not found in our CPC results in Fig. 5.

The value of p_i in this study was determined by Eq. (11) using the experimental average specific cake resistance obtained by filtration at 1.0×10^5 Pa as follows.

$$p_i=92\ \text{(Pa)}$$

Tiller and Leu [1982] indicated that p_i is small enough to be ignored for a moderately compressible cake, and is about 10^3-10^4 Pa for a very compressible cake. The cake formed by bentonite floc is extremely compressible, but the p_i is much smaller than expected. It is not possible to measure the value of p_i, i.e., 92 Pa, with CPC. The p_i of Tiller's experiment in Fig. 1 is 1,390 Pa.

To know the α_o and p_a of Eq. (5) proposed by Shirato, we assume that the p_a is p_i of Tiller. Then α_o and ε_o can be fixed with the CPC results measured at high solid compressive pressures.

$$\alpha=4.55\times10^9(1+p_s/92)^{1.125} \tag{25}$$

$$1-\varepsilon=0.9829\times(1+p_s/92)^{0.317} \tag{26}$$

Our new concepts suggested in this study do not need the above constants.

The three constitutive equations and experimental CPC results are shown in Fig. 6.

The blank circles that are a little smaller than experiment one connected by a line represent Tiller's constitutive equation. The specific resistance is constant under 92 Pa. Shirato's constitutive equation is expressed with rhombuses. Almost all of the calculated values coincide well with Tiller's equation except at the vicinity of 92 Pa, i.e., p_i. The small black points represent the constitutive equation proposed in this study. The three constitutive equations and experimental CPC results coincide well at higher solid compressive pressure. It means that the Eqs. (23), (25), and p_i, p_a are properly suggested.

To verify what is the correct constitutive equation among the equations of Tiller, Shirato, and ourselves, it must be known whether

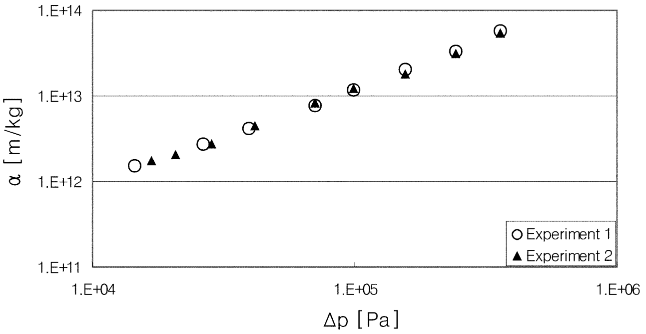

Fig. 5. Experimental results of compression-permeability cell with bentonite floc flocculated with cationic polymer flocculant.

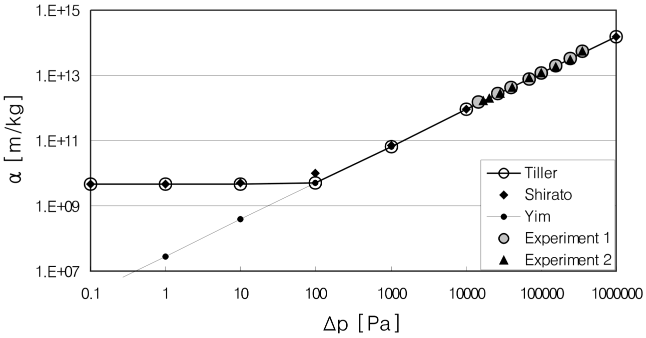

Fig. 6. Experimental results and theoretical constitutive equations proposed by Tiller, Shirato, and Yim.

