

## Deposition of Submicron Particles in Deep Bed Filtration under Unfavorable Surface Conditions

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**Abstract**—Deterioration in the filter removal efficiency of submicron particles ( $\lambda/\lambda_0$ ) under unfavorable surface conditions is affected by the number of deposited particles per filter grain. In the case of above micron particles, the deterioration of filter removal efficiency has been mainly due to the blocking effect of deposited particles and not by the number of deposited particles. Deposition of large number of submicron particles changed the surface characteristics of collectors (filter grain associated with deposited particles) and enhanced unfavorable surface conditions. Filtration experiments were conducted with monodispersed suspensions of known sizes of submicron latex particles at different ionic strengths, using glass beads as filter grains. The filtration performance was predicted by using a mathematical model, assuming a linear relationship between  $\lambda/\lambda_0$  and  $\sigma$  (i.e.  $\lambda/\lambda_0 = 1 - k\sigma$ ). For both particles,  $k$  was found to decrease and  $\lambda_0$  was found to increase with the increase in the ionic strength. A comparison was made of the importance of blocking effect for the filtration of submicron particles.

Key words: Blocking Effect, Deep Bed Filtration, Filter Coefficient, Submicron Latex Particles, Specific Deposit

### INTRODUCTION

A number of methods have been proposed in the literature [Tien, 1989; Vaidyanathan and Tien, 1991] to calculate the removal of particles of above micron size during the transient stage of deep bed filtration under unfavorable surface conditions (due to the repulsive double-layer interaction between particles and filter grains). When a filter run starts, the filter bed will be clean; this stage of filtration is called clean bed filtration. From then onwards the filter bed will start to accumulate particles, and this stage of filtration is called transient stage. One of those methods is to relate the deterioration of the particle removal to the accumulation of particle deposition in the filter bed. Another method is to take the surface charge accumulation on collector (filter grain associated with deposited particles) into account. This causes unfavorable surface conditions, which in turn reduces the particle removal. The third approach is to consider the blocking effect of already deposited particles which prevents the deposition of particles in the suspension. Prediction of particle removal based on the blocking effect is by far the best method for above-micron particles [Vaidyanathan and Tien, 1991; Song et al., 2002; Ryu et al., 1987]. However, it underestimates the reduction in deposition that was observed in experiments with above-micron particles. The assumption that the fluid flow is unaffected by the presence of previously deposited particles may be a cause for this discrepancy between theoretical and experimental results. In this study, an attempt was made to predict transient stage of deep bed filtration of submicron particles at different ionic strengths, assuming a linear relationship between filter coefficient ( $\lambda$ ) and specific deposit ( $\sigma$ ). The trend of variation of the mathematical model

coefficients was also investigated. A comparison was made of the importance of the blocking effect for the transient stage filtration of submicron particles.

### THEORETICAL FORMULATIONS

#### 1. Accumulation of Particle Deposition

Particle removal in a deep bed filter is expressed as [Iwasaki, 1937]:

$$\partial C / \partial x = -\lambda C \quad (1)$$

where,  $C$  is the particle concentration in the suspension,  $x$  is the axial distance along the filter depth and  $\lambda$  is the filter coefficient. The filter coefficient  $\lambda$  is a local variable (a function of time and filter depth). For unfavorable surface interaction between particles and filter grains,  $\lambda$  can be related to another local variable, the specific deposit  $\sigma$  by the following Eq.:

$$F = 1 - k\sigma \quad (2)$$

Where  $F = \lambda/\lambda_0$  and  $k$  is a constant for given conditions of filtration;  $\lambda_0$  denotes the initial filter coefficient. In order to find the change in  $F$  during a filter run,  $k$  and  $\sigma$  must be computed from experimental data. When  $F$  is described by Eq. (2), the particle concentration in the suspension can be given as:

$$C_{out}/C_{in} = \exp[U\lambda_0 C_{in} k \Theta] / \{ \exp[\lambda_0 L] + \exp[U\lambda_0 C_{in} k \Theta] - 1 \} \quad (3)$$

