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Determination of the Nusselt versus Graetz Correlation for Heat Transfer in Channels of Sinusoidal Cross-Section

Cu Phan, Daniel L. Holgate and Gregory J. Griffin†

School of Engineering, James Cook University, Townsville, Qld, 4811 Australia
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Abstract−Metallic catalytic converters are composed of monoliths through which pass hundreds or thousands of parallel channels. Their mode of manufacture is such that each channel has the cross-section of a sinusoidal curve closed off by a nearly straight edge. During the operation of such converters, heat generated at the channel walls is transferred to the gases flowing through the channels. To understand the overall heat transfer characteristics of the monolith, it is necessary to understand the heat transfer rate between the channel walls and the fluid contained within them. With the use of the computational fluid dynamics package FIDAP, a three-dimensional model of a single channel was used to determine the local Nusselt number (Nu) versus Graetz number (Gz) correlation for heat transfer between the fluid and the walls of the channel. Flow through the channel was laminar and developing from a flat velocity profile at the channel inlet to the fully developed flow towards the outlet. Three different models were developed which corresponded to sinusoid height to width aspect ratios of 5 : 2, 3 : 2, and 1 : 1, respectively. The Nu vs. Gz correlations for the straight edge, curved sinusoidal edge and entire perimeter were calculated and are reported.

Key words : Heat Transfer, Channels, Sinusoidal Cross-section, Nusselt Number, Graetz Number, Monolith, CFD, Catalytic Combustion

INTRODUCTION

The monolithic reactor has been widely used in automotive exhaust systems to reduce emissions of undesired products and by-products of combustion. The monolith supports are usually cylindrical blocks made from a ceramic or metallic substrate with numerous straight parallel channels that have cross-sectional area from one to a few square millimetres. Metallic monoliths are normally manufactured by winding two layers of plain and corrugated metal sheet to form a compact spiral structure (see Fig. 1). This structure contains numerous channels each with a sinusoidal cross section closed off by a (near) straight edge. The walls of the channels are coated with a wash coat and catalyst preparation so as to initiate and sustain the reaction of gaseous fluid flowing through the channels. The monolith configuration offers many advantages when compared with traditional packed bed reactors, which has led to its use in other applications such as in catalytic combustors for gas turbines and also in steam reforming of hydrocarbons. These advantages include low pressure drop, improved radial heat transfer, fast response during transients and high surface to volume ratio [Flytzani-Stephanopoulos et al., 1986].

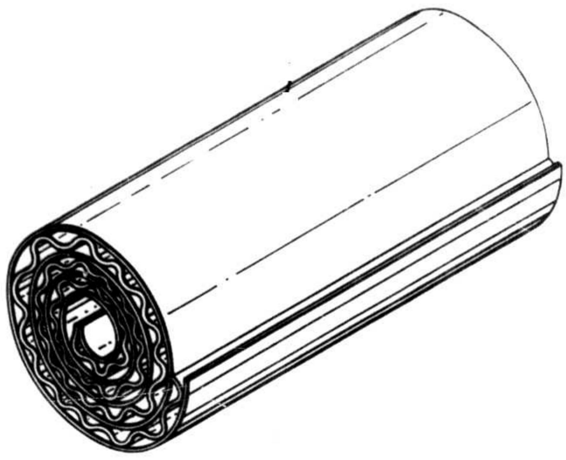

Fig. 1. A metal monolith with sinusoidal channels.