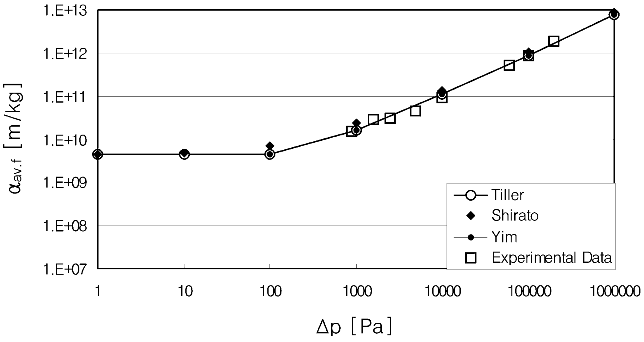

Fig. 7. Average specific cake resistances by experiments and by various constitutive equations.

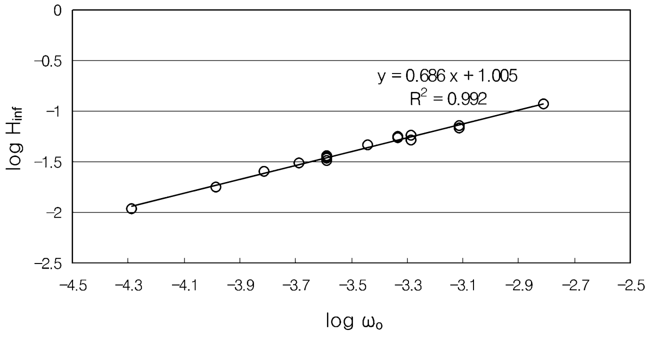

Fig. 8. Relation between final equilibrium height and total solid volume per unit area.

the specific resistance or porosity would or would not change at pressure lower than 92 Pa.

3. Average Specific Cake Resistances by Experiments and Constitutive Eqs.

The average specific cake resistances calculated by equations on Table 1 and measured with constant pressure filtration at various filtration pressures are shown in Fig. 7.

The p_f suggested in this study is the solid compressive pressure originating from the drag force by flow and weight of the first solid layer. The value of p_f can be calculated with Eq. (22) and average specific cake resistance measured by filtration experiment.

$$p_f=36\ \text{(Pa)}$$

Inversely, the average specific cake resistances at various pressures can be calculated by Eq. (22) and this p_f. Average specific cake resistances calculated by the equations of Tiller, Shirato, us and that by constant pressure filtration are shown in Fig. 7. We can see in Fig. 7 that all of the calculated values by three different equations coincide well, and they also coincide with the eight experimental average specific resistances measured at the pressures from 0.91 kPa to 202 kPa. Although a filtration experiment was performed at the lowest pressure possible, the pressure was still too high to prove the concepts of Tiller and Shirato.

4. Measurement of Porosity at Low Pressures by Sedimentation

If the concept of Tiller or that of Shirato is true, the porosity remains constant under the solid compressive pressure p_i or p_a, i.e., 92 Pa in this case. It means that the sediment of the floc has uniform porosity when the bottom solid compressive is smaller than 92 Pa. Then the height must be directly proportional to the floc mass. But Shirato et al. [1983] proved that this is not true and proposed Eq. (27).

$$H_\infty=a'\omega_o^b \tag{27}$$

where H_∞ [m] is the final height of the sediment, and ω_o [m^3/m^2] is solid volume per unit area. The a' and b are constants defined by this equation. The value of b is 1 only when the porosity does not change with the height of sediment.

The equilibrium heights of bentonite floc in a cylindrical cell of 8.5 cm diameter are represented in Fig. 8. The maximum solid compressive pressures of the solids in the bottom calculated from the mass of floc are from 0.88 Pa to 26.3 Pa. These solid compressive pressures are much smaller than 92 Pa, i.e., the p_i of the floc, but the value of b is 0.686 instead of 1. The values of B and β of Eq. (2) are obtained from the constants in Eq. (27) as follows [Shirato et al., 1983].

$$B=\frac{1}{a'b\{(\rho_s-\rho)g\}^{1-b}} \tag{28}$$

$$\beta=1-b \tag{29}$$

Hence, the relationship between the porosity and solid compressive pressure of the floc sediment can be written with B and β as Eq. (30).

