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Onset of Buoyancy-Driven Convection in the Horizontal Fluid Layer Heated from Below with Time-Dependent Manner

Min Chan Kim†, Tae Joon Chung* and Chang Kyun Choi*

Department of Chemical Engineering, Cheju National University, Cheju 690-756, Korea
*School of Chemical Engineering, Seoul National University, Seoul 151-744, Korea
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Abstract−The onset of buoyancy-driven convection in an initially isothermal, quiescent fluid layer heated from below with time-dependent manner is analyzed by using propagation theory. Here the dimensionless critical time τ_c to mark the onset of convective instability is presented as a function of the Rayleigh number Ra_ϕ and the Prandtl number Pr. The present stability analysis predicts that τ_c decreases with increasing Pr for a given Ra_ϕ. The present predictions compare reasonably well with existing experimental results. It is found that in deep-pool systems the deviation of temperature profiles from conduction state occurs starting from a certain time $\tau \cong (2\sim4)\tau_c$.

Key words: Buoyancy-driven Instability, Critical Time, Propagation Theory, Ramp Heating

INTRODUCTION

When an initially quiescent, horizontal fluid layer is cooled from above or heated rapidly from below, the basic temperature profile of heat conduction develops with time and buoyancy-driven convection sets in at a critical time. In this transient system the critical time t_c to mark the onset of convective instability becomes an important question. This instability problem may be called an extension of classical Rayleigh-Benard problems. The related instability analysis has been conducted by using the frozen-time model [Morton, 1957], propagation theory [Choi et al., 1998; Yang and Choi, 2002; Kim et al., 2002], maximum-Rayleigh-number criterion [Tan and Thorpe, 1999], amplification theory [Foster, 1965], and stochastic model [Jhaveri and Homsy, 1982]. The first two models are based on linear theory and yield the critical time t_c. The last two models require the initial conditions at the heating time t=0 and the criterion to define manifest convection, which yields the characteristic time t_m to mark the first detection of manifest convection.

For transient conduction systems cooled from above or heated from below, propagation theory has been used to analyze the instability problems. This model assumes that at $t=t_c$ infinitesimal temperature disturbances are propagated mainly within the thermal penetration depth and with this length-scaling factor all the variables and parameters having the length scale are rescaled. The resulting stability criteria have compared well with experimental data in solidification [Hwang and Choi, 1996] and Marangoni-Benard convection [Kang et al., 2000]. Also, a similar approach is found to be successful in cases of forced convection flow [Choi and Kim, 1994; Kim et al., 1999, 2003] and porous media [Lee et al., 2000; Chung et al., 2002].

Here we will concentrate on the instability problem in an initially isothermal, quiescent fluid layer. Starting from time t=0, the lower boundary is heated with a constant temporal heating rate. For this specific system the instability criteria will be obtained by using propagation theory. Our predictions will be compared with available experimental and theoretical results.

THEORETICAL ANALYSIS

1. Mathematical Formulations

The system considered here is a Newtonian fluid layer with an initial temperature T_i. For time t>0 the horizontal layer of fluid depth d is heated from below with the constant temporal heating rate ϕ and its upper boundary is kept isothermally. The schematic diagram of the basic system of pure conduction is shown in Fig. 1. The governing equations of flow and temperature fields are expressed by employing the Boussinesq approximation as

$$\nabla \cdot \mathbf{U} = 0, \tag{1}$$

$$\left\{\frac{\partial}{\partial t} + \mathbf{U}\cdot\nabla\right\}\mathbf{U} = -\frac{1}{\rho_r}\nabla P + \nu\nabla^2\mathbf{U} + g\beta T\mathbf{k}, \tag{2}$$

$$\left\{\frac{\partial}{\partial t} + \mathbf{U}\cdot\nabla\right\}T = \alpha\nabla^2 T, \tag{3}$$

where **U**, T, P, ν, g, ρ, β, **k** and α represent the velocity vector, the temperature, the dynamic pressure, the kinematic viscosity, the gravitational acceleration constant, the density, the thermal expansion

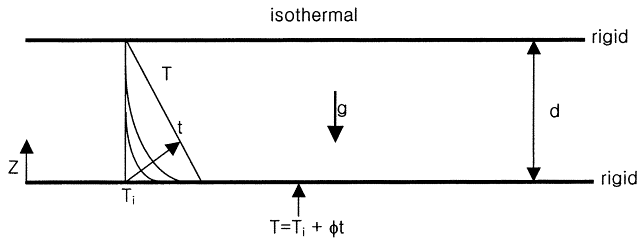

Fig. 1. Sketch of the basic conduction state considered here.