The mathematical modeling of the temperature and fuel concentration distribution for combustion within a metallic monolith can be extremely difficult due to the channel interaction between neighboring channels. This interaction can arise due to non-uniform distribution of fuel into the monolith, poor distribution of catalyst loading between monolith channels [Hayes and Kolaczkowski, 1999], or operation of the monolith in a non-adiabatic mode, i.e., heat transfer occurs between the monolith and external heat sinks or sources. Due to the large number of channels present within the monolith it is computationally unfeasible to model each channel individually by using the appropriate 2 or 3 dimensional form of the equations of change. Instead, previous researchers [Flytzani-Stephanopoulos et al., 1986; Kolaczkowski and Worth, 1995; Zygourakis, 1981] have used one-dimensional, lumped parameter model equations for each channel, and then used these as a basis for developing a model that includes channel interactions. For such models to be accurate it is necessary that the local mass and heat transfer coefficients between channel wall and bulk fluid be known. The heat transfer coefficients are usually calculated by using Nusselt number (Nu) versus Graetz number (Gz) correlations (with an analogous equation used to calculate the Sherwood and mass transfer coefficients). Such correlations have been theoretically and experimentally investigated for channels of circular [Flytzani-Stephanopoulos, 1986; Haw-

†To whom correspondence should be addressed.
E-mail: greg.griffin@csiro.au

thorn, 1974; Votruba et al., 1975], square [Kolaczkowski and Serbetcioglu, 1996] and triangular cross-section [Groppi and Tronconi, 1997]. These have shown that the geometry can significantly influence the mass and heat transfer characteristics of the monolith. Furthermore, it has been shown that the developing flow down the channels results in a significant change in heat transfer resistance with axial distance down the channel. Despite this, there is little data on heat and mass transfer coefficients for channels of sinusoidal cross-section. Sherony and Solbrig [1970] performed numerical calculations of the average Nusselt number for fully developed laminar flow in sinusoidal ducts. They reported numerical results for the effect of sinusoid aspect ratio on the Nusselt number. They compared their results with the experimental results reported by Howard [1965], although these results were rather sparse.

In this paper, the local Nusselt number versus Graetz number correlation is determined numerically for developing, laminar flow through a channel of sinusoidal cross section with a constant wall temperature. Local values are computed for the heat transfer through the straight edge of the channel, the curved sinusoidal edge of the channel and around the entire perimeter of the channel.

THEORETICAL MODEL

In developing the 3D model of the single sinusoidal channel the following assumptions were made:

- Steady-state laminar flow
- The velocity profile entering the channel was flat and develops down the channel
- Constant physical properties of the fluid phase
- Negligible heat transfer by radiation
- Natural buoyancy effects were negligible
- Wall temperature was constant
- At the outlet of the channel, flow is fully developed

Under these conditions the dimensionless equations of change are:

$$\text{(continuity)}\qquad (\nabla^*\cdot\vec{v}^*)=0 \tag{1}$$

$$\text{(motion)}\qquad \frac{1}{\mathrm{Re}}\nabla^{*2}\vec{v}^*-\nabla^*\mathrm{P}^*=\vec{v}^*\nabla^*\cdot\vec{v}^* \tag{2}$$

$$\text{(energy)}\qquad \frac{1}{\mathrm{RePr}}\nabla^{*2}\mathrm{T}^*=\vec{v}^*\cdot\nabla^*\mathrm{T}^* \tag{3}$$

The nomenclature used is placed at the end of this paper.

The boundary conditions are:

At the inlet of the channel,

$$z^*=0;\ T^*=0,\ v_z^*=1,\ v_x^*=v_y^*=0 \tag{4}$$

At the curved wall of the channel,

$$y^*=\frac{1}{2}A[1+\cos(\pi x^*)],\ 0\le x^*\le 1;\ v_x^*=v_y^*=v_z^*=0,\ T^*=1 \tag{5}$$

At the straight wall of the channel,

$$y^*=0,\ 0\le x^*\le 1;\ v_x^*=v_y^*=v_z^*=0,\ T^*=1 \tag{6}$$

At the line of symmetry,

$$x^*=0,\ 0\le y^*\le A;\ v_x^*=\frac{\partial v_y^*}{\partial x^*}=\frac{\partial v_z^*}{\partial x^*}=0,\ \frac{\partial T^*}{\partial x^*}=0 \tag{7}$$

