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Computation of multiloop controllers having desired closed-loop responses

Byung Chul Ji, Eun Mi Lee, Young A Han and Jietae Lee*,†

Department of Textile System Engineering, *Department of Chemical Engineering,
Kyungpook National University, Daegu 702-701, Korea
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Abstract−For n-by-n multivariable processes, multiloop controllers have n degrees of freedom and hence the n diagonal elements of closed-loop transfer functions can be designed to have desired closed-loop responses. Multiloop controllers having desired closed-loop responses can be considered as an extension of the single-input single-output internal model control and they can be used as reference controllers. However, computations of such multiloop controllers have not been well developed. The Newton-Raphson method and the iterative sequential loop closing method can be used, but they can suffer from a divergence problem for some processes. Here, the continuation method is applied to obtain multiloop control systems with desired closed-loop responses for a robust computation. The multiloop controllers with desired closed-loop responses can be used to obtain dynamic interaction measures and design multiloop PID controllers.

Key words: Desired Closed-loop Response, Multiloop Control System, Continuation Method, Dynamic Interaction Measure, Multiloop PI/PID Controller

INTRODUCTION

A multiloop control system where multiple single-input single-output (SISO) controllers are used to control interacting multivariable processes is often chosen in the chemical industry because of its simplicity. For n×n processes, the multiloop control system has n controllers. Hence, it can be designed to have desired closed-loop responses between n paired inputs and outputs. As shown later, multiloop controllers providing desired closed-loop responses are usually infinite dimensional and hence it is impossible to obtain the parametric models of transfer functions exactly. Here computational methods to obtain frequency responses of such multiloop controllers are investigated.

The direct synthesis method [1] and internal model control method [2] design control systems for SISO processes by specifying desired closed-loop responses. They are very simple and, by approximating them, excellent PID control systems can also be obtained [2,3]. The proposed controllers having desired closed-loop responses can be considered as an extension of SISO internal model controllers and can be applied to analyze and design multiloop control systems.

For 2×2 processes, analytic solutions for the multiloop control systems with desired closed-loop responses are available. For general processes, Jung et al. [4] proposed a design method based on the Newton-Raphson iteration in the frequency domain. However, the method is complex and suffers from a convergence problem. The sequential loop closing method [5,6] is one of the systematic methods for designing a multiloop control system. The method designs each loop sequentially, that is, the first loop is designed for the first pair of inputs and outputs and it is closed. The second loop is designed while the first loop has been closed. In this manner, all loops are designed. Each controller is designed based on the transfer function between the paired input and output while the former loops have been closed. Here, by applying the desired closed-loop response method for each step in the sequential loop closing method and repeating the design steps, we can obtain multiloop control systems with desired closed-loop responses. Simulations show that this iterative sequential loop closing method has better convergence property than the Newton-Raphson method for various process models. However, the method can also suffer from a convergence problem. To overcome the convergence problem of the above methods, the continuation method is applied to obtain frequency responses of multiloop controllers. By fitting frequency responses of the proposed multiloop controllers, practical multiloop PI/PID controllers can be obtained.

MULTILOOP CONTROL SYSTEMS WITH DESIRED CLOSED-LOOP RESPONSES

Consider a multivariable process transfer function:

$$y(s)=G(s)u(s)=\begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix} u(s) \tag{1}$$

where y(s) and u(s) are n output and input vectors, respectively, and G(s) is a n×n process transfer function matrix. A multiloop controller,

$$C(s)=\begin{bmatrix} c_1(s) & 0 & \cdots & 0 \\ 0 & c_2(s) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_n(s) \end{bmatrix} \tag{2}$$

is designed. The closed-loop transfer function matrix becomes y(s)

†To whom correspondence should be addressed.
E-mail: jtlee@knu.ac.kr

$=(I+G(s)C(s))^{-1}G(s)C(s)r(s)$, where r(s) is the set-point variable. Since the number of controllers is n, n elements in the closed-loop transfer function matrix can be designed to be desired ones like

$$\mathrm{diag}((I+G(s)C(s))^{-1}G(s)C(s))=\begin{bmatrix} h_1(s) \\ h_2(s) \\ \cdots \\ h_n(s) \end{bmatrix},\quad h_i(s)=\frac{\exp(-\theta_{ii}s)}{(\lambda_i s+1)^{r_i}} \tag{3}$$

where θ_{ii} and r_i are the time delay and the relative order in $g_{ii}(s)$, respectively, and λ_i is a desired closed-loop time constant. The term of diag(A) means a column vector whose elements are equal to the diagonal elements of a square matrix A. Eq. (3) is a nonlinear function for C(s) and hence iterative methods are required to solve.