$$1-\varepsilon=6.76\times10^{-3}\,p_s^{0.314} \tag{30}$$

The β value 0.314 of Eq. (30) by sedimentation coincides well with 0.317 of Eq. (24) determined by CPC within the experimental error limits. But the values of B are 0.00676 and 0.00409, respectively. The difference is not small. We think that further study about the difference is necessary. Anyhow, it is evident that the porosity changes continuously until 0.88 Pa according to Eq. (30). The porosity variation signifies that the specific resistance also varies at a low solid compressive pressure as Eq. (23).

Shirato et al. [1983] proved the variation of porosity for ferric oxide, Mitsukuri-Gairome clay, and zinc oxide at solid compressive pressures from 100 to 1,000 Pa, but they did not apply this effect to the calculation of average specific resistance.

5. Calculation of Sediment Height by New Concept

When Eq. (30) is valid until very small solid compressive pressure and the first solid layer pushes the following solids, the height of the sediment could be calculated with following procedures.

Eq. (31) is given by the filtration area A, the porosity ε in a very thin layer of solids dL, the mass of solid in the thin layer dm_c, and the solid density ρ_p.

$$A(dL)(1-\varepsilon)\rho_p=dm_c \tag{31}$$

The downward force exerted by the particles of mass dm_c in water can be expressed with gravitational and buoyant force as Eq. (32).

$$(dm_c)g-(dm_c)\frac{\rho}{\rho_p}g=dF \tag{32}$$

In this case, the solid compressive pressure, p_s, is defined as Eq. (33).

$$dp_s=\frac{dF}{A}=\frac{\left(1-\frac{\rho}{\rho_p}\right)g\,dm_c}{A} \tag{33}$$

Hence,

$$\frac{dm_c}{A}=\frac{1}{(1-\rho/\rho_p)g}dp_s \tag{34}$$

Substituting Eq. (34) into Eq. (31) yields

$$dL=\frac{1}{(\rho_p-\rho)g}\int_{p_f}^{p_{smax}}\frac{dp_s}{(1-\varepsilon)} \tag{35}$$

The p_{smax} is the solid compressive pressure exerted by the total weight of solid in water, and p_f is that by the first solid layer pressing down the next solid layer. Substituting Eq. (30) into Eq. (35) at the place of $(1-\varepsilon)$, and taking differentials of Eq. (35) leads to

$$L=\frac{1}{(\rho_p-\rho)gB}\frac{1}{1-\beta}\frac{(p_{smax}^{1-\beta}-p_f^{1-\beta})}{} \tag{36}$$

The p_{smax} can be calculated by the mass and solid density of sediment. The equilibrium height of the sediment was 1.1 cm with 0.8 g floc in 8.5 diameter cylindrical cell. It was the smallest mass that had been tested, and the p_{smax} was 0.88 Pa. The p_f is 0.01 Pa calculated by Eq. (36) with equilibrium height L, p_{smax}, B, and β. With this p_f, the equilibrium heights for the various floc mass are calculated and illustrated in Fig. 9 with the experimental sedimentation results.

The calculated results based on our new conceptions coincide well with the experimental results. The fact connotes that the porosity changes at the low solid compressive pressure range from 0.01 Pa to 26.3 Pa, i.e., Eqs. (2) and (30) are valid at this range of solid compressive pressure. It was not possible to find the experimental evidence for proving the existence of p_i. The authors successfully applied our new conceptions to the expression of oil from rapeseed at a high pressure of 9×10^6 Pa [Yim and Kwon, 1997].

6. Discussion about p_f

6-1. Analysis of p_f

The solid compressive pressure of the first solid layer, p_f, in floc sedimentation is calculated as 0.01 Pa, and that in cake filtration is calculated as 36 Pa. The difference is caused by the drag force by the flow through the particles in the first solid layer in filtration. Naturally, a drag force does not exist in the equilibrium sediment. Proving the existence of p_f by experimental means is difficult, because the two solid compressive pressures are too small to be measured. We think that p_f originates in the drag force, particle mass and buoyancy.