Where,  $C_{in}$  and  $C_{out}$  are the particle concentrations in the influent to the filter bed and in the effluent from the filter bed, respectively,  $U$  is the filtration velocity,  $\Theta$  is the time corrected for the initial filtration at different filter depths and  $L$  is the depth of the filter. For a given depth  $X$ ,  $\Theta$  can be written as:

$$\Theta = t - \int_0^x dx / (U/f) \quad (4)$$

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where  $f$  is the porosity of the filter. Details on the derivation of Eq. (3) are given elsewhere [Tien, 1989]. Eq. (3) can be rearranged to the following form:

$$\ln[C_{out}/(C_{in}-C_{out})]=U\lambda_o C_{in} k \Theta - \ln[\exp(\lambda_o L)-1] \quad (5)$$

From the gradient and intercept of the graph of  $\ln[C_{out}/(C_{in}-C_{out})]$  versus  $\Theta$ , one can calculate the values of  $\lambda_o$  and  $k$ . Once the values of  $\lambda_o$  and  $k$  are known, the specific deposit,  $\sigma_i$  at the top layer of the filter can be calculated by using the following Eq.:

$$1-k\sigma_i=\exp[-U\lambda_o C_{in} k \Theta] \quad (6)$$

Then the specific deposit,  $\sigma$  at a depth  $x$  can be calculated from the following Eq.:

$$\sigma/\sigma_i=C/C_{in} \quad (7)$$

## 2. Blocking Effect

Vaidyanathan and Tien [1991] describe the blocking effect of

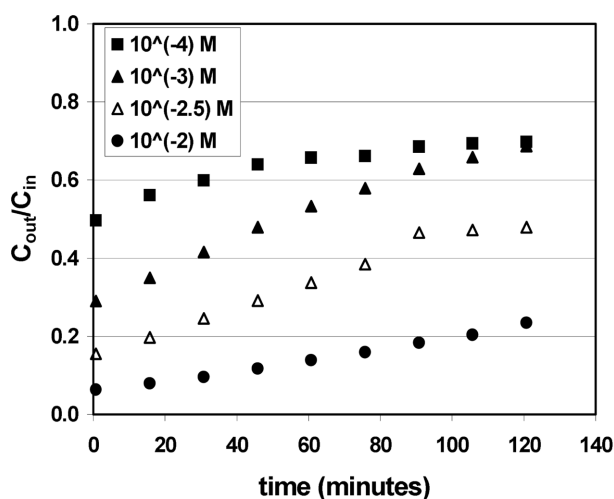
already deposited particles (on a collector). From their work, the following relationship was obtained:

$$\lambda/\lambda_o=1-m\langle\Delta\rangle/[\pi R_{lim}^2] \quad (8)$$

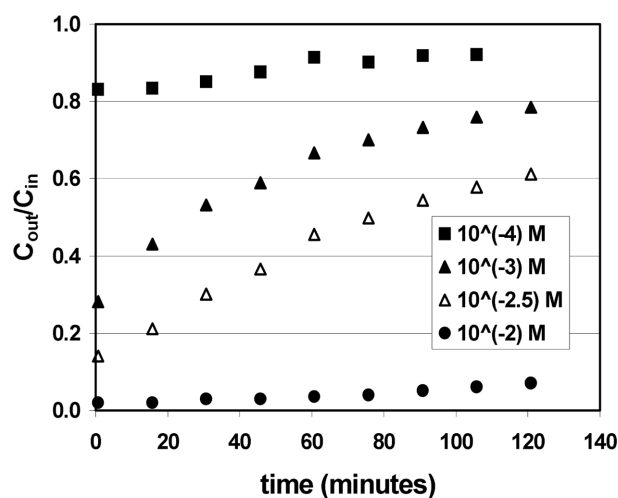
where  $m$  is the number of deposited particles,  $\langle\Delta\rangle$  is the projected area on the filter grain that inhibits particle deposition due an already deposited particle and  $R_{lim}$  is the radius of the region on the Happel cell that forms limiting trajectories. The derivation is given elsewhere [Vaidyanathan and Tien, 1991; Cho et al., 2002].