†To whom correspondence should be addressed.
E-mail: mckim@cheju.ac.kr
‡This paper is dedicated to Professor Hyun-Ku Rhee on the occasion of his retirement from Seoul National University.

coefficient, the vertical unit vector and thermal diffusivity, respectively. The subscript r represents the reference state. The surface temperature T_s at the vertical distance Z=0 increases with time during the conduction period.

The important parameters for characterizing the onset of motion in the present system are the Prandtl number (Pr), the Rayleigh number based on the temperature difference (Ra), and Rayleigh number based on the temporal heating rate (Ra_ϕ): $Pr=\nu/\alpha$, $Ra=g\beta\Delta T d^3/(\alpha\nu)$ and $Ra_\phi=g\beta\phi d^5/(\alpha\nu)$. Under linear theory the nondimensionalized conservation equations are given as usual:

$$\left\{\frac{1}{Pr}\frac{\partial}{\partial\tau}-\overline{\nabla}^2\right\}\overline{\nabla}^2 w_1=-\overline{\nabla}_1^2\theta_1, \tag{4}$$

$$\frac{\partial\theta_1}{\partial\tau}+Ra_\phi w_1\frac{\partial\theta_0}{\partial z}=\overline{\nabla}^2\theta_1, \tag{5}$$

where $\overline{\nabla}^2$ is the three-dimensional Laplacian, and $\overline{\nabla}_1^2$ is the horizontal one with respect to x and y. Here z, τ, w_1, θ_0 and θ_1 are the dimensionless vertical distance, time, vertical velocity disturbance, basic temperature and perturbed temperature, respectively. Each variable has been nondimensionalized by using d, d^2/α, α/d, $\phi d^2/\alpha$ and $\alpha\nu/g\beta d^3$, respectively. The proper boundary conditions of no slip and no perturbation of temperature are

$$w_1=\frac{\partial w_1}{\partial z}=\theta_1=0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=1. \tag{6}$$

For the basic state of heat conduction the dimensionless temperature profile is represented by

$$\frac{\partial\theta_0}{\partial\tau}=\frac{\partial^2\theta_0}{\partial z^2}, \tag{7}$$

with the following initial and boundary conditions,

$$\theta_0(0, z)=0,\ \theta_0(\tau, 0)=\tau,\ \theta_0(\tau, 1)=0. \tag{8}$$

The exact solutions of Eqs. (7) and (8) can be easily obtained as

$$\theta_0=4\tau\sum_{n=0}^{\infty}[i^2\mathrm{erfc}A-i^2\mathrm{erfc}B]. \tag{9}$$

Here $A=n/\sqrt{\tau}+\zeta/2$ and $B=(n+1)/\sqrt{\tau}-\zeta/2$, where ζ is the similarity variable ($=z/\sqrt{\tau}$). The "i^nerfc" means the n-th integral of the com- plementary error function. For deep-pool systems of small τ the above temperature profile is approximated by

$$\theta_0 = 4\tau i^2\mathrm{erfc}\left(\frac{\zeta}{2}\right) \quad \text{as } \tau\to 0. \tag{10}$$

2. Propagation Theory

For a given Ra_ϕ and Pr the time to mark the onset of convective instability should be found under the principle of the exchange of stabilities from Eqs. (4) and (5), subjected to the boundary conditions of Eq. (6). According to the normal mode analysis convective motion is assumed to exhibit the horizontal periodicity [Chandrasekhar, 1961]. Then the perturbed quantities are written in terms of dimensionless wavenumbers a_x and a_y as

$$\begin{aligned}&[w_1(\tau,x,y,z),\ \theta_1(\tau,x,y,z)]\\ &\quad=[w_1^*(\tau,z),\ \theta_1^*(\tau,z)]\exp[i(a_x x+a_y y)]\end{aligned} \tag{11}$$

where 'i' is the imaginary number. Substitution of the above equation into Eqs. (4)-(6) produces the usual amplitude functions in terms of the dimensionless horizontal wavenumber $a=(a_x^2+a_y^2)^{1/2}$.