6-2. Cake Thickness and p_f

Depending on the new concepts, cake thickness is determined by p_f, filtration pressure, and cake mass per unit area. The p_f mostly arises from the rate of flow. It has relatively high value at the initial period of filtration due to the high rate of flow, and has small values during the rest of the filtration period. The thin cake at the initial period of filtration has denser structure resulting from the higher p_f. The dense cake has high average specific cake resistance. This phenomenon is experimentally measured by the first author [Yim and Kim, 2000]. The period governed by the phenomenon is very short compared to the whole filtration time. We presumed that p_f is constant during the filtration process. All of the calculated average specific cake resistances in Fig. 7 are from the same value of p_f, i.e., 36 Pa. In spite of the wide difference in filtration pressures, the calculated results coincide well with experimental results.

The thickness of cake changes in relation to p_f. The calculated thickness of cake by the equation of Tiller and Cooper [1962] have been modified according to our p_f, and the calculated results are shown on Table 2. The cake mass per unit filter area, W, is 3.2 kg/m^2, and the filtration pressure is 10^5 Pa.

The experimental cake thickness was 3.8 cm at the pressure and the cake mass, and it coincides with the calculated thickness 3.85 cm based on p_f of 36 Pa. The p_f of 36 Pa was calculated by the experimental average specific cake resistance and Eq. (22). It signifies that the cake thickness can be predicted with p_f.

For Filtration, it is not possible to change the value of p_f at our own will. In the equilibrium state of the CPC test, the piston pushes directly on the upper part of the preformed cake. Thus the value of p_f at the equilibrium state is 10^5 Pa, and that of p_{smax} is also 10^5 Pa. The calculated cake thickness in this case is 7.14 mm (in fact we used 99999.9 Pa as p_f for convenience of calculation). The measured cake thickness in CPC was 7.1 mm.

The two cases, filtration and CPC, which can be verified experimentally, give the coincidence between calculations and experiments. The other thicknesses calculated are assumed to be correct. The new theories for hindered sedimentation [Yim et al., 1995] and expression [Yim and Kwon, 1997] were proposed on the basis of the relation between cake thickness and solid compressure of the first solid layer.

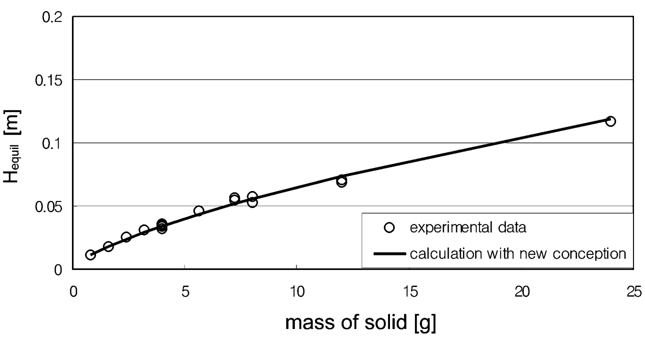

Fig. 9. Calculated sedimentation thickness by new conception and experimental results.

Table 2. Cake thickness, average porosity and solid wt percent in function of p_f (W=3.2 kg/m^2, p_{smax}=10^5 Pa)

p_f [Pa]	Cake thickness [m]	ε_{avg}	Solid wt%
0.01	3.85E-01	0.997	0.82%
0.1	1.95E-01	0.994	1.61%
1	1.01E-01	0.989	3.08%
10	5.38E-02	0.979	5.70%
36	3.85E-02	0.971	7.85%
100	2.97E-02	0.962	10.01%
1000	1.73E-02	0.935	16.46%
10000	1.07E-02	0.896	24.94%
99999.9	7.14E-03	0.844	34.57%