## EXPERIMENTAL

Latex particles of 0.46  $\mu\text{m}$  and 0.816  $\mu\text{m}$  were used as suspension at a concentration of 5 mg/L. Spherical glass beads of 0.175 mm were used as filter medium and were packed into the cylindrical filter column to a specified depth. KCl of predetermined quantity was used to control the ionic strength. The solution was allowed to flow at constant head through the filter downward at required flow rate. Head loss and effluent turbidity ( $C_{out}$ ) were measured at predetermined time intervals. The pH of influent and effluent were measured as well. A Coulter counter (Delsa 440) was used to measure the zeta potential of the latex suspension and glass beads at different ionic strengths. Particle aggregation was not observed in the range of ionic strengths used. The residence time of the filtration system was found by tracer experiments, and the clean bed removal was chosen as the particle removal at a time corresponding to a com-

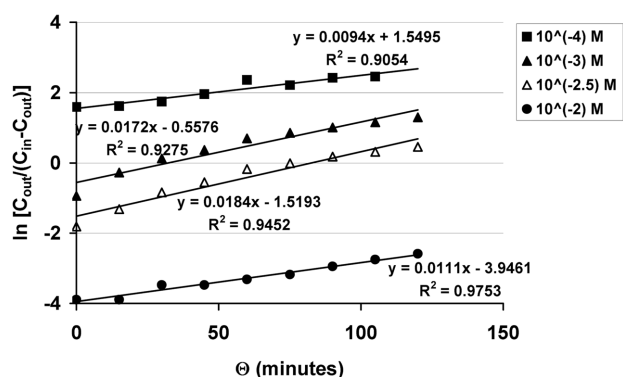


(a) 0.460  $\mu\text{m}$  Latex Particles

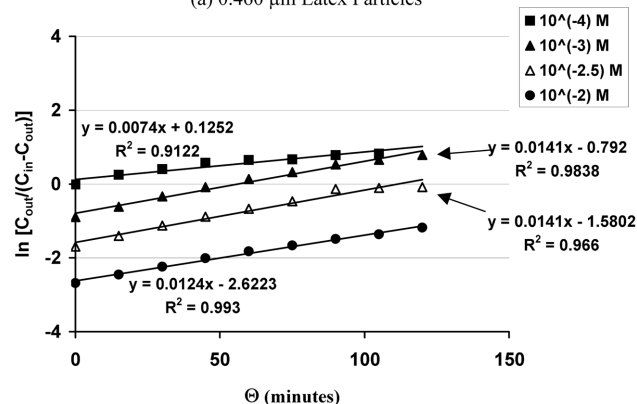


(b) 0.816  $\mu\text{m}$  Latex Particles

Fig. 1. Variation of  $C_{out}/C_{in}$  with time for different ionic strengths (filter grain size=0.175 mm, filter depth=10 cm filtration velocity=2.5 m/h, influent concentration=5 mg/L).



(a) 0.460  $\mu\text{m}$  Latex Particles



(b) 0.816  $\mu\text{m}$  Latex Particles

Fig. 2. Fitting experimental data to Eq. (5) to find  $k$  and  $\lambda_o$  (filter grain size=0.175 mm, filter depth=10 cm filtration velocity=2.5 m/h, influent concentration=5 mg/L).

