**SHORT COMMUNICATION**

# Bilinear model predictive control of grade change operations in paper production plants

**Yeong-Koo Yeo† and Yeon-Uk Choo**

Department of Chemical Engineering, Hanyang University, Seoul 133-791, Korea

**Abstract**–In this work the bilinear model predictive control method is applied to control the grade change operations in paper production plants. Because of the high nonlinearity of the grade change processes, control of the grade change operations has been performed manually by experienced engineers in the plants. In some cases the bilinear model can be very effective to represent nonlinear processes. In this study a bilinear model for paper plants is identified first. It is found that the bilinear model tracks the plant without significant discrepancy. Based on the multivariable bilinear plant model the optimal input variables are computed using the one-step ahead prediction method. Even for frequent changes in paper grades the bilinear model predictive control scheme exhibits good control performance.

Key words: Grade Change, Bilinear Model Predictive Control, One-step-ahead Prediction

## INTRODUCTION

The dynamics of paper grade change operations exhibit highly nonlinear behavior, and most of the grade change operations are executed manually by skilled operators. Because of the lack of proper control tools for the nonlinear grade change operations, the process performance during grade change operations has not been satisfactory. Specific nonlinear control schemes are not suitable in actual operations because paper grade change operations are performed frequently (2 to 5 times per day).

The bilinear model can be effectively used to represent nonlinear chemical processes. Many successful application results of bilinear model-based controllers to practical systems have been reported [Ruberti et al., 1972; Mohler, 1973]. Some investigators considered the optimal control problem for commutative bilinear system and obtained constant optimal input vectors for a time invariant system by using a quadratic cost function [Wei and Pearson, 1978]. Results of applications of bilinear control method in nuclear rocket engines and aircraft control were presented [Mohler, 1991]. A new iterative optimal control scheme based on successive approximations has been proposed [Aganovic and Gajic, 1995]. In the scheme a linear control law was represented in terms of a given state trajectory. One of the authors developed a bilinear model predictive controller based on autoregressive moving average process model [Yeo and Williams, 1987; Oh and Yeo, 1995]. The basic idea of their algorithm is conveyed in the present study with a little modification to control paper grade change operations.

The key controlled variables in grade change operations are the basis weight, the ash content and the moisture content of the paper being produced. As the manipulated variables it is common practice to choose the flow rate of the thick stock, the filler flow, the machine speed and the steam pressure. These controlled variables and manipulated variables are shown in Fig. 1. It is desired to control all the three controlled variables simultaneously, but most of the control methods proposed so far have concentrated on the control
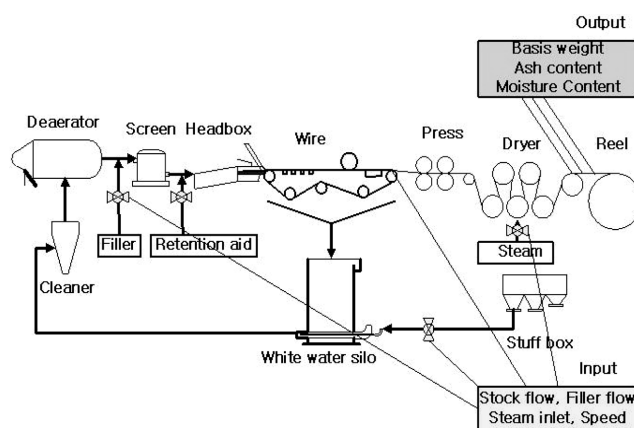
of only one or two key variables. The PID control method presented by Tang and Shi focused on the basis weight [Tang and Shi, 2002]. A transition control scheme based on the process model was proposed by some researchers [Skoglund and Brudin, 2000; Murphy and Chen, 1999]. The method tries to manipulate the steam pressure to make the moisture content follow the predetermined trajectory.

In the present study the multivariable bilinear model predictive control method is proposed to control the grade change operations in paper production plants. As the first step a bilinear model for grade change operations is identified. The so-called "parametric" bilinear model [Sen, 1986] was found to be suitable to represent the dynamics of grade change operations. From the numerical simulations the neural model identified showed reliable agreement with the plant data, which justifies the use of a bilinear model in the predictive control scheme. A one-step ahead multivariable bilinear model predictive control scheme is proposed next to control all the key controlled variables (basis weight, the ash content and the moisture content) simultaneously followed by numerical simulations.