6-3. Average Cake Porosity and p_f

Using the concept of p_i, Tiller and Leu [1982] presented the calculated results that average porosity of a cake hardly changes with the increment of filtration pressure. Although this description is right, it is possible to obtain a cake having small average porosity, i.e., a cake of higher solid content, by increasing p_f according to Table 2. The methods of increasing the p_f are, for example, pushing the cake surface directly with a piston as CPC, or centrifugal filtration. Table 2 indicates that the average porosity would decrease to a large extent when p_f increases only a hundred Pa. This phenomenon cannot be calculated with the conceptions of Tiller nor Shirato.

7. Maximum Compressibility Measured by Experimental Filtration

Instead of CPC test, compressibility of a cake is generally determined with Eq. (9) by filtrations at several pressures. But the compressibility determined by filtration experiments has theoretical limitations. According to the constitutive equations suggested by Tiller, Shirato, and this study, the compressibility of a very compressible cake by filtrations is always smaller than 1. In fact, the compressibility measured by CPC is frequently larger than 1.

In Eq. (11) derived from the Tiller's constitutive equation, when compressibility is 3 and p_i is 1 kPa, the value Δp_c^{1-n} is 10^{-10} and np_i^{1-n} is 3×10^{-6}. Thus in the denominator $(\Delta p_c^{1-n}-np_i^{1-n})$, np_i^{1-n} is greater than Δp_c^{1-n} about 30,000 times. Neglecting Δp_c^{1-n} in Eq. (11) yields

$$\alpha_{av}=\frac{a(1-n)}{-np_i^{1-n}}\Delta p_c^1 \tag{37}$$

The p_i^{1-n} in Eq. (37) is not related to the filtration pressure but an inherent property of a cake. Then the average specific cake resistance is directly proportional to the filtration pressure, which means that the compressibility measured by the filtration of various pressures is one. So the compressibility 3 by CPC changes into 1 when determined by filtration experiments.

At the same hypothesis, the part of the denominator in Eq. (12), which is derived from the Shirato's constitutive equation, can be omitted.

$$\alpha_{av}=\frac{a_o(1-n)}{-p_a}\Delta p_c^1 \tag{38}$$

The average specific cake resistance is directly proportional to filtration pressure, too.

Rearranging Eq. (22), which is derived from the notion proposed in this study, with the same method leads to

$$\alpha_{av}=\frac{a(1-n)}{-p_f^{1-n}}\Delta p_c^1 \tag{39}$$

Average specific cake resistance relates to filtration pressure by a power function of power 1.

The three Eqs. (37), (38), and (39) having three different constitutive equations induce the same theoretical result that the compressibility obtained by filtration experiments cannot exceed 1, even in the extremely high compressible cake measured by CPC. There exist some experiments that the n value by CPC is larger than 1 [Grace, 1953; Yim and Kwon, 1997]. However, we have not found compressibility by filtration experiments greater than 1.

Tiller and Leu [1980] proposed a technique for obtaining the value of n by filtration experiments. We think that this technique can be applied to cakes which have the compressibility below about 0.6, but it may be inaccurate for materials having larger compressibility.

CONCLUSION

By analyzing the constitutive equations of a cake, the real meaning of the equations was studied. The variation of porosity at very low compressive solid pressure is confirmed by the equilibrium height of the sediment. Based on the phenomenon, a new constitutive equation was proposed. And a new boundary condition of cake was also suggested by analyzing the structure of filter cake. The new constitutive equation and new boundary condition were proved with filtration experiments from 0.009 atm to 2 atm. The height of sediment was calculated with the above new concepts, and proved experimentally. The meaning of the solid compressive pressure of first solid layer, p_f, proposed in this study was suggested. The influence of p_f to the cake thickness is analysed theoretically. Finally, the fact is proved that determination of compressibility by filtration experiments is not possible in case of a very compressible cake using three constitutive equations.

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