Korean J. Chem. Eng., 25(6), 1273-1278 (2008)

Enhanced frequency response estimator to guarantee pre-specified phase angle and static disturbance rejection with all harmonics removed

Su Whan Sung and Jietae Lee†

Department of Chemical Engineering, Kyungpook National University, 1370 Sankyeok-dong, Buk-gu, Daegu 702-701, Korea
(*Received 2 April 2008 • accepted 30 June 2008*)

Abstract−We propose a new frequency response estimation method in order to guarantee a pre-specified phase angle of the estimated model under static disturbance circumstances with rejecting harmonics completely. The proposed method uses one cycle of the conventional relay feedback signal followed by a sinusoidal signal. The sinusoidal signal in a cyclic steady state has no harmonics, resulting in exact frequency response estimates. Also, it guarantees the pre-specified phase angle and removes the effects of static disturbances by adjusting the reference value of the sinusoidal signal.

Key words: Relay Feedback, Frequency, Phase Angle, Sine Signal, Disturbance, Harmonics

INTRODUCTION

Various relay feedback methods have been developed since the original relay feedback method was first proposed [1,2]. Shen et al. [3] proposed a biased relay feedback method to identify the zero and the ultimate frequency data of the process from one relay test. Modified relay feedback methods using a two-level signal, a saturation-relay or a preload relay have been proposed to increase the accuracy of the identified frequency data set, by reducing the high order harmonic terms of the relay output [4-6]. Also, a modified relay-based technique and a phase-locked loop identifier using sinusoidal signals have been proposed to remove the harmonics completely [7-9].

Several relay feedback methods using an artificial time delay [10, 11], a hysteresis [12] or a two-channel relay [13] have been proposed to obtain the set of the frequency data of the process for a lower frequency region than the ultimate frequency.

Many relay feedback methods have been proposed to reject static disturbances [14-17], which bias the reference value of the relay on-off as much as the static disturbance, in order to achieve the same accuracy in the case of no disturbance. Recently, two relay feedback methods have been proposed in order to guarantee the pre-specified phase angle and reject static disturbances simultaneously [18, 19]. Also, a new relay feedback method has been proposed to guarantee symmetric responses under static nonlinearity and static disturbances [20]. This can be successfully applied to identify Hammerstein-Wiener-type nonlinear processes.

The above-mentioned previous approaches have been substantially contributing towards the improvement of the accuracy and effectiveness of the frequency response estimation methods. But, there is still room to improve the previous approaches. All previous relay feedback methods except two [7,9] were unable to completely reject the effects of the harmonics, inevitably resulting in some errors in estimating the frequency data of the process. Also, the two previous approaches [7,9] have some drawbacks even though they can remove the harmonics completely in ideal situations. One method [7] cannot guarantee a desirable cycle under the circumstances of static disturbances. For example, if there are some errors (equivalently, static disturbances) in determining the deviation variables, it fails to provide the continuous cycling of the desired frequency because the previous approach has no instruments to change the direction of the continuous cycling when it starts deviating from the desired position. The other method [9] has the following problem. Inherently, the phase-locked loop identifier takes a long time for identification because the phase measurement and the feedback control loop introduce significant time lags. Furthermore, the slow dynamics of an input disturbance are not controlled by a fast closed-loop form, resulting in a long identification time.

Also, no previous methods can provide the frequency response estimates while guaranteeing the desired phase angle, rejecting static disturbances and removing all harmonics simultaneously.

In this research, a new frequency response estimation method is proposed to remove the effects of the harmonics completely, resulting in exact frequency response estimates. Also, this method guarantees the pre-specified phase angle and removes the effects of static disturbances automatically, resulting in exact estimates under even static disturbance circumstances. The three core functions of removing harmonics, rejecting static disturbances and guaranteeing the desired phase angle can be implemented independently. It is straightforward to deactivate one or two of the functions. The proposed each technique can be applied to improve the previous approaches.