The propagation theory employed to find the onset time of convective instability, i.e., the critical time t_c, is based on the assumption that in deep-pool systems the infinitesimal temperature disturbances are propagated mainly within the thermal penetration depth $\Delta_T(\propto\sqrt{\alpha t}$ at the onset time of convective instability and the following scale relations are valid for perturbed quantities from Eqs. (2) and (3):

$$\nu\frac{W_1}{\Delta_T^2}\sim g\beta T_1, \tag{12}$$

$$W_1\frac{\partial T_0}{\partial Z}\sim\alpha\frac{T_1}{\Delta_T^2}, \tag{13}$$

from the balance between viscous and buoyancy terms in Eq. (2) and also from the balance among terms in Eq. (3). Now, based on the relation (12), the following amplitude relation is obtained in dimensionless form:

$$\frac{w_1^*}{\theta_1^*}\sim\delta_T^2\sim\tau, \tag{14}$$

where $\delta_T(=\Delta_T/d\propto\sqrt{\tau})$ is the usual dimensionless thermal penetration depth following $\theta_0^*=0.01$ at $z=\delta_T$. The relation (13) yields

$$Ra^*w^*D\theta_0^*\sim\theta_1^*, \tag{15}$$

where $Ra^*=Ra_\phi\tau^{5/2}$, $w^*=w_1^*/\tau$ and $D=d/d\zeta$.

With the above reasoning the dimensionless amplitude functions of disturbances, based on the relation (14), are assumed to have the form of

$$[w_1^*(\tau,z),\ \theta_1^*(\tau,z)]=[\tau w^*(\zeta),\ \theta^*(\zeta)]. \tag{16}$$

The similarity variable ζ is introduced to take into account the position and temporal dependencies of disturbances. By using Eqs. (10), (11) and (16), the following dimensionless stability equations are obtained for $\tau\to 0$:

$$\left\{(D^2-a^{*2})^2+\frac{1}{2Pr}(\zeta D^3-a^{*2}\zeta D+2a^{*2})\right\}w^*=a^{*2}\theta^*, \tag{17}$$

$$\left(D^2+\frac{1}{2}\zeta D-a^{*2}\right)\theta^*=Ra^*w^*D\theta_0^*, \tag{18}$$

with boundary conditions,

$$w^*=Dw^*=\theta^*=0 \quad \text{at} \quad \zeta=0 \quad \text{and} \quad \zeta\to\infty, \tag{19}$$

where $a^*=a\sqrt{\tau}$. Here a^* and Ra^* have been assumed to be eigenvalues. In the limiting case of $Pr\to 0$, the viscous terms are negligible, and the above stability equations reduce to

$$\left(\frac{\zeta}{2}D^3-\frac{\zeta}{2}a^{*2}D+a^{*2}\right)w^*=a^{*2}\theta^*, \tag{20}$$

$$\left(D^2+\frac{1}{2}\zeta D-a^{*2}\right)\theta^*=PrRa^*w^*D\theta_0^*. \tag{21}$$

The boundary conditions at the heated surface are given as

$$w^*=\theta^*=0 \quad \text{at} \quad \zeta=0. \tag{22}$$

The above equations involve the time dependency implicitly.

For a given Pr the minimum value of Ra^* should be found in the plot of Ra^* vs a^* under the principle of the exchange of stabilities. In other words, the minimum value of τ, i.e., τ_c and its corresponding wavenumber α_c are obtained for a given Pr and Ra_ϕ. Since time is frozen by letting $\partial(\cdot)\partial\tau\equiv 0$ under the frame of amplitude coordinates τ and ζ instead τ and z (see Eqs. (10), (17) and (18)), the propagation theory may be called the relaxed frozen-time model by implicitly treating τ_c as the parameter but it considers the time dependency.