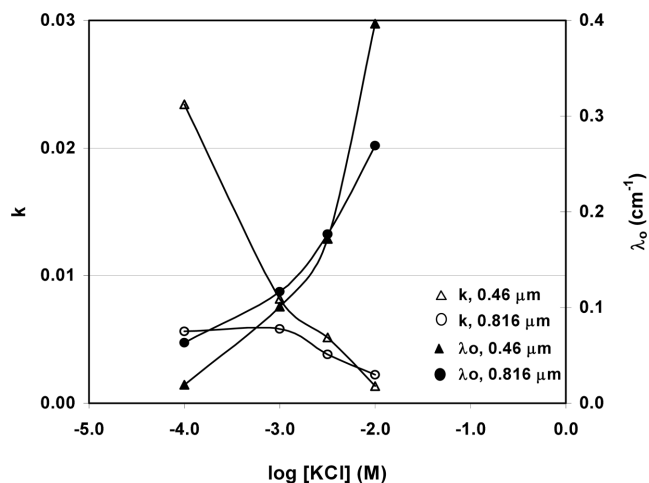


Fig. 3. Variation of  $k$  and  $\lambda_0$  with ionic strength (filter grain size=0.175 mm, filter depth=10 cm filtration velocity=2.5 m/h, influent concentration=5 mg/L).

plete breakthrough of the inert tracer during the tracer experiments.

## RESULTS AND DISCUSSION

### 1. Accumulation of Deposition of Submicron Particles

The removal of 0.46 and 0.816  $\mu\text{m}$  particles during the transient stage of filtration is shown in Fig. 1(a) and 1(b), respectively. Since the surface interaction between those particles and filter grains (glass beads) was unfavorable from the beginning, a significant increase in  $C_{out}/C_{in}$  with filtration time (from the beginning of a filter run) was observed in all cases. The values of  $\lambda_0$  and  $k$  were obtained by fitting experimental data to Eq. (5). The fittings are shown in Fig. 2(a) and 2(b).

Fig. 3 shows the variation of  $\lambda_0$  and  $k$  with ionic strength, for both 0.46 and 0.816  $\mu\text{m}$  particles. In general, for both particles  $k$  was found decrease with increase in the ionic strength. Further, for both particles  $\lambda_0$  was found to increase with the increase in ionic strength.

Figs. 4(a) and 4(b) show the relationship between the specific deposit ( $\sigma$ ) and  $F (= \lambda/\lambda_0)$  at the top layer of the filter. From these figures, it can be seen that the variation of  $F$  with  $\sigma$  is similar for both 0.46 and 0.816  $\mu\text{m}$  particles at ionic strengths below  $10^{-2.5}$  M KCl. But, at  $10^{-2}$  M KCl ionic strength, the rate of increase in  $\sigma$  is larger for 0.46  $\mu\text{m}$  particles compared to that of 0.816  $\mu\text{m}$  particles.

Further, Vaidyanathan [1986] proposed a method to find the ultimate specific deposit,  $\sigma_u$  when the filter bed is saturated with deposited particles. Assuming that only the top portion of the filter bed is saturated during filtration, he proposed the following relationship:

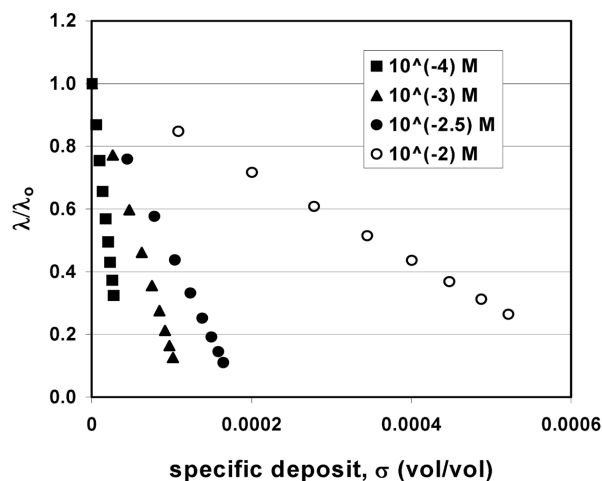
$$F = 1 - [1/\sigma_u]\sigma \quad (9)$$