**Fig. 1. Schematic diagram of paper machine.**

## GRADE CHANGE OPERATIONS

†To whom correspondence should be addressed.
E-mail: ykyeo@hanyang.ac.kr

A modern paper machine can be considered as a production line consisting of a stock preparation system, wire section, wet pressing, drying and coating units as shown in Fig. 1. In stock preparation, different raw materials such as chemical pulp from pulp mills, mechanical pulp from chip refiners, chemicals and additives are mixed together. Pulps are usually refined in order to achieve the required product quality. After it is cleaned and diluted, pulp stock is fed into the white-water system in the wire section of the paper machine. The wet-end section of paper mills exhibit highly nonlinear behavior [Yeo et al., 2005].

A white-water system consists of the headbox, wire and the circulated white-water that is a filtrate from the wire. White-water is used to dilute the stock to the desired consistency (0.3-1%) for the paper web forming process. The headbox spreads the stock flow on the wire across the width of the machine and the paper web is formed. After that, water is removed first by wet pressing and then contact drying on steam-heated cylinders. Often, modern paper machines have on-machine coaters that apply pigment-coating color on the paper. Coating may be on both sides and even double or triple layered. In addition to these principal systems, there are several support systems such as for broke handling, chemical preparation and so on.

A grade change is a product quality change on a paper machine. In a big grade change, several inputs to the paper machine are changed, for example proportioning of raw materials, refiner loads, stock flows, headbox settings, machine speed, lineal pressures in wet pressing, steam pressures and coating settings. So far the most common way to execute a grade change is by ramping. An open-loop method such as ramping suits grade change well because there are no exact target values for the basis weight and moisture. It is satisfactory if the target values hit inside an acceptance range after the grade change. Most grade changes are basis weight transitions. The basis weights in a production schedule are run in a cycle. The cycle is optimized so that the basis weight changes are as small as possible. The ideal condition is that the allowed ranges of sequential grades overlap.

A typical grade change consists of a calculation of target values and a dynamic co-ordination of paper machine speed, pulp stock flow and steam. It is crucial to a successful grade change that the new target values are accurately known. Stock flow and machine speed together control both the production rate and basis weight. Drying is controlled by steam pressure, but often only the last steam groups are used for control purpose. Because of long time constants and dead times in the drying process, the target values for steam pressures are the most important. Also, raw material properties, the condition of the paper machine, basis weight, moisture and speed all affect the drying rate. The machine tender usually gets the initial target values from the records of the previous runs.

## BILINEAR MODEL PREDICTIVE CONTROL

The paper production process is highly nonlinear, and model predictive control methods based on linearized process models do not exhibit satisfactory control performance. Exact description of the nonlinearity of the paper process is very difficult, and representation of nonlinear dynamics of the drying section of the paper machine can be found elsewhere [Yeo et al., 2005]. To avoid the complicated nonlinear representation as well as the impracticable use of simple

linear models, we introduce bilinear representation for the grade change operation. The grade change operation can be characterized as a typical multivariable nonlinear process. For the specific paper plants considered in the present study, there are four input variables (machine speed, stock flow, steam pressure, filler flow) and three output variables (basis weight, moisture content, ash content).

The bilinear model has similar structure with linear models in that the bilinear model is linear with respect to model parameters. The bilinear model can be classified into three types: state-space type, polynomial type and parametric type. The polynomial and parametric bilinear models are usually used in adaptive control methods based on model identification algorithms. In the present study we employed the parametric bilinear model structure. For a single variable system, the parametric bilinear model can be represented as

$$y(k) = \phi^T \theta \tag{1}$$

where

$$\phi = [a_1, a_2, \ldots, a_{n-1}, a_n, b_1, b_2, \ldots, b_{n-1}, b_n, c_1, c_2, \ldots, c_{n-1}, c_n]^T \tag{2}$$

$$\theta = [y(k-1), \ldots, y(k-n), u(k-\tau-1), \ldots, u(k-\tau-n), \\ y(k-1)u(k-\tau-1), \ldots, y(k-n)u(k-\tau-n)]^T \tag{3}$$

The estimate of the parameter vector $\hat{\theta}$ can be obtained from the minimization of the following quadratic cost function:

$$\min\left\{ V(\theta) = \frac{1}{N}\sum_{t=1}^{N}(y(t) - \hat{y}(t))^2 \right\} \tag{4}$$

The following recursive parameter estimation method can be effectively used [Ljung, 1987]:

$$P(k)^{-1} = \lambda P(k-1)^{-1} + \phi(k)\phi(k)^T \tag{5}$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\phi(k)[y(k) - \hat{\theta}(k-1)^T\phi(k)] \tag{6}$$