The conventional frozen-time model neglects the terms involving $\partial(\cdot)/\partial\tau$ in Eqs. (4) and (5) in amplitude coordinates τ and z. This results in $(D^2-a^{*2})^2w^*=a^{*2}\theta^*$ and $(D^2-a^{*2})\theta^*=Ra^*w^*D\theta_0^*$ instead of Eqs. (17) and (18). The resulting stability criteria become independent of Pr and τ_c is obtained for a given Ra_ϕ

3. Solution Method

In order to integrate the stability Eqs. (17) and (18), the trial value of the eigenvalue Ra^* and the boundary conditions D^3w^* and $D\theta^*$ at $\zeta=0$ are assumed properly for given Pr and a^*. Since the boundary conditions represented by Eq. (19) are all homogeneous, the value of Dw^* at $\zeta=0$ can be assigned arbitrarily. This procedure is based on the outward shooting method in which the boundary value problem is transformed into the initial value problem. The trial values, together with the four known conditions at the heated boundary, give all the information to perform the numerical integration.

The integration based on the 4th-order Runge-Kutta method is done from $\zeta=0$ to a fictitious distance to satisfy the infinite boundary conditions. With the Newton-Raphson iteration the trial values of Ra^*, D^3w^* and $D\theta^*$ are corrected until the stability equations satisfy the infinite boundary conditions within the maximum relative tolerance of 10^{-8}. Then, by increasing the distance step by step the above integration is repeated. Finally, the value of Ra^* is decided through the extrapolation. Using the similar procedure, the results from the frozen-time model are obtained.

RESULTS AND DISCUSSION

The predicted values based on the propagation theory constitute the stability curve, as shown in Fig. 2. From the minimum Ra^*-value

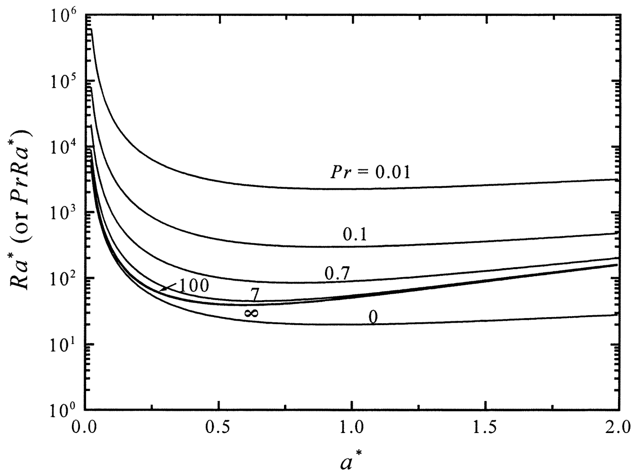

Fig. 2. Marginal stability curves of various Pr-values in present deep-pool systems.

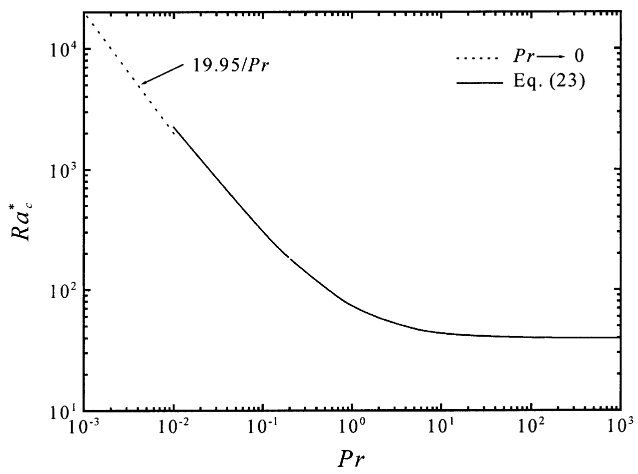

Fig. 3. Effect of Pr on the critical condition in present deep-pool systems.