From Eqs. (2) and (9),  $\sigma_u$  and  $k$  can be related by the following Eq.:

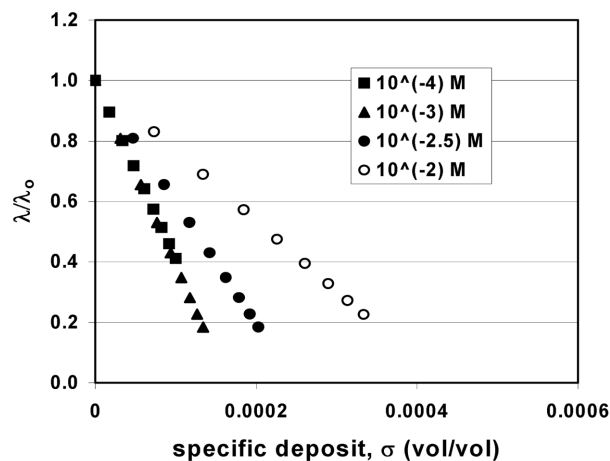
$$\sigma_u = 1/k \quad (10)$$

Thus for both 0.46 and 0.816  $\mu\text{m}$  particles, the ultimate specific deposit  $\sigma_u$  increases with the increase in the ionic strength.

Since the number of particles ( $m$ ) attached to a filter grain will increase with the increase of  $\sigma$ ,  $\lambda/\lambda_0$  can be related to  $m$  as well.



(a) 0.460  $\mu\text{m}$  Latex Particles



(b) 0.816  $\mu\text{m}$  Latex Particles

Fig. 4. Deterioration of  $\lambda/\lambda_0$  with specific deposit  $\sigma$ , at the top layer of the filter (filter grain size=0.175 mm, filter depth=10 cm filtration velocity=2.5 m/h, influent concentration=5 mg/L).

The following expression can be used to calculate  $m$ :

$$m = \sigma[(1-f)N_R^3] \quad (11)$$

where,  $N_R$  is the ratio between particle radius and filter grain radius ( $a_p/a_c$ ). Thus, the number of particles attached to a filter grain at the

Table 1. Particle deposition on to filter a grain in the top layer of the filter, at different ionic strengths

		Number of particles deposited/filter grain			
	Time (minutes)	$10^{-4}$ M	$10^{-3}$ M	$10^{-2.5}$ M	$10^{-2}$ M
0.460 $\mu\text{m}$	30	802	3770	6305	16113
	60	1406	6021	9935	27662
	90	1862	7365	12025	35940
	120	-	8167	13229	41874
0.816 $\mu\text{m}$	30	485	813	1233	1924
	60	874	1346	2041	3251
	90	1186	1695	2571	4165
	120	-	1924	2918	4796

top layer of the filter can be calculated by using Eqs. (6) and (11). Similarly, the number of particles attached to a filter grain at the bottom layer of the filter can be calculated by using Eqs. (6), (7) and (11). Table 1 shows the accumulation of particles on a filter grain of the top layer of the filter during the transient stage filtration.

## 2. Blocking Effect of Submicron Particles

When  $\lambda/\lambda_0$  was calculated for 0.46 and 0.816  $\mu\text{m}$  particles by using Eq. (8),  $\lambda/\lambda_0$  was found to be zero as soon as the filtration started. In calculating  $\lambda/\lambda_0$ ,  $\theta_{lim}$  was taken as  $\pi/2$  and corresponding  $\langle\Delta\rangle/[\pi R_{lim}^2]$  was obtained by using the following relationship [Vaidyanathan and Tien, 1991]:

$$\langle\Delta\rangle/[\pi R_{lim}^2] = 0.5N_R \quad (12)$$

The derivation of Eq. (12) is given below:

The limiting trajectory of the depositing particles describes a circle CL of radius  $R_{lim}$  at the Happel cell boundary (Fig. 5). Thus, the collection efficiency of the spherical collector can be given by the following equation:

$$\eta = R_{lim}^2/b^2 \quad (13)$$

Where,  $b$  is the radius of the fluid envelope in Happel's model. Deposition of a particle  $P$  will form a shadow on the collector on which no other particle may deposit. The inhibition of particle deposition by  $P$  can be quantified by determining the projected area  $\Delta$  of the source region  $S$  relative to that of CL. In general,  $P$  may be located anywhere in the region  $\theta_p \in [0, \theta_{lim}]$ . The initial collection efficiency

in the region  $[0, \theta]$  is given by [Mackie et al., 1987]:

$$\eta(\theta) = 2p^2\psi(1+N_R, \theta) \quad (14a)$$

$$\text{where, } \psi(1+N_R, \theta) = J(1+N_R)\sin^2\theta \quad (14b)$$

$$\text{and } p = 1/b \quad (14c)$$

$J_r$  is given as:

$$J_r = (1/2)[(k_1/r) + k_2r + k_3r^2 + k_4r^4] \quad (14d)$$

$$\text{Where, } k_1 = 1/w \quad (14e)$$

$$k_2 = -(3+2p^5)/w \quad (14f)$$

$$k_3 = (2+3p^5)/w \quad (14g)$$

$$k_4 = -p^5/w \quad (14h)$$

$$w = 2 - 3p + 3p^5 - 2p^6 \quad (14i)$$

The expected value of  $\Delta$  can be given as:

$$\langle\Delta\rangle = \int \Delta(\theta) d\lambda(\theta) / \eta(\theta_{lim}) \quad (15a)$$

$$= \int 2\Delta(\theta) \sin\theta \cos\theta d\theta / \sin^2(\theta_{lim}) \quad (15b)$$

Thus, the decrease in filter coefficient due to  $m$  deposited particles can be given by Eq. (8).

In their work, Vaidyanathan and Tien [1991] have taken  $\theta_{lim}$  as  $\pi/2$  for 6.4  $\mu\text{m}$  particles and have suggested  $\theta_{lim}$  to be less than  $\pi/2$  for larger particles (22  $\mu\text{m}$ ). The major assumption made in calculating  $\lambda/\lambda_0$  was that  $m$  was small so that the blocking of each particle is mutually exclusive.

The lower values of  $\lambda/\lambda_0$  are due to the over-estimation of the blocking effect of already deposited particles. The blocking effect of individual particles (deposited) is not mutually exclusive for submicron particles as they deposit in large numbers. In addition, in the case of submicron particles, the effect of Brownian motion must be included along with trajectory analysis. Thus, Eq. (8) should also incorporate an additional term  $[\lambda/\lambda_0]_{\text{Brownian}}$  to account for the effect of Brownian motion.

## CONCLUSIONS

The function  $F(=\lambda/\lambda_0)$  characterizing the filter performance was determined by matching the experimental effluent concentration history. An analytical solution of fluid phase concentration was made possible when  $F$  was considered to be  $(1-k\sigma)$ . In general, for both particles  $k$  was found to decrease with the increase in the ionic strength. However, the decline in the  $k$  value for 0.46  $\mu\text{m}$  particles was steeper compared to that for 0.816  $\mu\text{m}$  particles with the increase in the log ionic strength, in the range of  $10^{-4}$  M KCl to  $10^{-3}$  M KCl. Thus, a decrease in  $F$  with the increase in specific deposit  $\sigma$  was found to be larger for 0.46  $\mu\text{m}$  particles compared to that for 0.816  $\mu\text{m}$  particles at lower ionic strengths (less than  $10^{-2.5}$  M KCl). But, this trend was found to reverse at higher ionic strengths ( $10^{-2}$  M KCl). For both particles,  $\lambda_0$  increased with the increase in the ionic strength. The increase in  $\lambda_0$  value for 0.46  $\mu\text{m}$  particles was steeper compared to that for 0.816  $\mu\text{m}$  particles with the increase in the log ionic strength, when the ionic strength of the suspension increased from  $10^{-2.5}$  M KCl to  $10^{-2}$  M KCl. The values of  $k$  and  $\lambda_0$  at different ionic strengths are useful in predicting the filter removal efficiency of 0.46  $\mu\text{m}$  and 0.816  $\mu\text{m}$  particles.

