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The Sensitivity and Stability of Spinning Process Using Frequency Response Method

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Abstract−The sensitivity and stability by frequency response of the final filament to several sinusoidal disturbances have been investigated in viscoelastic spinning by using various novel numerical algorithms. Amplitudes, or gains of the spinline cross-sectional area at the take-up, show resonant peaks, which are frequently encountered in hyperbolic systems. To effectively solve the complex system of the frequency response equation, alternative ways have been performed and compared. Interestingly, in the one-dimensional systems considered, integrating the linearized equations over the spinline length to shoot at the take-up boundary condition using two initial guesses ("two-shot" method) proved far more efficient than modal analysis using eigenfunction data or solving the matrix problem from the entire length by a direct method or an iterative one (GMRES). Also, the methodology to determine the stability of the system by using frequency response data, as suggested in Kase and Araki [1982], has been revamped to viscoelastic spinning system.

Key words: Frequency Response, Sensitivity, Stability, Viscoelastic Spinning, Modal Analysis, Iterative Method (GMRES), Two-shot Method, Direct Method

INTRODUCTION

In polymer processes such as fiber spinning, film casting, and tubular film blowing, manufacturing uniform and thin products (e.g., fibers or films) is not a trivial task at a high-speed operation. This is primarily caused by the extensional flow with free surfaces, which is very susceptible to many kinds of unexpected disturbances that affect the uniformity of fibers or films. Therefore, many efforts to explore the stability and sensitivity issues in these processes, closely related to productivity/profitability of the final products, have been increasing in academia and industry. Stability issues, especially focused on a self-sustained periodic oscillatory instability called as "draw resonance," have been scrutinized by many researchers during the last four decades [Pearson and Matovich, 1969; Gelder, 1972; Kase, 1974; Fisher and Denn, 1976; Hyun, 1978; Liu and Beris, 1988; Petrie, 1988; Larson, 1992; Kim et al., 1996; Jung et al., 1999a, 2000, etc].

Also, sensitivity issues about the propagation of disturbances, which influences the uniformity of fibers or films have been recently attractive [Kase and Araki, 1982; Devereux and Denn, 1994; Jung et al., 1999b, 2002; Lee et al., 2001, 2003a]. Sensitivity of the system to disturbances is generally analyzed by frequency response method, measuring the sinusoidal output of the linearized system subjected to small ongoing sinusoidal inputs or disturbances [Devereux and Denn, 1994; Park et al., 2001; Jung et al., 2002; Lee et al., 2003a, b]. Most information about the linear dynamical behavior, including the amplitude or gain, and the phase angle in Bode plots, can be drawn from this analysis. Kase and Araki [1982] designed Newtonian spinning as a feedback loop controlled by a spinline tension and examined the sensitivity and stability of their system from the transfer function data between disturbances and state variables after directly solving linearized transient governing equations. Also, other endeavors to investigate the sensitivity of the system have been performed by Jung et al. [1999b] and Lee et al. [2001] using tension sensitivity analysis. The above literature consistently emphasized that the spinline tension acting on a spinline became the key link in relaying disturbances to the draw resonance instability.

This study has focused on the frequency response method exhibited by Jung et al. [2002] to investigate the stability as well as the sensitivity of viscoelastic spinning to any sinusoidal disturbances. Especially, to seek an effective algorithm for solving complex frequency response equations, several novel numerical schemes have been suggested and compared. Also, the stability of a viscoelastic spinning has been efficiently expected from the Nyquist plot of frequency response data, by revamping the feedback loop concept of Kase and Araki [1982].

GOVERNING EQUATIONS FOR SPINNING FLOWS

Dimensionless governing equations of the isothermal spinning with PTT fluid model are as follows [Jung et al., 2002]. PTT model is well known for its robustness and accuracy in portraying extensional deformation processes for both extensional thickening and extensional thinning fluids [Khan and Larson, 1987; Kwon and Leonov, 1995].