Table 1. Stability criteria predicted from the propagation theory for deep-pool systems

Pr	0	0.01	0.1	0.7	1	7	10	100	∞
Ra^*_c	$\frac{19.95}{Pr}$	2240	297	85.2	72.9	45.0	43.4	39.7	39.3
a^*_c	0.94	0.95	0.91	0.79	0.76	0.64	0.63	0.60	0.59

the earliest time to mark the onset of thermal instability, i.e., τ_c is obtained for a given Pr and Ra_ϕ. The critical conditions predicted for deep-pool systems are shown in Fig. 3 and also in Table 1. Based on these results, the critical conditions are correlated as

$$\tau_c=4.34\left[1+\left(\frac{0.508}{Pr}\right)^{7/10}\right]^{4/7}Ra_\phi^{-2/5}\ \text{for}\ \tau_c<0.01, \tag{23}$$

within the error bound of 2%. It is believed that for a given Ra_ϕ and Pr a fastest growing mode of infinitesimal disturbances would set in at $t=t_c$ with $a=a_c$. The above equations show that τ_c decreases with an increase in Ra_ϕ and also Pr. Eq. (23) can be rewritten as a function of Ra:

$$\tau_c=11.55\left[1+\left(\frac{0.508}{Pr}\right)^{7/10}\right]^{20/21}Ra^{-2/3}\ \text{for}\ \tau_c<0.01, \tag{24}$$

where $Ra=Ra_\phi\tau_c$. This equation is less convenient in predicting τ_c but it is more useful for comparison with isothermally heated systems. The Pr-effect on τ_c becomes pronounced for Pr<1, that means the inertia terms make the system more stable. For Pr>100 τ_c is almost independent of Pr, as shown in Fig. 3. The critical wavenumber a_c decreases with increasing Pr for a given Ra_ϕ, as featured in Table 1.

Now, the domain of time is extended to $\tau_c>0.01$ by keeping Eqs. (17) and (18) and using the basic temperature profiles of Eq. (9). The upper boundary $\zeta\rightarrow\infty$ is replaced with z=1, i.e., $\zeta=1/\sqrt{\tau_c}$ instead of Eq. (19) and in Eqs. (17) and (18) Ra^* and a^* are replaced with $Ra\tau_c^{5/2}$ and $a\sqrt{\tau_c}$. Also, in Eq. (9) τ is replaced with τ_c but ζ is maintained. Since τ_c is the fixed parameter, the resulting stability equations are a function of ζ only and the physics of Eqs. (14) and (15) is still alive. For a given Pr and τ_c the minimum Ra_ϕ-value and its corresponding wavenumber a_c are obtained. This extension of

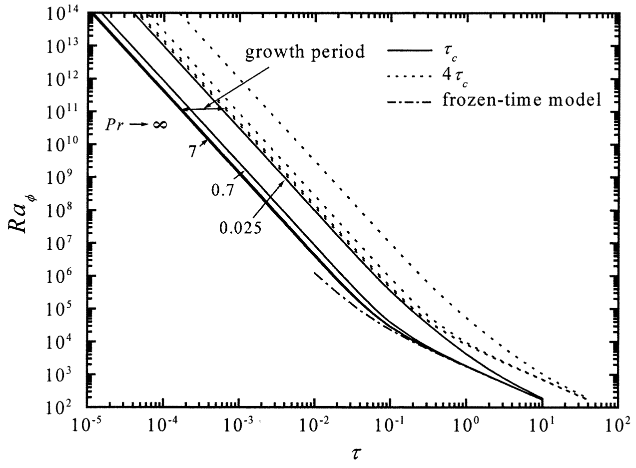

Fig. 4. Characteristic times with respect to Ra_ϕ for a given Pr.

the propagation theory is well described in the work of Yang and Choi [2002] and Kim et al. [2002]. In the present system the numerical procedure is almost the same as that in the previous section. The results are summarized in Fig. 4, wherein those obtained from the conventional frozen-time model are also shown. For $\tau_c<0.01$ the former ones are the same as those of deep-pool systems (Eq. (23)). For large τ_c they approach the well-known critical conditions of $Ra_\phi\tau_c=1708$ in the isothermal heated system since the basic temperature profiles become linear. It is known that for small τ the frozen-time model yields the lower bound of τ_c and the terms involving $\partial(\cdot)/\partial\tau$ in Eqs. (4) and (5) stabilize the system. It is interesting that the propagation theory yields smoothly the stability criteria over the whole domain of time.