For a 2×2 process, Eq. (3) is

$$\frac{g_{11}(s)c_1(s)+(g_{11}(s)g_{22}(s)-g_{12}(s)g_{21}(s))c_1(s)c_2(s)}{1+g_{11}(s)c_1(s)+g_{22}(s)c_2(s)+(g_{11}(s)g_{22}(s)-g_{12}(s)g_{21}(s))c_1(s)c_2(s)}=h_1(s)$$

$$\frac{g_{22}(s)c_2(s)+(g_{11}(s)g_{22}(s)-g_{12}(s)g_{21}(s))c_1(s)c_2(s)}{1+g_{11}(s)c_1(s)+g_{22}(s)c_2(s)+(g_{11}(s)g_{22}(s)-g_{12}(s)g_{21}(s))c_1(s)c_2(s)}=h_2(s) \tag{4}$$

Eq. (4) is quadratic for $c_1(s)$ or $c_2(s)$. Hence $c_1(s)$ and $c_2(s)$ have the square roots of polynomial functions of s and cannot be represented by rational transfer functions. Frequency responses of $c_1(j\omega)$ or $c_2(j\omega)$ can be solved analytically.

NEWTON-RAPHSON METHOD AND SEQUENTIAL LOOP CLOSING METHOD

To solve the Eq. (3), Jung et al. [4] proposed a method based on the Newton-Raphson iterations in the frequency domain. They solve

$$\begin{aligned} Z(j\omega)=&\mathrm{diag}((I+G(j\omega)C(j\omega))^{-1}G(j\omega)C(j\omega)) \\ &-[h_1(j\omega), h_2(j\omega), \cdots, h_n(j\omega)]^T=0 \end{aligned} \tag{5}$$

for $C(j\omega)$ at each frequency ω by the Newton-Raphson method and obtain PID controllers by fitting $C(j\omega)$. The method is somewhat complex and can have a convergence problem for some processes.

The sequential loop closing (SLC) method is one of the well-known methods to tune the multiloop control systems systematically [5,6]. The method designs multiloop controllers sequentially. The first loop is designed for the first pair of inputs and outputs and it is closed. The second loop is designed while the first loop has been closed. Since the first loop is closed, the transfer function of the second pair is changed and hence design of the second loop should be done with the changed transfer function. In this manner, all loops are designed. Each controller is designed based on the transfer function between the paired input and output while former loops have been closed. Here we iterate the design sequence until converges. When each loop is designed to have a desired closed-loop response and the iteration has been converged, multiloop control systems will have desired closed-loop responses.

Since each loop is designed under all other loops being closed, an effective transfer function should be obtained. Consider the control system that the *m*-th loop is open while all other loops are closed as in Fig. 1. The transfer function between u_m and y_m under all other loops closed becomes

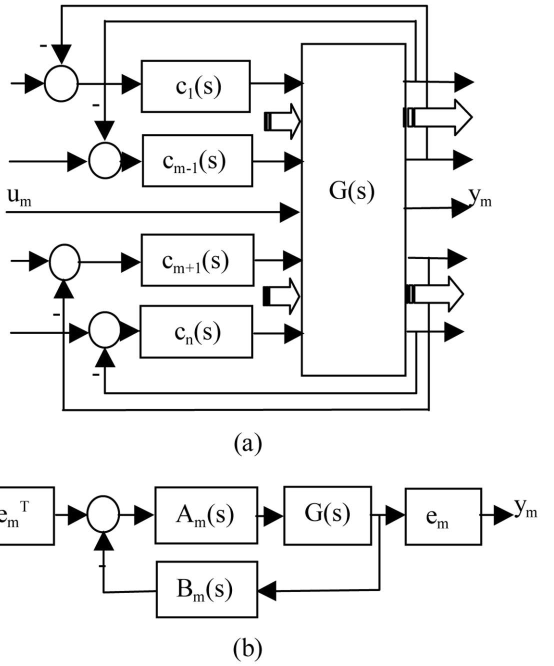

Fig. 1. Control system with the m-th loop open (a) and its equivalent system (b).