Continuity equation:

$$\frac{\partial a}{\partial t}+\frac{\partial}{\partial x}(av)=0 \tag{1}$$

where $a\equiv\frac{A}{A_0}, v\equiv\frac{V}{V_0}, t\equiv\frac{\tilde{t}V_0}{L}, x\equiv\frac{\tilde{x}}{L}$

Equation of motion:

$$C_{in}\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right)=\frac{1}{a}\frac{\partial(a\tau)}{\partial x} \tag{2}$$

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where $\tau \equiv \frac{\sigma L}{2\eta_0 V_0}, C_{in} \equiv \frac{\rho V_0 L}{2\eta_0}$

Constitutive equation: PTT fluids

$$K\tau + De\left(\frac{\partial \tau}{\partial t} + v\frac{\partial \tau}{\partial x} - 2(1-\xi)\tau\frac{\partial v}{\partial x}\right) = \frac{\partial v}{\partial x} \tag{3}$$

where $K \equiv \exp(2\varepsilon De\,\tau), De \equiv \frac{\lambda V_0}{L}$

(Notations appearing here are given in the Nomenclature.)

These dimensionless equations are subject to the following boundary conditions:

$$a=a_0=1,\ v=v_0=1,\ \tau=\tau_0 \text{ at } x=0 \text{ for all } t \tag{4}$$

$$v=v_L=r \text{ at } x=1 \text{ for all } t \tag{5}$$

In the above equations, several assumptions have been incorporated in order to simplify the model and to focus on the extensional deformation which constitutes dominant dynamics in spinning. First, the thin filament approximation simplifies this system to a one-dimensional model. Second, the origin of the spinning distance coordinate is chosen at the die (extrudate) swell position, meaning all the pre-spinneret deformation history of the liquid is not included in the model. Third, inertial and rheological forces are more dominant than other secondary forces.

FREQUENCY RESPONSE METHOD

Frequency response represents the sensitivity of the linearized system by inspecting the amplitudes of output variables to ongoing sinusoidal perturbations around a base flow. For convenience, above nonlinear governing equations [Eqs. (1)-(3)] are compactly reduced to the following simple vector form:

$$\underline{R}(\underline{q}, \dot{\underline{q}}, p) = \underline{0} \tag{6}$$

where $\underline{q}$ is the solution vector of spinline cross-section, spinline velocity, and axial stress, $\dot{\underline{q}}$ time derivative of $\underline{q}$, and p any parameter to be perturbed. Then, introducing small perturbation and linearizing Eq. (6) around the steady state ($\underline{q} = \underline{q}_s$) leads to a transient linearized equation set:

$$\underline{\underline{M}}(\underline{q}_s, p)\Delta\dot{\underline{q}} - \underline{\underline{J}}(\underline{q}_s, p)\Delta\underline{q} - \underline{F}(\underline{q}_s, p)\Delta p = \underline{0} \tag{7}$$

where $\underline{\underline{J}} \equiv (\partial\underline{R}/\partial\underline{q})_{q_s, p_s}$ is Jacobian matrix at the steady state, $\underline{\underline{M}} \equiv -(\partial\underline{R}/\partial\dot{\underline{q}})_{q_s, p_s}$ mass matrix, and $\underline{F} \equiv (\partial\underline{R}/\partial p)_{q_s, p_s}$ forcing vector of the residuals to the parameter p evaluated at steady state.

A particular solution of Eq. (7) responds to a steady oscillation with the same frequency as a sinusoidal disturbance (i.e., $\Delta p = \zeta \exp(i\omega t)$) as follows:

$$\Delta\underline{q} = \zeta\underline{k}\exp(i\omega t) \tag{8}$$

where ω is the frequency of the ongoing disturbance, ζ the complex value of the amplitude of the imposed disturbance, $i=\sqrt{-1}$, $\underline{k}$ the complex value representing the amplitude and phase lag of solutions relative to the imposed disturbance.

Substituting Eq. (8) into Eq. (7) leads to the linear complex equation

$$(i\omega\underline{\underline{M}} - \underline{\underline{J}})\underline{k} = \underline{F} \tag{9}$$

Amplitude or gain, G_i, and phase angle, θ_i of a state variable i, can be determined from the complex response of Eq. (9). (But, the phase lag is not examined in this study.)

$$G_i \equiv \sqrt{(\mathrm{Re}(k_i))^2 + (\mathrm{Im}(k_i))^2},\ \theta_i \equiv \tan^{-1}\frac{\mathrm{Im}(k_i)}{\mathrm{Re}(k_i)} \tag{10}$$

Some possible ways to solve the complex linear system, Eq. (9), are described in the next sections.