This algorithm requires the inversion of the matrix $P(k)$ at each sampling interval. With the use of matrix inversion lemma [Goodwin and Sin, 1984; Ljung, 1987], we can avoid the calculation of the inverse matrix. We can introduce the time varying forgetting factor [Fortescue et al., 1981] as

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)[y(k) - \hat{\theta}(k-1)^T\phi(k)] \tag{7}$$

$$K(k) = P(k-1)\phi(k)/[1 + \phi(k)^T P(k-1)\phi(k)] \tag{8}$$

$$\lambda(k) = 1 - \{y(k) - \hat{y}(k)^2\}/\{\sigma(1 + \phi(k)^T P(k-1)\phi(t))\} \tag{9}$$

$$W(k) = P(k-1) - K(k)\phi(k)^T P(k-1) \tag{10}$$

To ensure an upper bound on $P(k)$, we update it as

$$P(k)=W(k)/\lambda(k) \text{ if trace}(W(k)/\lambda(k)) \leq C, \text{ else } P(k)=W(k) \text{ (i.e. } \lambda(k)=1) \tag{11}$$

where $\sigma \approx 1000*\sigma_w$ and $\sigma_w$ is the variance of any process output measurement noise. In this work, the lower bound on $\lambda(k)$ was set to 0.9.

Various well-known identification techniques can be employed for the identification of a grade change operation. In this study, four typical methods (bilinear model, subspace closed-loop identification, neural network with prediction error method and 13-level pseudo-random sequence (PRS) method) were applied for the identification of typical sequence of grade change operations. Results are shown
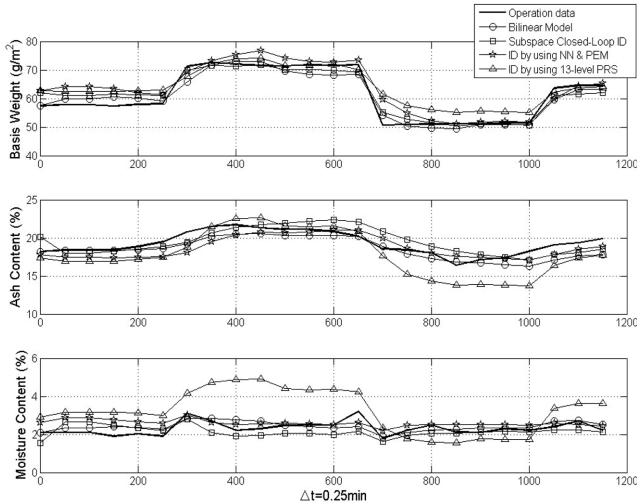
**Fig. 2. Identification of grade change operation (58→71→51→ 63 g/m².).**

in Fig. 2. As can be seen from Fig. 2, the bilinear model shows excellent tracking performance compared to other identification methods. This fact fortifies our choice of the bilinear model for the predictive control of grade change operations.

The bilinear model predictive control algorithm is obtained from the optimal control law that results from the minimization of some specific cost function. The present control law is based on the bilinear ARMA model predictive controller which was originally developed by one of the author [Yeo and Williams, 1987]. The model order of 2 was found to suffice to represent the grade change operations. Assuming that the time delay is known, the 4-input 3-output bilinear model can be written as

$$\underline{y}(t-\tau+1) = Q + H\underline{u}(t) \qquad (12)$$

where

$$\underline{y}(t) = [y_1(t) \quad y_2(t) \quad y_3(t)]^T, \quad \underline{u}(t) = [u_1(t) \quad u_2(t) \quad u_3(t) \quad u_4(t)]^T$$
$$Q = a_1\underline{y}(t+\tau) + a_2\underline{y}(t+\tau-1) + b_2\underline{u}(t-1) + c_{21}y_1(t+\tau-1)\underline{u}(t-1)$$
$$\quad + c_{22}y_2(t+\tau-1)\underline{u}(t-1) + c_{23}y_3(t+\tau-1)\underline{u}(t-1)$$
$$H = b_1 + c_{11}y_1(t+\tau) + c_{12}y_2(t+\tau) + c_{13}y_3(t+\tau)$$

In the input vector, $u_1$, $u_2$, $u_3$ and $u_4$ represent machine speed, stock flow, steam pressure and filler flow, respectively, and the elements of the output vector $y_1$, $y_2$ and $y_3$ denote basis weight, moisture content and ash content, respectively. The objective function for the one-step ahead predictive control is defined as

$$J = [\underline{y}_d(t+\tau+1) - Q]^T R\{\underline{y}_d(t+\tau+1) - Q - H\underline{u}(t)\}$$
$$\quad - \underline{u}^T(t)H^T R[\underline{y}_d(t+\tau+1) - Q] + \underline{u}^T(t)[H^T RH + S]\underline{u}(t) \qquad (13)$$

where $\underline{y}_d$ represents the desired output vector and R and W are weighting matrices for output error and inputs, respectively. From the minimization of the objective function, we have

$$\underline{u}(t) = [H^T RH + S]^{-1} H^T R[\underline{y}_d(t+\tau+1) - Q] \qquad (14)$$