$$y_m(s)=p_m(s)u_m(s) \tag{6}$$

where

$$p_m(s)=e_m(I+G(s)A_m(s)B_m)^{-1}G(s)A_m(s)e_m^T$$
$$A_m(s)=\mathrm{diag}\{c_1(s)\cdots c_{m-1}(s)\ 1\ c_{m+1}(s)\cdots c_n(s)\}$$
$$B_m=\mathrm{diag}\{\underbrace{1\cdots 1}_{m-1}\ \ 0\ \ \underbrace{1\cdots 1}_{n-m}\}$$
$$e_m=\{\underbrace{0\cdots 0}_{m-1}\ \ 1\ \ \underbrace{0\cdots 0}_{n-m}\}$$

To obtain multiloop control systems with desired closed-loop responses, a controller at each step such that

$$c_m(s)=\frac{h_m(s)}{1-h_m(s)}\frac{1}{p_m(s)} \tag{7}$$

is calculated. Frequency responses of $c_m(s)$ can be obtained easily. If the iteration converges, controllers satisfy Eq. (5). It is remarked that this sequential loop closing method can find $c_m(s)$, a rational function of s. The order of $c_m(s)$ increases as iteration progresses and hence model reduction is needed at each step.

For 2×2 processes, Eq. (7) becomes

$$c_1(s)=\frac{h_1(s)}{1-h_1(s)}\frac{1}{g_{11}(s)-\dfrac{g_{12}(s)g_{21}(s)c_2(s)}{1+g_{22}(s)c_2(s)}} \tag{8a}$$

$$c_2(s)=\frac{h_2(s)}{1-h_2(s)}\frac{1}{g_{22}(s)-\dfrac{g_{12}(s)g_{21}(s)c_1(s)}{1+g_{11}(s)c_1(s)}} \tag{8b}$$

With initial values of $c_2(j\omega)=0$, we calculate $c_1(j\omega)$ and $c_2(j\omega)$ from Eqs. (8a) and (8b) successively. Convergence conditions of this fixed-point iteration can be derived. However, obtaining tight convergence condition is as complex as solving equations themselves and, on

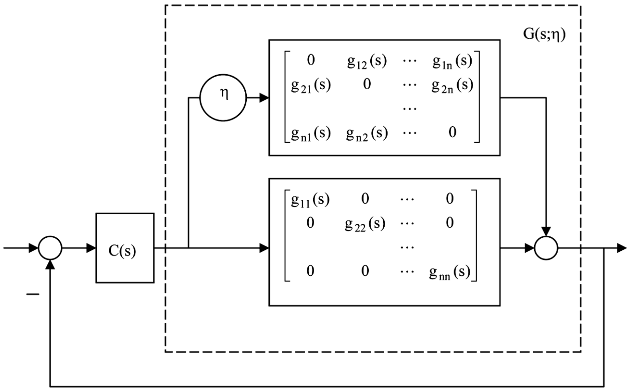

Fig. 2. Continuation model for designing multiloop control systems.

the other hand, simple conditions are usually too conservative.

CONTINUATION METHOD

To solve the nonlinear problem of Eq. (5), the continuation method [7] can be applied. For this, we decompose G(s) as (see Fig. 2):

$$G(s;\,\eta)=G_d(s)+\eta G_0(s) \tag{9}$$

where $G_d(s)$ and $G_0(s)$ are the diagonal and off-diagonal parts of G(s), respectively. For a G(s; η) and a frequency ω, Eq. (5) becomes

$$Z(j\omega;\,\eta)=\mathrm{diag}((I+G(j\omega;\,\eta)C(j\omega;\,\eta))^{-1}G(j\omega;\,\eta)C(j\omega;\,\eta)) - [h_1(j\omega), h_2(j\omega), \ldots, h_n(j\omega)]^T=0 \tag{10}$$

C(jω; η), solution of Eq. (10), is traced from η=0 to 1. When η=0, G(s; η) is diagonal and we can obtain each element of the controller C(s; η=0) easily by the SISO internal model control method. By applying the continuation method, we can obtain the controller C(jω; η) for the process $G(j\omega;\,\eta)|_{\eta=1}=G(j\omega)$ at a given frequency ω.