Davenport and King [1972, 1974] measured the bottom temperature and defined its deviation from the conduction state as the characteristic time. This time represents the detection time of manifest convection (t_m). Their experimental data are compared with the present predictions from the propagation theory in Fig. 5. In Fig. 5(a) the experimental τ_m-values approaches the predicted τ_c-values with time but with increasing Ra_ϕ the difference between τ_c and τ_m becomes relatively large. In Fig. 5(b) it is shown that for air $\tau_m\cong 4\tau_c$ is kept.

The relation of $\tau_m\cong 4\tau_c$ was suggested by Foster [1969]. This means that a fastest growing mode of instabilities, which set in at $t=t_c$, will grow with time until manifest convection is near the whole bottom boundary detected at $t\cong 4t_c$, as illustrated in Fig. 4. This scenario is supported to a certain degree by the above experimental and theoretical results for water. The validity of $t_m\cong 4t_c$ requires a further study but this relation is kept even in other transient diffusive systems [Choi et al., 1998; Kang et al., 2000]. It seems evident that convective motion is relatively weak during $t_c\leq t<t_m$ since the related heat transport is well represented by the conduction state.

Recently Tan and Thorpe [1999] suggested a very simple instability analysis assuming that at the onset of manifest convection the following relation is maintained, based on Eq. (10):

$$\text{Maximum of}\left\{\frac{g\beta Z^4}{\alpha\nu}\left(\frac{\partial T_0}{\partial Z}\right)\right\}=960,\quad \frac{a_c\times 7.664\times Z_{max}}{2\pi}=2.45, \qquad (25)$$

which are satisfied by $\partial T_0/\partial Z=-2\phi\sqrt{t/\alpha}$ ierfc $(Z/\sqrt{4\alpha t})$ at

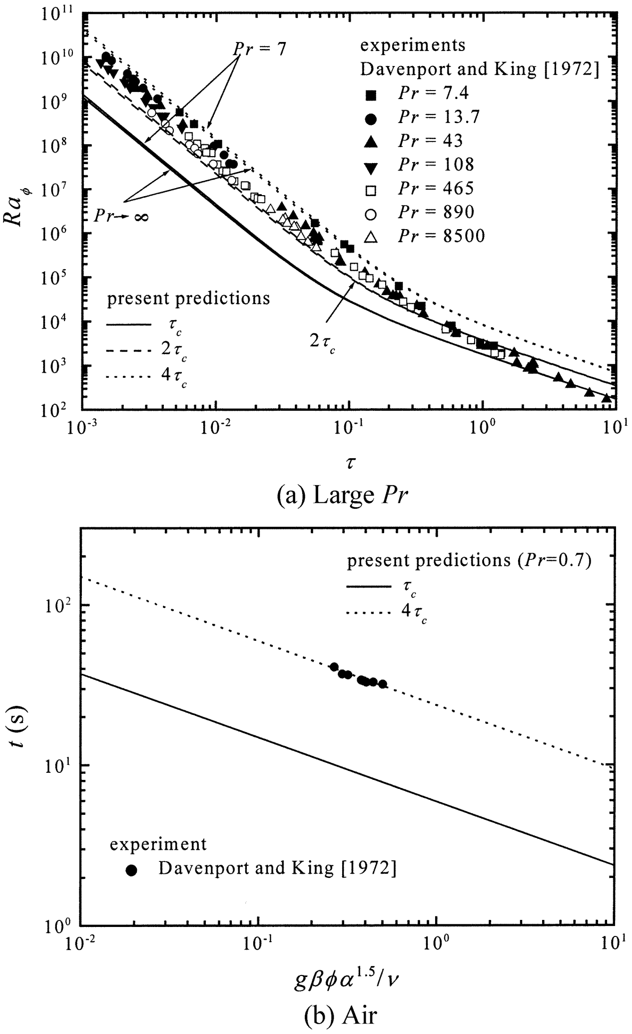

Fig. 5. Comparison of critical Rayleigh numbers with available experimental data.