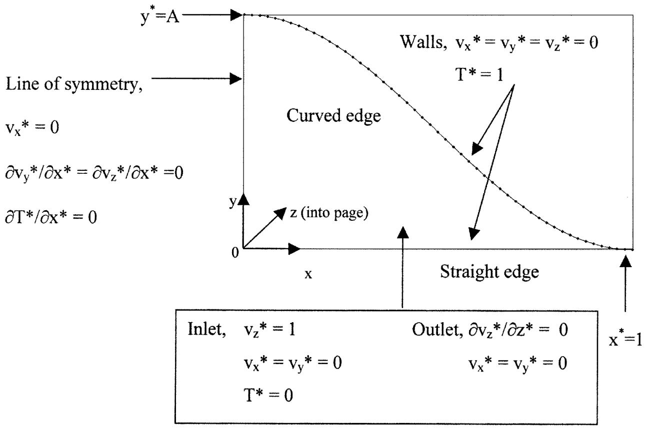

Fig. 2. Schematic diagram of channel cross section with boundary conditions.

At the outlet of the channel,

$$z^*=L;\ \frac{\partial v_z^*}{\partial z^*}=0,\ v_x^*=v_y^*=0 \tag{8}$$

Heat conduction in the axial direction was considered negligible, i.e., $\partial^2T^*/\partial z^{*2}=0$, and transverse pressure gradients are neglected, i.e., $\partial P^*/\partial x^*=\partial P^*/\partial y^*=0$.

The channel geometry is shown in Fig. 2. Note that the characteristic length used in these equations is half the base length of the sinusoid. However, the final correlations developed use the hydraulic diameter of the channel as the characteristic length.

1. Model Solution

The model described above was implemented by using the computation fluid dynamics package FIDAP on an 84 processor Silicon Graphics™ Origin 3800 System. This used a finite element method coupled with a segregated solver iterative process to obtain a solution. An example of the element mesh used is shown in Fig. 3.

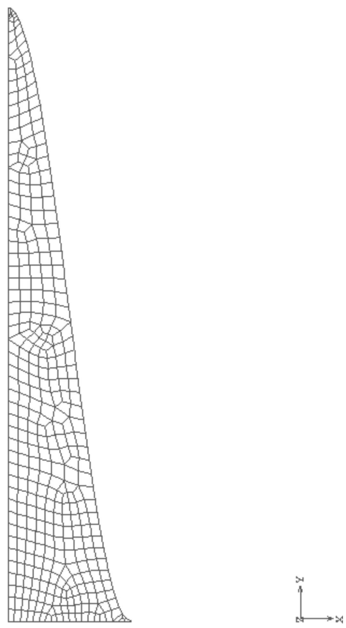

Fig. 3. Plan view of numerical mesh used for channel of 5 : 2 aspect ratio.

Typical time for solution was 2 CPU hours. Solutions were obtained for Re ranging from 24 to 800, with Pr=0.67. This represents a range of 5-45 in Gz. Solutions were obtained for three models corresponding to three different sinusoid height to width aspect ratios: 5 : 2 (Model 1), 3 : 2 (Model 2) and 1 : 1 (Model 3). The dimensionless length of the channel model was fixed at 19 (Model 1), 21 (Model 2) and 25 (Model 3).