Continuation method solves

$$dZ(j\omega;\,\eta)=\Theta[dc_1, dc_2, \ldots, dc_n]^T+\varphi d\eta=0$$

$$\Theta=\begin{bmatrix} \frac{\partial z_1(j\omega;\,\eta)}{\partial c_1} & \frac{\partial z_1(j\omega;\,\eta)}{\partial c_2} & \cdots & \frac{\partial z_1(j\omega;\,\eta)}{\partial c_n} \\ \frac{\partial z_2(j\omega;\,\eta)}{\partial c_1} & \frac{\partial z_2(j\omega;\,\eta)}{\partial c_2} & \cdots & \frac{\partial z_2(j\omega;\,\eta)}{\partial c_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial z_n(j\omega;\,\eta)}{\partial c_1} & \frac{\partial z_n(j\omega;\,\eta)}{\partial c_2} & \cdots & \frac{\partial z_n(j\omega;\,\eta)}{\partial c_n} \end{bmatrix},$$

$$\varphi=\begin{bmatrix} \frac{\partial z_1(j\omega;\,\eta)}{\partial \eta} \\ \frac{\partial z_2(j\omega;\,\eta)}{\partial \eta} \\ \vdots \\ \frac{\partial z_n(j\omega;\,\eta)}{\partial \eta} \end{bmatrix} \tag{11}$$

To obtain derivatives in Eq. (11), we apply the perturbation technique. Let $\eta=\bar{\eta}+\varepsilon\tilde{\eta}$ and $C(j\omega;\,\eta)=\bar{C}(j\omega;\,\eta)+\varepsilon\tilde{C}(j\omega;\,\eta)$. The closed-loop transfer function matrix becomes

$$\begin{aligned} &[I+G(j\omega;\,\eta)C(j\omega;\,\eta)]^{-1}G(j\omega;\,\eta)C(j\omega;\,\eta) \\ &=[I+(\bar{G}+\varepsilon\tilde{G}+O(\varepsilon^2))(\bar{C}+\varepsilon\tilde{C}+O(\varepsilon^2))]^{-1} \\ &\quad(\bar{G}+\varepsilon\tilde{G}+O(\varepsilon^2))(\bar{C}+\varepsilon\tilde{C}+O(\varepsilon^2)) \\ &=[I+\bar{G}\bar{C}]^{-1}[I-\varepsilon(\bar{G}\tilde{C}+\tilde{G}\bar{C})(I+\bar{G}\bar{C})^{-1}+O(\varepsilon^2)] \\ &\quad[\bar{G}\bar{C}+\varepsilon(\bar{G}\tilde{C}+\tilde{G}\bar{C})+O(\varepsilon^2)] \\ &=\bar{H}+\varepsilon[\bar{S}\bar{G}\tilde{C}\bar{S}+\bar{S}\tilde{G}\bar{C}\bar{S}\tilde{\eta}]+O(\varepsilon^2) \end{aligned} \tag{12}$$

where

$$\begin{aligned} \bar{G}&=G_d(j\omega)+\bar{\eta}G_0(j\omega) \\ \tilde{G}&=G_0(j\omega) \\ \bar{S}&=[I+\bar{G}\bar{C}]^{-1} \\ \bar{H}&=[I+\bar{G}\bar{C}]^{-1}\bar{G}\bar{C} \end{aligned}$$