1. Direct Method for Complex System

Eq. (9) can be directly solved from the following "doubled" matrix. To avoid complex arithmetic, the complex system is doubled with real and imaginary parts of the solution vector, $\underline{k}$

$$\begin{pmatrix} -\underline{\underline{J}} & -\omega\underline{\underline{M}} \\ \omega\underline{\underline{M}} & -\underline{\underline{J}} \end{pmatrix}\begin{pmatrix} \mathrm{Re}(\underline{k}) \\ \mathrm{Im}(\underline{k}) \end{pmatrix} = \begin{pmatrix} \underline{F} \\ \underline{0} \end{pmatrix} \tag{11}$$

where $\mathrm{Re}(\underline{k})$ and $\mathrm{Im}(\underline{k})$ are the real and imaginary parts of the solution vector, $\underline{k}$. An LU decomposition of the doubled system is used at each frequency.

2. Integrating over the Spinline Length (Two-shot Method)

Doubled linear Eq. (11), comprising real and imaginary parts separately, can be successively integrated along the spinline length by using only two initial guesses (in this case, two perturbed initial stresses at the spinneret) to shoot at the take-up boundary conditions. Due to the linearity of the system, the desired initial guess at each frequency is found by linear interpolation of two initial guesses without further iterations and then, the amplitude of spinline cross-sectional area at the take-up at each frequency can be easily obtained. The 4th-order Runge-Kutta method is used in this study as an initial boundary problem with shooting technique. It is, however, noted that using this method may be limited to one-dimensional systems. Other powerful techniques to surpass the direct method can be required for complicated multi-dimensional systems.

3. Modal Analysis

An alternative to directly solving Eq. (9) is to decompose the complex system into its independent dynamical normal modes using left/right eigenvectors of the system together with the corresponding eigenvalues evaluated from the linear stability analysis. This method is well known to modal analysis in the fields of vibration and structural engineering [Wahed and Bishop, 1976; Fawzy and Bishop, 1977; Claeyssen, 1990]. Chen [1992] first explored the application of modal analysis in the coating flow system. It has been suggested from his results that modal analysis is a very attractive alternative when stability has already been analyzed, because a large system can be effectively reduced by only a small number of the leading eigenmodes. A brief description of modal analysis follows.

Modal analysis finds the complex response of the inverse of the complex matrix, $(i\omega\underline{\underline{M}} - \underline{\underline{J}})^{-1}$ as a linear combination of the matrix products of the right and left eigenvectors, by using the well known spectral decomposition theorem [Saad, 1992]. So the response is constructed as follows:

$$\underline{k} = (i\omega\underline{\underline{M}} - \underline{\underline{J}})^{-1}\underline{F} = \sum_{j=1}^{N}\frac{\underline{\phi}_j\underline{\psi}_j}{i\omega - \lambda_j}\cdot\underline{F} \tag{12}$$

where $\underline{\phi}_j$ and $\underline{\psi}_j$ are the normalized right and left eigenvectors corresponding to same eigenvalue, λ_j, and N is the size of $\underline{\underline{J}}$ or $\underline{\underline{M}}$ matrix.

The total response of $\underline{k}$ can be divided into two contributions: k_p the principal response from the leading modes from 1 to N_p, and $\underline{k}_B$ the response from remaining modes from N_p+1 to N, so-called, "background" response.

$$\underline{k}=(i\omega\underline{\underline{M}}-\underline{\underline{J}})^{-1}\underline{F}=\underline{k}_p+\underline{k}_B=\sum_{j=1}^{N_p}\frac{\underline{\phi}_j\underline{\psi}_j}{i\omega-\lambda_j}\cdot\underline{F}+\sum_{j=N_p+1}^{N_p}\frac{\underline{\phi}_j\underline{\psi}_j}{i\omega-\lambda_j}\cdot\underline{F} \tag{13}$$

In general, $\underline{k}_B$ is very small compared to $\underline{k}_p$, so the total response is approximated to the response by the leading modes. However, a background term has been also considered here by simply approximating at zero frequency [Eq. (14)].

$$\underline{k}_B=\underline{k}_0+\sum_{j=1}^{N_p}\frac{\underline{\phi}_j\underline{\psi}_j}{\lambda_j}\cdot\underline{F} \tag{14}$$

where $\underline{k}_0=-\underline{\underline{J}}^{-1}\underline{F}$.