PROPOSED FREQUENCY RESPONSE ESTIMATOR

Fig. 1 shows the schematic diagram and a typical response of the process for the proposed estimation method. It uses one cycle of the conventional relay feedback signal followed by a sinusoidal signal combined with a pulse signal. The one cycle of the conventional relay feedback is to initialize the parameters of the sinusoidal signal. And, the sinusoidal signal is to activate the process without harmonics. The role of the pulse signal is to enforce the process output and the process input to converge to the complete sinusoidal signal. In Fig. 1(b), the first several cycles of incomplete sinusoidal signals clearly converge to the complete sinusoidal signal due to the pulse signals. Then, we reach the important conclusion that the process

†To whom correspondence should be addressed.
E-mail: jtlee@knu.ac.kr

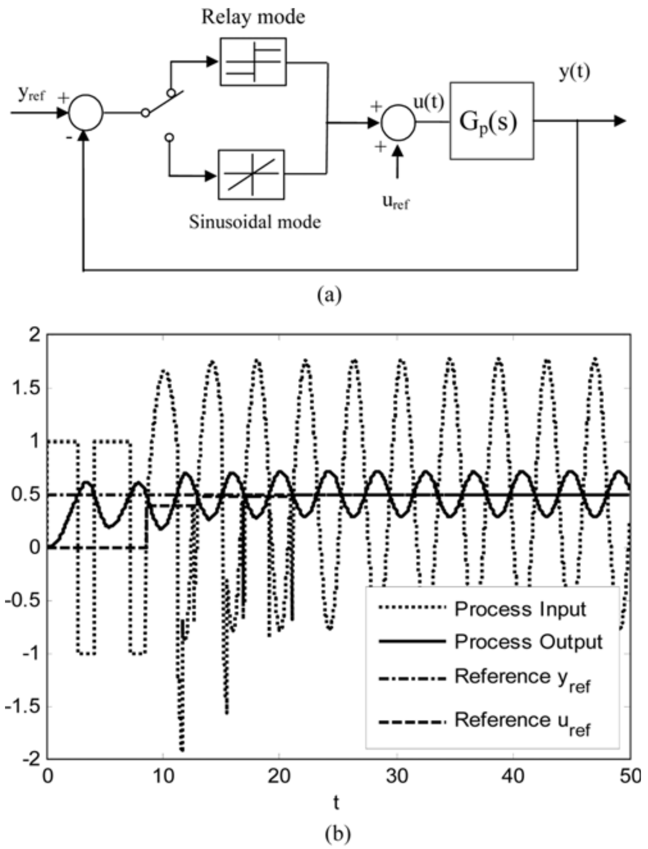

Fig. 1. Schematic diagram of the proposed method (a) and typical responses (b).

input and the process output in the cyclic-steady-state include no harmonics. Also, the proposed method uses the sinusoidal signal that leads the process output as much as the pre-specified phase angle. Then, the pre-specified phase angle between the process input and the process output is guaranteed. The reference (u_{ref}) for the process input is automatically updated by an updating rule (which will be discussed later) to enforce the oscillation of the process input and output to be symmetric by rejecting the effects of the static disturbance. In Fig. 1(b), the reference value for the process output in the relay feedback method is y_{ref}=0.5, which is equivalent to introducing a static disturbance. Nevertheless, the proposed method automatically raises the process output up to the reference value of y_{ref}= 0.5 and provides the symmetric oscillation of the complete sinusoidal signal by updating u_{ref}. Once we obtain the process input and the process output of complete sinusoidal signals, it is straightforward to estimate the frequency response data of the process by using the describing described function analysis method.

Consider the following steps to understand the working of the proposed method in more detaildetails. Step 1, obtain one cycle from t=0 to t≈10 using the conventional relay feedback method. That is, u(t)=d when y(t)<y_{ref}, u(t)=−d when y(t)≥y_{ref}, where d, u(t), y(t) and y_{ref} denote the magnitude of the relay, the process input, the process output and the reference value of the relay feedback, respectively. The role of Step 1 is to initialize the frequency. Step 2, change from the relay feedback mode to the sinusoidal mode. In the sinusoidal mode, the proposed method determines the process input as follows:

When y(t)≥y_{ref} (negative input status)

$$u(t) = -\frac{4d}{\pi}\sin(\omega_{k-1}^{off}(t-t_{off,k})-\phi)+u_{ref} \quad \text{if } t\le t_{on}^{expected} \tag{1}$$

$$u(t) = -\frac{4d}{\pi}\sin(\omega_{k-1}^{off}P_{off,k-1}-\phi)+u_{ref} \quad \text{if } t_{on}^{expected}<t\le t_{on}^{expected}+\delta \tag{2}$$

$$u(t) = -\frac{4d}{\pi}\sin(\omega_{k-1}^{off}P_{off,k-1}-\phi)-\frac{4d}{\pi}+u_{ref} \quad \text{if } t>t_{on}^{expected}+\delta \tag{3}$$

$$u(t) = -\frac{4d}{\pi}\sin(\omega_{k-1}^{off}(t-t_{off,k})-\phi)-\frac{4d}{\pi}+u_{ref} \quad \text{if } t\le t_{off}^{expected}-\delta \tag{4}$$

When y(t)<y_{ref} (positive input status)

$$u(t) = \frac{4d}{\pi}\sin(\omega_{k-1}^{on}(t-t_{on,k})-\phi)+u_{ref} \quad \text{if } t\le t_{off}^{expected} \tag{5}$$

$$u(t) = \frac{4d}{\pi}\sin(\omega_{k-1}^{on}P_{on,k-1}-\phi)+u_{ref} \quad \text{if } t_{off}^{expected}<t\le t_{off}^{expected}+\delta \tag{6}$$

$$u(t) == \frac{4d}{\pi}\sin(\omega_{k-1}^{on}P_{on,k-1}-\phi)+\frac{4d}{\pi}+u_{ref} \quad \text{if } t>t_{off}^{expected}+\delta \tag{7}$$

$$u(t) == \frac{4d}{\pi}\sin(\omega_{k-1}^{on}(t-t_{on,k})-\phi)+\frac{4d}{\pi}+u_{ref} \quad \text{if } t\le t_{on}^{expected}-\delta \tag{8}$$

Where, $\omega_{k-1}^{off}=\pi/P_{off,k-1}$, $\omega_{k-1}^{on}=\pi/P_{on,k-1}$, $t_{on}^{expected}=t_{off,k}+P_{off,k-1}$ and $t_{off}^{expected}=t_{on,k}+P_{on,k-1}$. $P_{off,k-1}$ and $P_{on,k-1}$ denote the time length of the half-cycle in the previous (k-1-th) cycle corresponding to the negative input status and the positive input status, respectively. $t_{on,k}$ is defined as the time at which the process input changes from the negative input status to the positive input status, which is the start of the on status of the k-th cycle. The negative input status of the k-th cycle starts at $t_{off,k}$. $t_{on}^{expected}=t_{off,k}+P_{off,k-1}$ and $t_{off}^{expected}=t_{on,k}+P_{on,k-1}$ are the expected subsequent t_{on} and t_{off} on the basis of the previous half periods of $P_{off,k-1}$ and $P_{on,k-1}$. δ is a small positive value to prevent undesirable activation of (3), (4), (7) or (8) due to uncertainties. Because the magnitude of the fundamental frequency term of the relay mode is $4d/\pi$, we choose the magnitude of the sinusoidal signal of $4d/\pi$ for the smoothing transit from the conventional relay mode to the sinusoidal mode without changing the magnitude of the fundamental frequency term. $-\pi+\phi$ corresponds to the pre-specified phase angle between the process input and the process output. From (1) and (5), the process output is y(t)=sin(ωt)+y_{ref} for the process input u(t)=4d sin(ω(t)+$\pi-\phi$)/π+u_{ref} in cyclic-steady-state. So, it is clear that the phase angle between the process input and the process output in cyclic-steady-state becomes the pre-specified phase angle of $-\pi+\phi$.

Let us explain the role of each equation from (1) to (8). If the process reaches a cyclic-steady-state (that is, $P_{off,k-1}=P_{off,k}$ and $P_{on,k-1}=P_{on,k}$), only (1) and (5) will be activated. It is clear that u(t) of (1) or (5) has no harmonics and the phase angle between u(t) and y(t) is exactly $-\pi+\phi$ in the cyclic-steady-state. Then, it is straightforward to estimate the exact frequency model, of which the phase angle is exactly $-\pi+\phi$.