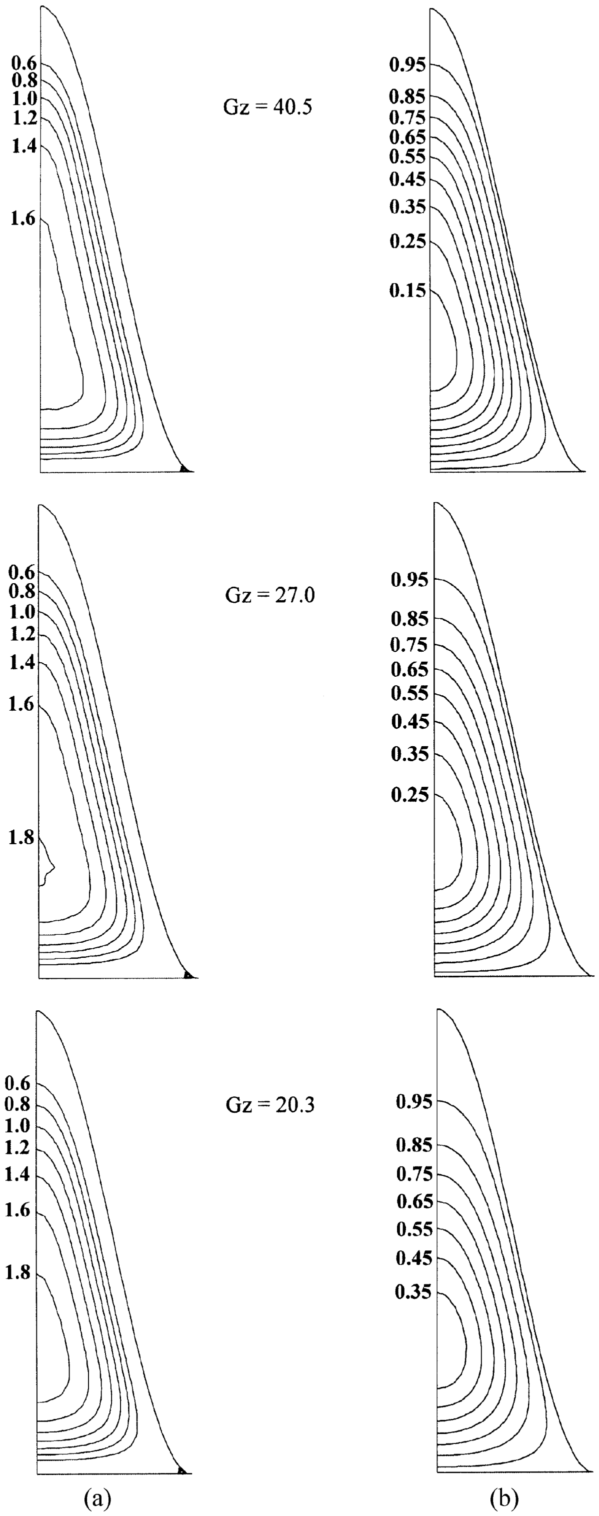

Fig. 4. Velocity (a) and temperature (b) contour plots across the channel at various distances (or Gz) down the channel. Results are for model 2, Re=300, Pr=0.67.

RESULTS AND DISCUSSION

Fig. 4 shows typical contour plots of temperature and fluid speed at various distances (or Gz) down the channel. As would be expected, the velocity profile shows a change from the flat velocity profile at the entrance to the fully developed profile as Gz decreases. The temperature of the fluid increased as it travelled down the channel and approached the wall temperature, $T^*=1$.

Using the computed temperature and velocity profiles across the channel cross section provided by FIDAP, we calculated the mixing cup average temperature at that cross-section (T^*_{av}) and the average temperature gradient along a wall, $(\partial T^*/\partial n^*)_{av}$. From these values, Nu was calculated by using:

$$\frac{hL}{k} = \mathrm{Nu} = \frac{\left(\dfrac{\partial T^*}{\partial n^*}\right)_{av}}{(1 - T^*_{av})} \tag{9}$$

Figs. 5, 6 and 7 show, respectively, the Nu versus Gz values derived for the straight edge of the channel, the curved edge of the

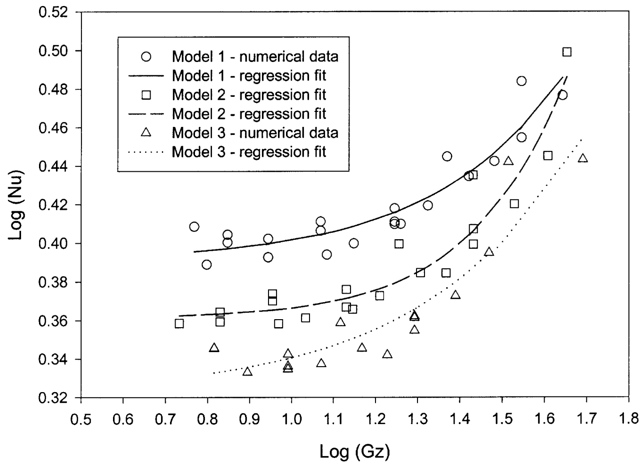

Fig. 5. Nu vs Gz values for straight edge of channel.