4. Iterative Method (GMRES)

As in the case of the direct method, the doubled matrix to solve the linear system is also considered here.

$$\underline{\underline{Q}}\underline{k}=\begin{pmatrix}-\underline{\underline{J}} & -\omega\underline{\underline{M}}\\ \omega\underline{\underline{M}} & -\underline{\underline{J}}\end{pmatrix}\begin{pmatrix}\mathrm{Re}(\underline{k})\\ \mathrm{Im}(\underline{k})\end{pmatrix}=\begin{pmatrix}\underline{F}\\ \underline{0}\end{pmatrix}=\underline{f} \tag{15}$$

The iterative method constructs the response on a m-th Krylov subspace of the doubled matrix, $\underline{\underline{Q}}$, defined by

$$\underline{\underline{U}}_m=\{\underline{u}_1,\underline{\underline{Q}}\underline{u}_1,\ldots,\underline{\underline{Q}}^{m-1}\underline{u}_1\} \tag{16}$$

where $\underline{u}_1=\underline{s}_0/\|\underline{s}_0\|_2$, $\underline{s}_0=\underline{f}-\underline{\underline{Q}}\,\underline{k}_0$, and $\underline{k}_0$ is the initial guess of the solution. This method approximates the response as a linear combination of basis vectors that span the Krylov subspace.

As a useful one among the iterative methods, the generalized minimal residual method (GMRES) developed by Saad and Schultz [1986] is implemented to solve the doubled linear system. The solution is expressed as a linear combination of the Krylov subspace basis vectors,

$$\underline{k}=\underline{k}_0+\underline{\underline{U}}_m\underline{y} \tag{17}$$

where $\underline{y}$ is a vector of size m, the Krylov subspace dimension. This method minimizes the residual norm over all vectors in Eq. (17). Residual $\underline{s}(\equiv\underline{f}-\underline{\underline{Q}}\underline{k})$ of Eq. (15) can be expressed into the following form, combined with Eqs. (16) and (17)

$$\underline{s}_0=\underline{s}_0-\underline{\underline{Q}}\underline{\underline{V}}_m\underline{y}=\beta\underline{u}_1-\underline{\underline{U}}_{m+1}\overline{\mathrm{H}}_m\underline{y}=\underline{\underline{U}}_{m+1}(\beta\underline{e}_1-\overline{\mathrm{H}}_m\underline{y}) \tag{18}$$

where $\beta\equiv\|\underline{s}_0\|_2$, $\underline{e}_i$ is unit vector, and $\overline{\mathrm{H}}_m$ the (m+1)×m augmented Hessenberg matrix. Since the column vectors of $\underline{\underline{U}}_{m+1}$ are orthogonal, then GMRES algorithm seeks $\underline{y}$ to minimize $\|\beta\underline{e}_1-\overline{\mathrm{H}}_m\underline{y}\|_2$. (A more detailed algorithm of this method is well described in Saad [1996].) Also, the performance of this method is effectively improved with a pre-conditioning step [Gates, 1999].

RESULTS AND DISCUSSION

1. Sensitivity Results

Frequency response of the spinline cross-sectional area at take-up to several ongoing disturbances was evaluated by several algorithms outlined above. Nodal points (np) by finite difference along the whole spinline are 401, guaranteeing acceptable accuracy. More detailed explanations about sensitivity results of PTT viscoelastic