The weighting matrices R and S are diagonal matrices of the form R=diag[$r_1$, $r_2$, K $r_n$] and S=diag[$s_1$, $s_2$, K $s_m$]. In our specific case, n=3 and m=4. In the numerical simulations constant weights were used such that $r_1$=$r_2$=$r_3$=1.0 and $s_1$=$s_2$=$s_3$=$s_4$=0.8. Choice of these

constant weighting parameters should be understood in trial-and error sense. There is no systematic way to choose "optimal" constant weighting parameters.

## RESULTS AND DISCUSSION

The basis weight of the product is the most important output variable in the grade change operation. In the actual operation, the grade change is considered to be achieved if the perturbations in the basis weight shrink within the range of ±10% of the difference between target values of the basis weights during the grade change. On-line measurements of the moisture and ash contents are highly contaminated by strong noises and only the trend of those variables is meaningful. In the actual grade change operations, the small variations or oscillations appearing in the moisture and ash contents are ignored and the basis weight is the primary post indicating the status of grade change operations.

Oscillations or variations in the basis weight cause severe problems in the product quality. Any perturbations in the basis weight should be suppressed quickly. PID controllers normally used in the operation have limits in the shortening of settling time. In fact, the amount of off-spec products is proportional to the grade change time. For these reasons most of the grade change operations have been performed by manual control mode to wind up with unsatisfactory control performance. This fact justifies the introduction of model based control schemes such as the bilinear model predictive control method developed in the present study.

Figs. 3 and 4 show results of numerical simulations of bilinear model predictive control operations compared with operational data for two different sequences of grade change operations (operation A: 58→72→51→63→72 g/m³, operation B: 72→76→51→ 47→51 g/m³). In the operation sequence, the target value (basis weight) of the new grade becomes the set point (i.e., grade change of 51→63 means that a step change of magnitude of 12 is introduced into the set point). The primary objective of the proposed bilinear model predictive control scheme is to eliminate unnecessary
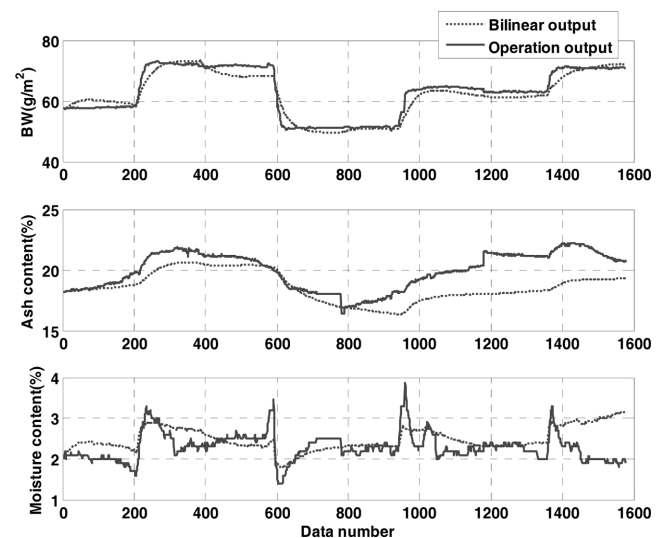


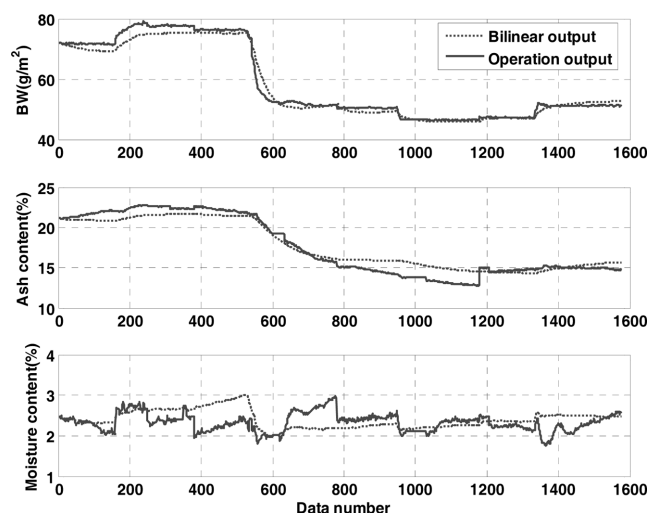**Fig. 3. Results of numerical simulations (Operation A: 58→72→ 51→63→72 g/m².).**

**Fig. 4. Results of numerical simulations (Operation B: 72→76→ 51→47→51 g/m².).**

oscillations in the key output variables, especially in the basis weight and the ash content. It is obvious that perturbations appearing in the actual operation were suppressed satisfactorily by the bilinear control action. This means that the bilinear control scheme can provide stable grade change operations considering the fact that even minor oscillations in basis weight may cause severe unstable operational status such as web break.