When the continuous-cycling of the process input/output deviates from the desired cyclic-steady-state, the proposed method forces the continuous-cycling to go back to the desired position by adding the additional step signals of $4d/\pi$ to the sinusoidal signals as shown

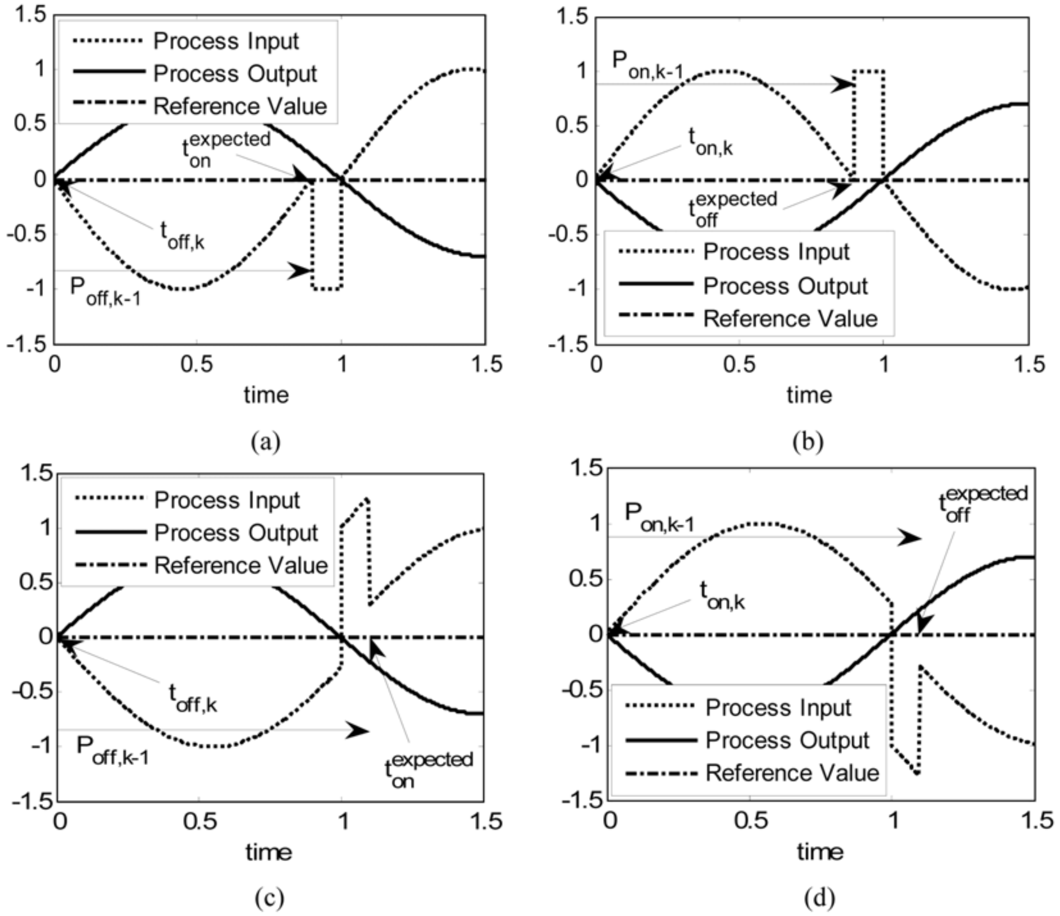

Fig. 2. Proposed mechanisms to achieve a cyclic-steady-state.

in (3), (4) and (7), (8). For simplicity, assume δ=0 and consider the mechanisms shown in Fig. 2. If $y(t)\geq y_{ref}$ and $t>t_{on}^{expected}$ as shown in Fig. 2(a) (that is, the process output does not cross the reference value yet although it is expected to cross at $t=t_{on}^{expected}$), the time length corresponding to the positive status of the process output should be shortened. This can be achieved by simply adding a negative additional signal of $-4d/\pi$ (which corresponds to the negative pulse in Fig. 2(a)) to the process input as shown in (3) and Fig. 2(a). Also, the time length of the negative status of the process output can be shortened by adding the positive additional signal of $4d/\pi$ (which corresponds to the negative pulse in Fig. 2(b)) to the process input as shown in (7) and Fig. 2(b). This is the same principle as one used in the conventional relay feedback method, which shortens the time length of the positive status of the process output cycle by entering a negative process input, and vice versa.