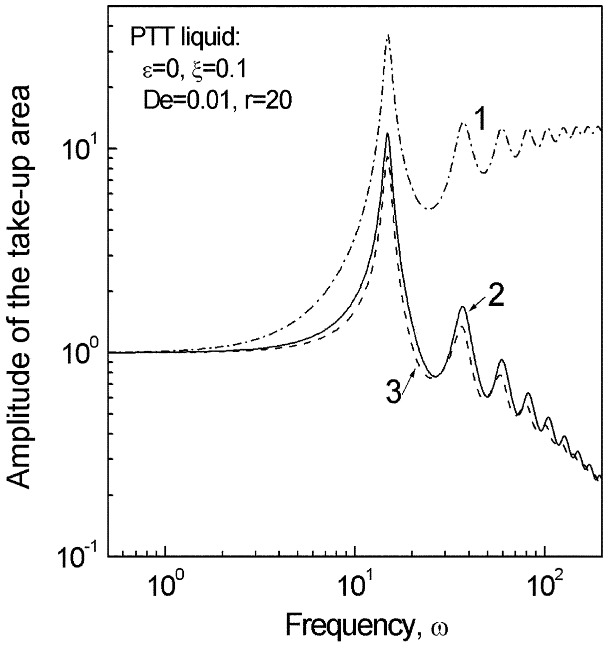

Fig. 1. Amplitudes of spinline cross-sectional area at the take-up for isothermal PTT spinning where various disturbances are introduced in 1: spinneret area, 2: take-up velocity, 3: extrusion velocity.

fluids are presented in Jung et al. [2002]. Some of the interesting results are introduced here, because the main focus of this study develops useful numerical methods for solving frequency response and determines the stability of the system by using these frequency data.

Fig. 1 shows predicted amplitudes or gains of the spinline cross-sectional area at the take-up as a function of the frequency to sinusoidal perturbations in spinneret cross-sectional area, take-up velocity, and extrusion velocity. In all cases, amplitudes of the solution give unity value in the low-frequency region, because the sinusoidal change of flow rate directly alters the final spinline cross-sectional area. In contrast, the change of other disturbances, not influencing the flow rate, such as viscosity, elasticity, and cooling conditions, results in almost zero amplitude at the low-frequency region [Jung et al., 2002].

One interesting thing is that the changes in sensitivity, i.e., amplitudes from frequency response, are well connected to some information from linear stability analysis. Amplitudes shown in Fig. 1 have resonant peaks along the frequency domain, closely related to the wave characteristics of the hyperbolic systems [Friedly, 1972]. In this case, frequencies at resonance peaks are exactly the same as the imaginary parts of successive leading eigenmodes from linear stability analysis.

Process sensitivity is directly scrutinized with the amplitude G_i. The more sensitive a process is, the higher the peaks of amplitude to sinusoidal disturbances. Therefore, it is possible to systematically analyze the effect of several process conditions such as cooling, viscoelasticity, inertial force, etc. on the sensitivity of the system [Jung et al., 2002].

2. Comparison of the Performance of Several Numerical Algorithms

Direct solving the full doubled matrix of Eq. (11) as a direct method will be tedious and time-consuming if unknowns or nodal points along the spinline are enormously increased. More efficient numerical techniques should be devised to solve the large-sized frequency response equations. It has been found that the two-shot method, which solves the frequency response of our one-dimensional system by integrating the linearized equations over the spinline length with only two initial guesses, is superior to all other methods considered here. The performance of each method is presented in Table 1 and explained later.

Fig. 2 portrays the frequency response results by modal analysis in Newtonian spinning. Over 0.45 N principal leading modes are required to closely agree with the exact solution. Interestingly, even

Table 1. The performance test of the direct, two-shot, modal, and iterative methods

Methods	Computation time (s)		Residuals $\|(i\omega\underline{M} - \underline{J})\underline{k} - \underline{F}\|_2$	
	ω=28	ω=80	ω=28	ω=80
Direct	132	132	9.59×10^{-8}	8.18×10^{-8}
Two-shot	<1	<1		
Modal: N_p=0.05 N	1-2	1-2	1.03	10.38
0.2 N			1.04	9.77
0.35 N			0.52	9.46
0.45 N			0.13	1.02
0.55 N			1.34×10^{-7}	8.78×10^{-8}
Iterative: m=5	50	51	17.87	500.84
20	65	73	8.85×10^{-9}	31.99
40	60	83	6.08×10^{-9}	1.41
50	65	92	5.40×10^{-9}	7.63×10^{-9}