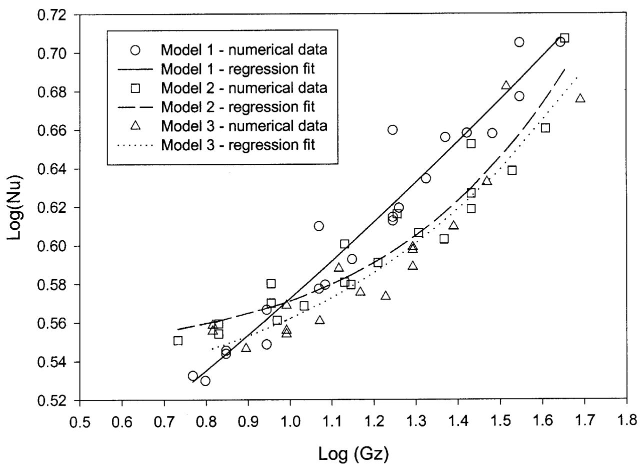

Fig. 6. Nu vs Gz values for curved edge of channel.

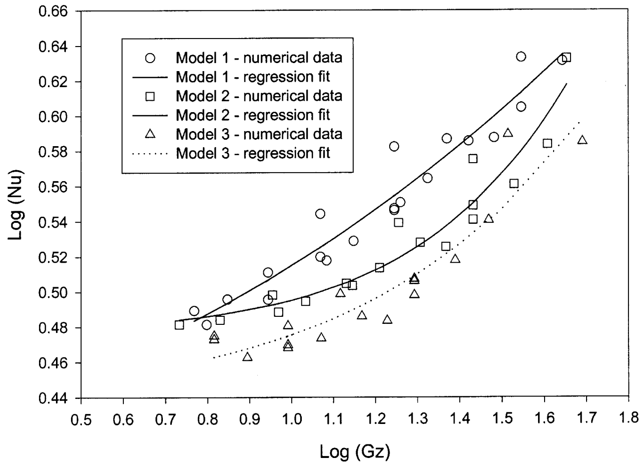

Fig. 7. Nu vs Gz values averaged for the entire perimeter of the sinusoidal channel.

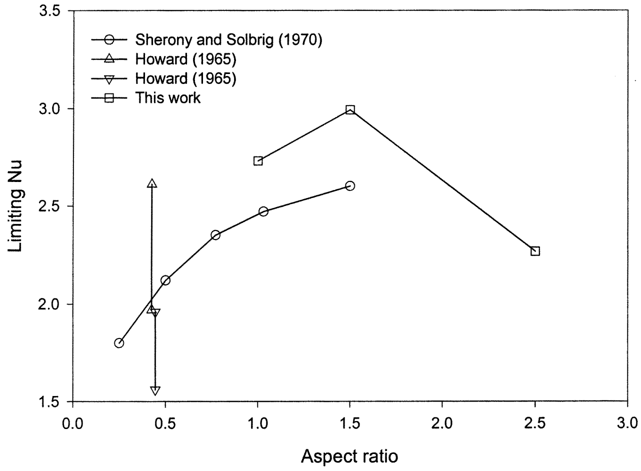

Fig. 8. Limiting Nu values.