$Z_{max}=2.35\sqrt{\alpha t}$ from Eq. (10). This results in $\tau_m=12.5Ra_\phi^{-2/5}$ and $a_c=0.24\ Ra_\phi^{1/5}$, independently of Pr. It may be stated that their predictions correspond to those in deep-pool systems of $Pr\rightarrow\infty$, which yield the relation of $\tau_m\cong 3\tau_c$. This relation compares well with the experimental data, as shown in Fig. 5(a). But their model lacks the phys- ics. For example, the constants in Eq. (25), which were estimated with the Biot number of 1, are not valid.

Considering the above results, it is known that for deep-pool systems of Pr>1 the relation of $\tau_m=(2\sim4)\tau_c$ (see Fig. 5(a)) and for Pr>1 $\tau_m\cong 4\tau_c$ (see Fig. 5(b)), based on the present τ_c-values from the propagation theory. But the first detection time of convective motion is very difficult experimentally. Furthermore, the dependency of physical properties on the temperature may cause a discrepancy between models and experimental data. In Davenport and King's [1972] experiments, the maximum temperature difference is 0.5 °C for Pr=7 (methanol) and 23.5 °C for Pr=8500 (1000 cS silicon oil). A temperature increase of 23.5 °C for 1000 cS silicon oil would reduce its viscosity by 30%, so the variable-viscosity effect should be considered [Davaille and Jaupart, 1993]. The effect of the dependency of physical properties on the temperature for high Pr-fluids may mitigate the discrepancy, as suggested by Tan and Thorpe [1999]. Also, the above models themselves are not valid ones. Even in the propagation theory the validity of Eq. (16) should be justified. Very recently Choi et al. [2003] reported that in isothermally heated systems the propagation theory yields approximate τ_c-values for small τ, and the relation of $\tau_m \cong 6\tau_c$, based on the τ_c-values obtained from the numerical simulation, is more preferred for constant-properties systems of large Pr.

CONCLUSIONS

The critical condition to mark the onset of convective instability in an initially quiescent, horizontal fluid layer heated from below with a constant temporal heating rate has been analyzed by using propagation theory and also the frozen-time model. The resulting relations of $\tau_m \cong 3\tau_c$ for Pr>1 and $\tau_m \cong 4\tau_c$ for Pr<1 compare reasonably well with Davenport and King's [1972] experimental data for deep-pool systems. For $\tau<\tau_m$, the velocity disturbances seem relatively weak. For $\tau>1$ decision of the first detection of manifest convection is very difficult experimentally; that shows the discrepancy between the present relation and the experimental data. The present results also show that the propagation theory can be applied to the stability analysis of diffusive systems without loss of generality.

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NOMENCLATURE

a : dimensionless wavenumber
d : depth of the fluid layer [m]
Pr : Prandtl number, ν/α
Ra : Rayleigh number based on the temperature difference, $g\beta\Delta Td^3/(\alpha\nu)$
Ra_ϕ : Rayleigh number based on the temporal heating rate, $g\beta\phi d^5/(\alpha^2\nu)$
T : temperature [K]
t : time [s]
(U, V, W) : velocities in Cartesian coordinates [m/s]
(u, v, w) : dimensionless velocity disturbances in Cartesian coordinates
(X, Y, Z) : Cartesian coordinates [m]
(x, y, z) : dimensionless Cartesian coordinates

Greek Letters

α : thermal diffusivity [m^2/s]
Δ_T : thermal boundary-layer thickness [m]
δ_T : dimensionless thermal boundary-layer thickness, Δ_T/d
ϕ : temporal heating rate [K/s]
θ : dimensionless temperature disturbance, $g\beta d^3T_1/(\alpha\nu)$
θ_0 : dimensionless basic temperature, $\alpha(T_0-T_i)/(\phi d^2)$
ν : kinematic viscosity [m^2/s]
τ : dimensionless time, $\alpha t/d^2$
ζ : similarity variable, $z/\sqrt{\tau}$

Subscripts

c : critical conditions
i : inlet condition
0 : basic quantities
1 : perturbed quantities

Superscript

* : transformed quantities

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