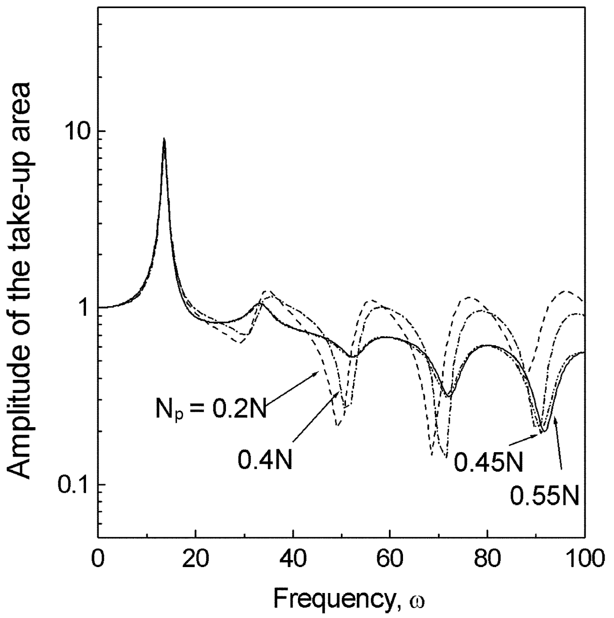

Fig. 2. The effect of the number of the leading modes (N_p) on the sensitivity in the modal analysis when a disturbance is introduced at the take-up velocity (De=0, C_{in}=0.05, and r=25).

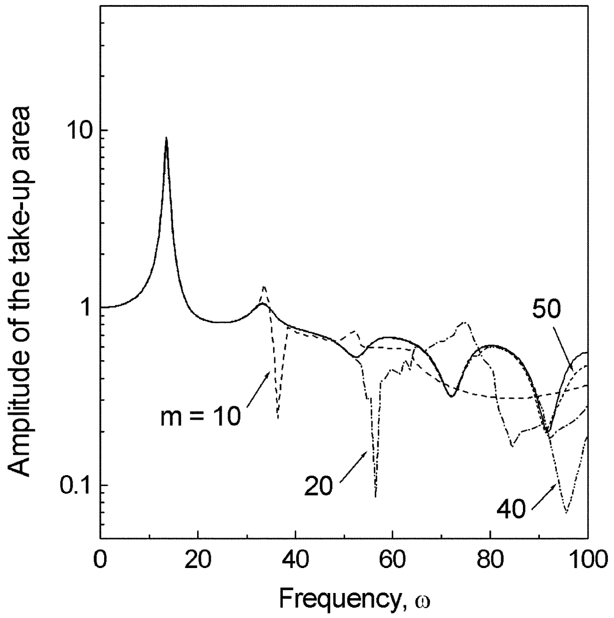

Fig. 3. The effect of the Krylov subspace dimension (m) on the sensitivity in the iterative method (GMRES) when a disturbance is introduced at the take-up velocity (De=0, C_{in}=0.05, and r=25).

with the small number of leading modes the first peak or the most dangerous one can be exactly predicted. Solving by this method is much faster than the direct method, because a leading part of all eigenmodes (N) are only used. The response by this method might depend on the forcing term as well as the number of leading modes.

The results in Newtonian spinning by iterative method (preconditioned GMRES) show that the frequency response is well estimated with around 50 Krylov subspace dimensions, bringing forth faster computation time than the direct method (Fig. 3). Also, the most dangerous peak in the low frequency region has been exactly obtained with only a small number of Krylov subspace dimensions, as mentioned in the case of modal analysis.

Table 1 compares the performance of the direct method, two-shot method, modal analysis, and iterative method at only two frequencies, ω=28 and 80, respectively. The time needed by the direct method is virtually the same at all frequencies. The two-shot method is the fastest among all methods, just requiring a few seconds to draw the whole frequency response of the spinline cross-sectional area at the take-up. The performance of the modal analysis with fast computation time is comparable to the two-shot method. The computation time for extracting the leading left/right eigenvectors from all eigenmodes and calculating the frequency response of the spinline cross-sectional area at the take-up only is recorded in Table 1 (In other words, computation time for obtaining the eigenmodes from linear stability analysis is not included. There is no need to calculate eigenmodes at every frequency, because these are input data before starting the frequency response calculation.). In the iterative method, the total time includes the time spent on the preconditioning operation in addition to the GMRES algorithm. This method also needs shorter computation time than the direct method. Because the preconditioning step is only conducted before starting the frequency response calculation, actual computation time at every frequency is shorter than that given in Table 1.