## NOMENCLATURE

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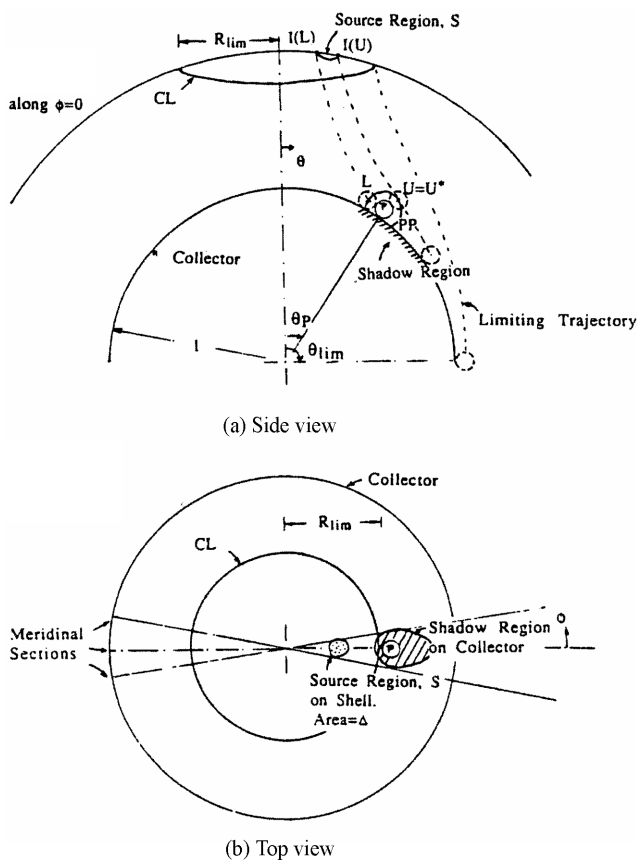


Fig. 5. Blocking effect of a particle on a Happel collector (adopted from Vaidyanathan and Tien, 1991).

$a_c, a_p$  : radius of filter grains and radius of particles, respectively [L]  
 $b$  : radius of fluid envelope in Happels model [L]  
 $C$  : particle concentration in the suspension [ $\text{ML}^{-3}$ ]  
 $C_{in}, C_{out}$  : particle concentration in the influent and the effluent, respectively [ $\text{ML}^{-3}$ ]  
 $f$  : porosity of the filter bed  
 $k$  : constant used in Eq. (2)  
 $L$  : filter depth [L]  
 $m$  : number of particles attached to a filter grain  
 $N_R$  : interception number [ $a_p/a_c$ ]  
 $R_{lim}$  : radius of a region on Happel cell that forms limiting trajectories [L]  
 $t$  : time  
 $U$  : filtration velocity [ $\text{LT}^{-1}$ ]  
 $x$  : axial distance along the filter depth [L]

### Greek Letters

$\langle \Delta \rangle$  : projected area on a filter grain that inhibits particle deposition  
 $\lambda, \lambda_o$  : transient stage and initial filter coefficients, respectively  
 $\Theta$  : corrected time for the initial filtration at different depths [T]  
 $\theta_{lim}$  : polar angle (originating from the center of the collector) of a particle deposited on the collector along the limiting trajectory  
 $\sigma, \sigma_u$  : specific deposit and ultimate specific deposit, respectively

$\sigma_i$  : specific deposit at the top layer of the filter

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