## CONCLUSIONS

Because of the high nonlinearity of the grade change processes in the paper production plants, control of the grade change operations has been performed manually by experienced engineers. A multivariable bilinear model predictive control method is proposed and applied to control the grade change operations in paper production plants. In this study a bilinear model for the paper plants is identified first followed by one-step ahead bilinear model predictive control. It is found that the bilinear model tracks the plant without significant discrepancy. Based on the multivariable bilinear plant model the optimal input variables are computed using the one-step ahead prediction method. Even for frequent changes in paper grades the bilinear model predictive control scheme exhibits good control performance in numerical simulations. Off-line application of the present control scheme is planned.

## NOMENCLATURE

$a_i$, $b_i$, $c_{ij}$ : bilinear model parameters ($a_i \in R^{4 \times 4}$, $b_i \in R^{3 \times 4}$, $c_{ij} \in R^{3 \times 4}$)
K       : update parameter defined by (8)
N       : prediction horizon
P       : update matrix defined by (5)
R       : weighting matrix (R=diag[$r_1$, $r_2$, K $r_n$])

S       : weighting matrix (S=diag[$s_1$, $s_2$, K $s_m$])
$\underline{u}$       : input vector
$u_i$       : element of input vector (i=1, 2, 3, 4)
$\underline{y}$       : output vector
$\underline{y}_d$       : desired output vector
$y_i$       : element of output vector (i=1, 2, 3)

**Greek Letters**
$\phi$       : parameter vector
$\lambda$       : update parameter
$\theta$       : past data vector
$\sigma_w$       : variance of output measurement noises
$\tau$       : time delay [min]

## REFERENCES

Aganovic, Z. and Gajic, Z., "Successive approximation procedure for steady-state optimal control of bilinear system," *Journal of Optimization Theory and Applications*, **84**, 273 (1995).
Fortescue,T. R., Kershenbaum, L. S. and Ydstie, B. E., "Implementation of self-tuning regulators with variable forgetting factors," *Automatica*, **17**, 831 (1981).
Goodwin, G. C. and Sin, K. S., *Adaptive filtering prediction and control*, Prentice-Hall, New Jersey, USA, p. 50-67 (1984).
Ljung, L., *System identification - theory for the user*, Prentice-Hall, New Jersey, USA, p. 303-326 (1987).
Mohler, R. R., *Nonlinear systems volume II: applications to bilinear control*, Prentice Hall, New Jersey, USA, p. 33-50 (1991).
Murphy, T. F. and Chen, S. C., "Transition control of paper-making processes: paper grade change," *Control Applications, 1999. Proceedings of the 1999 IEEE International Conference on control Applications*, **2**, 1278 (1999).
Oh, S. C. and Yeo, Y. K., "A study on the adaptive predictive control method for multivariable bilinear processes," *Korean J. Chem. Eng.*, **12**, 472 (1995).
Skoglund, A. and Brundin, A., "A multivariate process model for grade change in a paperboard machine," *Nordic Pulp & Paper Research Journal*, **15**, 183 (2002).
Tang, W. and Shi, S. J., "Autotuning PID control for large time-delay processes and its application to paper basis weight control," *Industrial & Engineering Chemistry Research*, **41**, 4318 (2002).
Wei, K. C. and Pearson, A. E., *On minimum energy control of commutative bilinear systems*, IEEE Transactions on Automatic Control, AC-23, 1020 (1978).
Yeo, Y. K. and Williams, D. C., "Bilinear model predictive control," *Industrial & Engineering Chemistry Research*, **26**, 2267 (1987).
Yeo, Y. K., Ryu, J. Y. and Yi, S. C., "Dynamics of the wet-end section in paper mills," *Korean J. Chem. Eng.*, **22**, 17 (2005).
Yeo, Y. K., Hwang, K. S., Kim, Y. G. and Kang, H., "Modeling of the drying process in paper plants," *J. of Chemical Eng. of Japan*, **38**(2), 120 (2005).