From the performance test among the methods considered in this study, it has been confirmed that the two-shot method is best suited to solve the frequency response of one-dimensional systems such as spinning, film casting process, and simplified curtain coating flow [Jung and Scriven, 2001]. (Performance: Two shot>modal analysis> iterative method>direct method.) Also, modal analysis and iterative method are more powerful tools than direct method. In particular, using these two methods will be notably favorable in multi-dimensional systems with large sparse matrix, as demonstrated in Gates [1999].

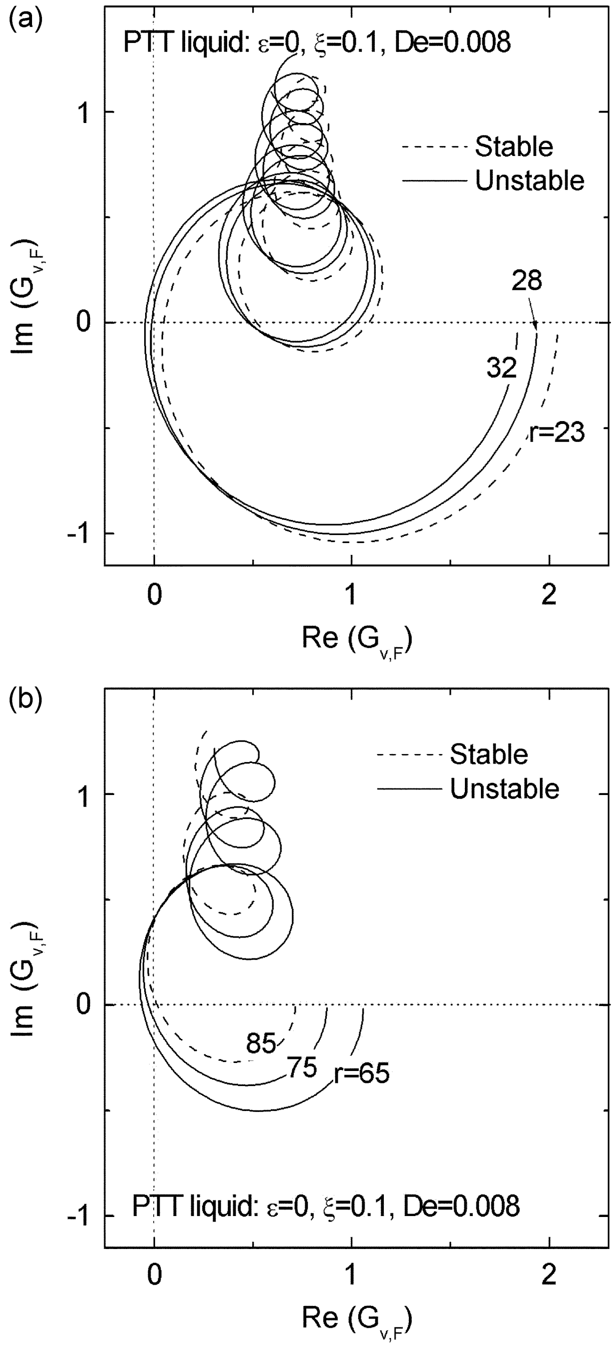

Fig. 4. Vector loci of transfer function between spinline tension and take-up velocity of a PTT liquid in (a) low drawdown ratio and (b) high drawdown ratio regions.

3. Stability Results by Frequency Response

As suggested in Kase and Araki [1982], frequency response has been evidently applied to determine the stability of a simple Newtonian spinning system. They devised the spinning system as a feedback loop controlled by the spinline tension acting on the spinline (Fig. 6 in their article) and derived the transfer function equation relating the cross-sectional area at the take-up to any disturbances. It has been demonstrated that the spinning system is unstable when the vector locus of the transfer function, $G_{v,F}$, connecting spinline tension and take-up velocity, "encircles" the origin of the complex plane in Nyquist plot. The above stability criterion has been extended to the viscoelastic system of our study. Furthermore, our method is much simpler and faster than Kase's in that we applied one of the frequency response methods suggested here (i.e., two-shot method for this study) to obtain the transfer function, $G_{v,F}$, whereas they used the transient numerical simulation. Fig. 4 shows $G_{v,F}$ loci for determining the stability of a PTT spinning system. The stable and unstable conditions have been exactly expected by inspecting whether the vector locus of $G_{v,F}$ includes the origin of the complex plane or not. Whole stability windows can be drawn from this method as shown in Fig. 5, giving the same results as those by the linear stability theory.