Terms of order ε in Eq. (12) are linear for $\tilde{C}(j\omega;\,\eta)$. To obtain an explicit equation for $\tilde{C}(j\omega;\,\eta)$, we use the relationship [8]

$$\mathrm{vec}(\bar{S}\bar{G}\tilde{C}\bar{S})=(\bar{S}^T\otimes(\bar{S}\bar{G}))\mathrm{vec}(\tilde{C}) \tag{13}$$

where $\otimes$ means the Kronecker product and vec(A) is a vector such that $[a_{11}, a_{21}, \ldots, a_{n1}, a_{12}, a_{22}, \ldots, a_{nn}]^T$. Removing terms corresponding to zeros, we have

$$\begin{aligned} &\mathrm{diag}(\bar{S}\bar{G}\tilde{C}\bar{S})=D^T(\bar{S}^T\otimes(\bar{S}\bar{G}))D\,\mathrm{diag}(\tilde{C}) \\ &D=(e_1, e_{n+2}, e_{2n+3}, \cdots, e_{n^2}),\ e_k=[\underbrace{0\cdots0}_{k-1}\ 1\ \underbrace{0\cdots0}_{n^2-k}]^T \end{aligned} \tag{14}$$

From Eqs. (12) and (14), we can obtain

$$\begin{aligned} \Theta&=D^T(\bar{S}^T\otimes(\bar{S}\bar{G}))D \\ \varphi&=\mathrm{diag}(\bar{S}G_0\bar{C}\bar{S}) \end{aligned} \tag{15}$$

To trace Z(jω; η)=0 with increasing η from 0 to 1 for a given frequency ω, we solve

$$\frac{d}{d\eta}\begin{bmatrix} c_1(j\omega;\,\eta) \\ c_2(j\omega;\,\eta) \\ \vdots \\ c_n(j\omega;\,\eta) \end{bmatrix}=-\Theta^{-1}\varphi \tag{16}$$

Initial value is

$$c_n(j\omega;\,\eta)_{\eta=0}=\frac{h_i(j\omega)}{1-h_i(j\omega)}\frac{1}{g_{ii}(j\omega)}, \qquad i=1, 2, \cdots, n \tag{17}$$

For a small tracking error, Newton-Raphson correction with the matrix can also be used [7].

APPLICATIONS OF FREQUENCY RESPONSES OF MULTILOOP CONTROLLERS

One of the dynamic interaction measures in Lee and Edgar [9] is defined as

$$q(\lambda) = \underset{\omega}{\mathrm{Max}}\,\sigma_{Max}\left((I+G(j\omega)C(j\omega))^{-1}G(j\omega)C(j\omega)-\begin{bmatrix} h_1(j\omega) & \cdots & 0 \\ & \vdots & \\ 0 & \cdots & h_n(j\omega) \end{bmatrix}\right) \tag{18}$$

It means the largest magnitude of difference between the actual closed-loop transfer function matrix and the desirable closed-loop transfer function matrix. If $q(\lambda)$ is small, controller performance will not differ much from desirable control performance. The dynamic interaction measure can be used to solve the pairing problem of determining the input-output pair with the lowest $q(\lambda)$. To calculate $q(\lambda)$, Lee and Edgar [9] used the controllers of Eq. (17) for simplicity. However, the controllers of Eq. (17) where interaction terms are ignored totally are not used in practice and may not be the best choice to calculate $q(\lambda)$. The proposed controllers having desired closed loop responses can be used for $q(\lambda)$.

Approximating frequency responses of controllers having desired closed loop responses, multiloop PI/PID controllers can be designed. The design procedure is as follows.

Step 1: Choose the design parameters, λ_i's, and 100 logarithmically equally spaced frequencies between $\min(1/\lambda_i)/10$ and $10*\max(1/\lambda_i)$.

Step 2: Set ε=0.1 and update $C(j\omega)$ with initial values of Eq. (17) until η=1.

Step 3: Approximate $c_i(j\omega)$ by PI or PID controller. For this, the weighted least squares method as the routine 'invfreqs' in MATLAB can be used.

Simulations show that this multiloop PI/PID controller design method is well comparable with other design methods.

CASE STUDIES

For process models in Luyben [11], frequency responses of multiloop controllers having desired closed loop responses are computed via the Newton-Raphson method [4], the repeated sequential loop closing method and the proposed continuation method. The Newton-Raphson method fails to converge for some 4×4 models. The sequential loop closing method converges for all the models in Luyben [11]. However, there may be other processes for which it fails to converge.