Table 1. Calculated parameters for Eq. (9)

Model	Edge	a	b	m
1	Straight	2.46	1.68×10^{-2}	1.55
2	Straight	2.30	1.87×10^{-4}	2.18
3	Straight	2.10	4.34×10^{-3}	1.32
1	Curved	1.54	1.04	0.326
2	Curved	3.49	0.0152	1.19
3	Curved	3.23	5.70×10^{-2}	0.865
1	Perimeter	2.27	0.333	0.48
2	Perimeter	2.99	5.06×10^{-3}	1.43
3	Perimeter	2.73	2.50×10^{-2}	0.999

channel and the entire perimeter of the channel. A line of best fit was calculated for each set of data by using the equation:

$$\mathrm{Nu}=a+b(\mathrm{Gz})^{m} \tag{10}$$

Table 1 provides the parameters for the line of best fit. Note there was some scatter in the data indicating that a simple Nu vs Gz correlation was not entirely appropriate. It can be seen that Nu increases as Gz increases although, for the straight edge of the channel, the change in value of Nu is much less sensitive to the Gz values tested. The limiting value for Nu (i.e., as Gz approaches 0 where the flow becomes fully developed down the channel length) shows a maximum at an aspect ratio of 3 : 2. Literature [Shah and London, 1978] reporting on the effect of aspect ratio on Nu for fully developed flow in ducts of triangular cross-section showed a similar trend. The Nusselt numbers for the straight edge were always less than that for the curved edge. This would agree qualitatively with the velocity contours shown in Fig. 4, which shows the velocity gradients are higher along the curved edge compared to the straight edge. All data shown in Figs. 5 to 7 show that fully developed flow had not been achieved over the range of Gz values tested and that the higher aspect ratio (Model 1) had a much longer entry length.

When compared with values provided for other geometries, it is interesting that the asymptotic values (i.e., Nu as Gz$\rightarrow$0) calculated is close to the values for triangular channels (Nu=2.35-2.50) and straddle the value quoted for square channels (Nu=2.98; Groppi and Tronconi, 1997). Fig. 8 shows a comparison between the calculated limiting values of Nu determined here and those given by other researchers. Where the results overlap, the values calculated here are up to 25% higher than the theoretical values that are published by Sherony and Solbrig, 1970. This can be partly ascribed to the uncertainty introduced in this work by extrapolating the numerical data. The experimental data of Howard did not overlap with this work and the limiting value of Nu could not be determined, only a range of values could be given.

CONCLUSIONS

A computer simulation of laminar flow in channels of sinusoidal cross-section has been developed. From the results computed by using this simulation, the local Nusselt number for the curved edge, straight edge and entire perimeter has been calculated at various Graetz numbers. Equations to describe the correlation between Nusselt number and Graetz number have been reported and the results compared with channels of other cross-sectional shape.

NOMENCLATURE

- C_p : heat capacity [W/kg·K]
- D : characteristic length [m]
- D_H : hydraulic diameter [m]
- h : heat transfer coefficient [$W/m^2\cdot K$]
- k : thermal conductivity, [W/m·K]
- L : dimensionless length of channel
- n : vector length normal to the surface [m]
- P : fluid pressure [Pa]
- P^* : dimensionless pressure$=\dfrac{P-P_0}{\rho V^2}$
- T : temperature [K]
- T^* : dimensionless temperature$=\dfrac{T-T_0}{T_1-T_0}$
- v : velocity [m/s]
- v^* : dimensionless velocity$=\dfrac{v}{V}$
- V : inlet velocity

x : rectangular coordinate [m]
y : rectangular coordinate [m]
z : rectangular coordinate [m]
x^* : dimensionless coordinate $=x/D_H$
y^* : dimensionless coordinate $=y/D_H$
z^* : dimensionless coordinate $=z/D_H$

Greek Letters

ρ : fluid density [kg/m^3]
μ : viscosity [kg/ms]

Subscripts and Superscripts

* : non-dimensional quantity
0 : inlet condition
$\rightarrow$: vector quantity
x, y, z : components in x, y, z coordinates

Commonly Used Dimensionless Groups

$Gz = \left(\frac{RePr}{z*}\right)$: Graetz number

$Nu = \left(\frac{hD}{k}\right)$: Nusselt number

$Pr = \left(\frac{C_p\mu}{k}\right)$: Prandtl number

$Re = \left(\frac{DV\rho}{\mu}\right)$: Reynolds number

Sh : Sherwood number

Mathematical Operations

∇ : the "del" or "nabla" operator

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