CONCLUSION

The sensitivity and stability of the viscoelastic spinning system to several sinusoidal disturbances have been investigated by using the frequency response method. Due to the hyperbolic characteristics of the system, amplitudes of the spinline cross-sectional area at the take-up show resonant peaks along the whole frequency domain, where the frequencies at local maxima of amplitudes are equal to the imaginary parts of the leading eigenmodes. To effectively solve

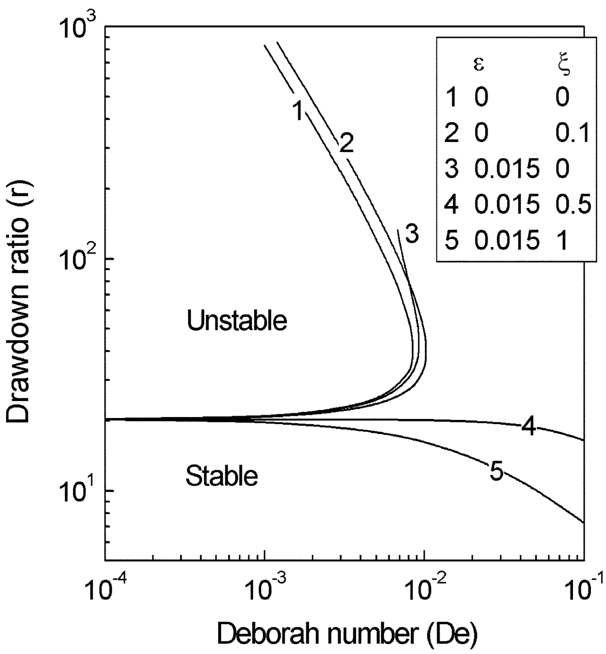

Fig. 5. Neutral stability curves of PTT fluids in isothermal spinning.

the frequency response equation system, alternative methods have been developed and compared. Interestingly, in the cases examined, the two-shot method proved far more efficient than modal analysis using the eigenfunctions or solving the matrix problem from the entire spinline length by a direct method or an iterative method (GMRES). The two-shot method can also be implemented to analyze the sensitivity of other one-dimensional systems such as sheet casting and simplified curtain coating. Modal analysis and iterative method are also attractive tools in solving the frequency response, especially for multi-dimensional systems with large sparse matrix. Also, the stability of the system using frequency response data is successfully determined by revamping the stability criterion from the feedback control system, as devised in Kase and Araki [1982].

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NOMENCLATURE

A : spinline cross-sectional area
a : dimensionless spinline cross-sectional area
C_{in} : coefficient representing the inertia force
De : Deborah number
$\underline{F}$: forcing vector
$\underline{G}_i$: amplitudes or gains of state variable i to the imposed disturbance
$\overline{H}_m$: (m+1)×m augmented Hessenberg matrix
$\underline{\underline{J}}$: Jacobian matrix
$\underline{k}$: complex value representing the amplitude and phase lag of solutions relative to the imposed disturbance
L : spinning distance between the spinneret and the take-up
$\underline{\underline{M}}$: mass matrix
m : Krylov subspace dimension
N : matrix size of $\underline{\underline{J}}$ and $\underline{\underline{M}}$
np : number of mesh points in the discretized spinning distance coordinate
$\underline{q}$: solution vector of state variables
r : draw-down ratio
$\tilde{t}$: time
t : dimensionless time
V : spinline velocity
v : dimensionless spinline velocity
$\tilde{x}$: distance from the spinneret
x : dimensionless distance from the spinneret
$\underline{z}$: solution vector of state variables

Greek Letters

ε : material parameter of PTT fluids
ϕ : normalized right eigenvector
$\overline{\eta}_0$: liquid viscosity at zero strain-rate
λ : material relaxation time
ρ : liquid density
σ : spinline axial stress
τ : dimensionless spinline axial stress
ω : frequency
ξ : material parameter of PTT fluids
ψ : normalized left eigenvector
$\overline{\zeta}$: complex value of the amplitude of the imposed disturbance

Subscripts

0 : values at the spinneret
L : values at the take-up
s : values at steady state

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