Process 1: Consider the Wood-Berry column model [13]:

$$G(s)=\begin{bmatrix} \dfrac{12.8\exp(-s)}{16.7s+1} & \dfrac{-18.9\exp(-3s)}{21s+1} \\ \dfrac{6.6\exp(-7s)}{10.9s+1} & \dfrac{-19.4\exp(-3s)}{14.4s+1} \end{bmatrix}$$

Fig. 3 shows step set point responses for controllers with desired closed-loop response. Since the proposed C(s) is calculated in the form of frequency responses, we use the FFT method [10] to simulate step set point responses. We can see that step responses between paired inputs and outputs are identical to desired closed-loop responses.

Fig. 4 shows approximations of the proposed controllers by PI controllers. PI controller parameters for λ=(5, 5) are in Table 1. Fig. 5 shows step set point responses for PI controllers obtained by fitting the proposed controllers with desired closed-loop responses. Fig. 4 indicates that approximations by PI controllers are not so accurate.

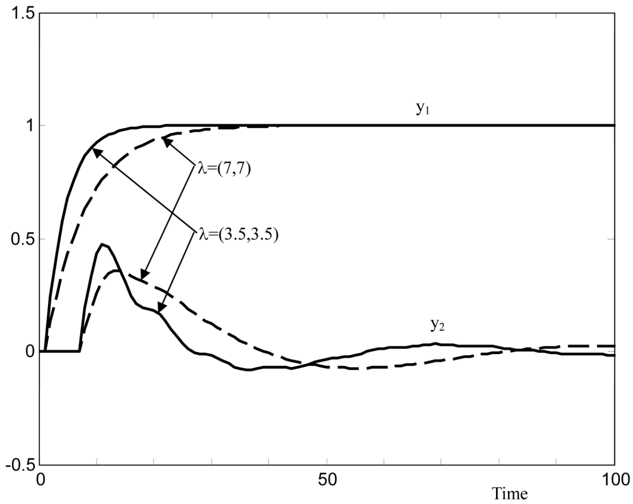

Fig. 3. Step set point responses of the proposed control systems for the Wood-Berry column model.

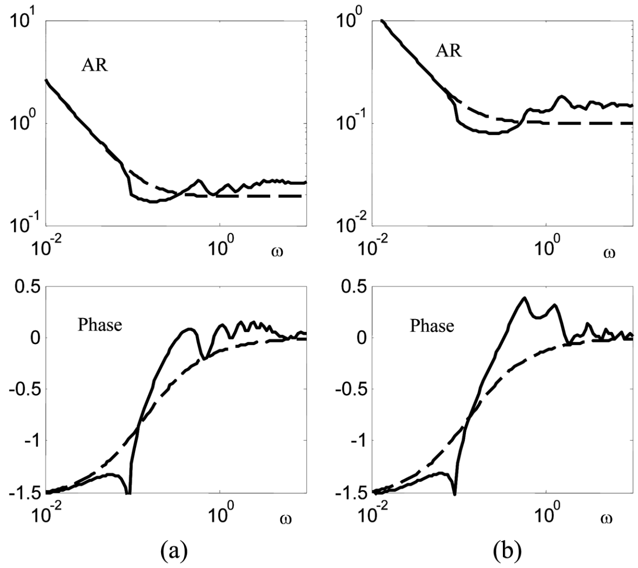

Fig. 4. Frequency responses of the proposed controllers (solid line) and their approximate PI controllers (dashed line) for the Wood-Berry column model (a: for the case of λ=(7, 7), b: for the case of λ=(3.5, 3.5)).

Table 1. Tuning results for the Wood-Berry and Ogunnaike-Ray column models

Process	BLT method	Proposed PI controller
Wood-Berry column	Kc={0.375, −0.075} τ_I={8.29, 23.6}	λ={5, 5} Kc={0.1896, −0.099} τ_I={7.2283, 7.6794}
Ogunnaike-Ray column	Kc={1.51, −0.295, 2.63} τ_I={16.4, 18, 6.61}	λ={15, 15, 5} Kc={0.2327, −0.0378, 1.1851} τ_I={1.3769, 0.9057, 4.1179}

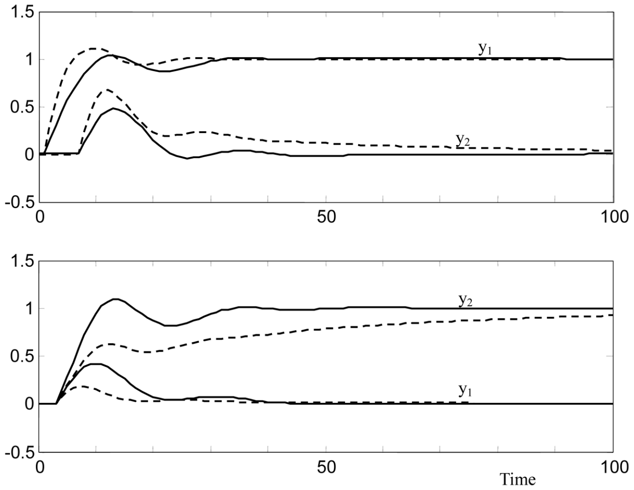

Fig. 5. Unit step set point responses of multiloop PI control systems for the Wood-Berry column model (solid line: the proposed multiloop controller, dotted line: the multiloop controller designed by BLT method).

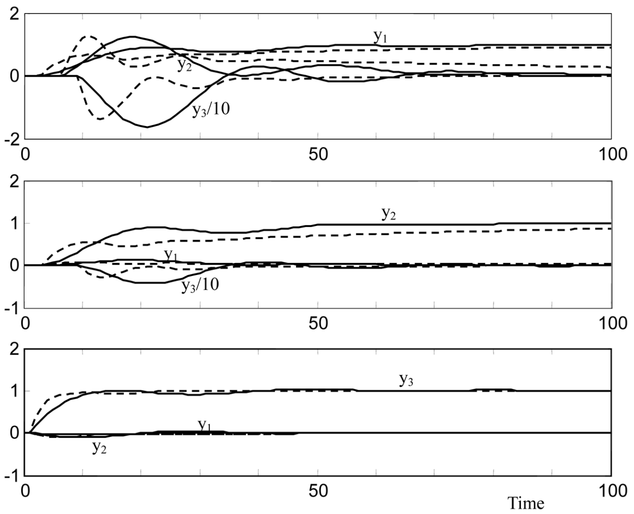

Fig. 6. Unit step set point responses of multiloop PI control systems for the Ogunnaike-Ray column model (solid line: the proposed multiloop controller, dotted line: the multiloop controller designed by BLT method).

However, control performance is far better than that of control systems designed by the biggest log-modulus tuning method [BLT; 11].

Process 2: Consider the Ogunnaike-Ray column model [12].

$$G(s) = \begin{bmatrix} \frac{0.66\exp(-2.6s)}{6.7s+1} & \frac{-0.61\exp(-3.5s)}{8.64s+1} & \frac{-0.0049\exp(-s)}{9.06s+1} \\ \frac{1.11\exp(-6.5s)}{3.25s+1} & \frac{-2.36\exp(-3s)}{5s+1} & \frac{-0.01\exp(-1.2s)}{7.09s+1} \\ \frac{-34.68\exp(-9.2s)}{8.15s+1} & \frac{46.2\exp(-9.4s)}{10.9s+1} & \frac{0.87(11.61s+1)\exp(-s)}{(3.89s+1)(18.8s+1)} \end{bmatrix}$$

Frequency responses of the proposed controllers for λ=(15, 15, 5) are calculated by the continuation method and their approximate PI controller parameters are in Table 1. Fig. 6 shows step set point responses for PI controllers obtained by approximating the proposed controllers with desired closed-loop responses. We can see that control performance is far better than that of control systems designed by the BLT method.

CONCLUSION

The continuation method is applied to calculate multiloop controllers having desired closed-loop responses. The convergence problem of iterative methods such as the Newton-Raphson method and the sequential loop closing method for finding multiloop controllers with desired closed-loop responses can be avoided. The proposed controllers having desired closed-loop responses can be used to obtain dynamic interaction measures and design multiloop PID controllers.

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