If $y(t)<y_{ref}$ and $t\leq t_{on}^{expected}$ as shown in Fig. 2(c) (that is, the process output crosses the reference value earlier than expected), the time length corresponding to the positive status of the process output will be lengthened by adding the positive additional signal of $4d/\pi$ (which corresponds to the positive pulse in Fig. 2(c)) to the process input as shown in (8) and Fig. 2(c). The same mechanism is applied to the opposite case as shown in Fig. 2(d).

For the proposed method, (2) and (6) of the proposed method are to prevent undesirable activation of (3), (4) and (7), (8) due to cycle-to-cycle uncertainties, such as measurement noises and high frequency disturbances. δ will be a small positive value. If the overload of the actuator is of no concern, (2) and (6) can be removed.

u_{ref} is updated in each cycle by $u_{ref}(k)=u_{ref}(k-1)+\alpha(2d/\pi)(P_{on}(k-1)-P_{off}(k-1))/P(k-1)$, which shifts u_{ref} as much as the magnitude of the zero-frequency quantity to reject the effects of the disturbance and to obtain a symmetric input signal. Here, $P_{on}(k-1)$ and $P_{off}(k-1)$ are the time length corresponding to the positive status and the negative status of the process input signal, respectively. $P(k-1)=P_{on}(k-1)+P_{off}(k-1)$ corresponds to the time length of the (k−1)-th cycle and $(2d/\pi)(P_{on}(k-1)-P_{off}(k-1))/P(k-1)$ is the time-average value (equivalently, zero-frequency quantity) of the previous cycle. Note that $(2d/\pi)(P_{on}(k-1)-P_{off}(k-1))/P(k-1)$ monotonically decreases as the process input of u_{ref}+D increases, where D is the input static disturbance. So, the update of $u_{ref}(k)-u_{ref}(k-1)=\alpha(2d/\pi)(P_{on}(k-1)-P_{off}(k-1))/P(k-1)$ will remove the effect of the static input disturbance (equivalently, making $u_{ref}+D=0$), resulting in symmetric responses of the process input and output. The convergence of the proposed update method will be discussed later in the next section.

In this research, we recommend $\alpha=1.5\exp(-(n_c-1)/\tau_\alpha)$. Here, n_c is the number of the sinusoidal cycles. τ_α determines the convergence rate of u_{ref}. A small τ_α means a fast convergence. But, too small τ_α will not provide enough time for u_{ref} to converge to the right value. Therefore $3\leq\tau_\alpha\leq 5$ is recommended. Roughly speaking, this requires 4-6 cycles to obtain a symmetric process input and output. A typical performance is shown in Fig. 1(b) with τ_α=3. Here, the reference value of the process output is 0.5. It confirms that the proposed method provides a symmetric cycle for the nonzero reference value. Previous relay feedback approaches cannot provide a symmetric cycle for the nonzero reference value if they have no mechanisms to reject static disturbances.

From the activated process data by the proposed method, we can estimate the frequency data in a straightforward manner. Because the process output in cyclic-steady-state is $y(t)=\alpha\sin(\omega t)+y_{ref}$ for the process input $u(t)=(4d/\pi)\sin(\omega t-(-\pi+\phi))+u_{ref}$, the identified frequency data of the process is

$$|G(j\omega)|=\frac{\pi a}{4d},\ \ \angle G(j\omega)=-\pi+\phi \tag{9}$$

Where, a is the measured oscillation magnitude of the process output.

The proposed method removes the effects of static disturbances and provides the oscillation corresponding to the pre-specified phase angle. Also, it can remove completely harmonics, resulting in exact frequency response estimates of (9).

DISCUSSIONS ON CONVERGENCE

There are insurmountable limitations in proving the convergence in a rigorous way for all possible situations in operating the proposed method. So, discussion of the convergence of the proposed method will be confined to the two restricted situations of Case 1 and Case 2 with several assumptions.

Case 1) Assume that there is no entry of static disturbances (no update of u_{ref}) and that a small perturbation is added to the amplitude or frequency of signal during the cyclic-steady-state operation. The necessary condition (Loeb condition) for a stable limit cycle in Case 1 is as follows [21].

$$\left(\frac{\partial U}{\partial\omega}\frac{\partial Q}{\partial a}-\frac{\partial V}{\partial\omega}\frac{\partial P}{\partial a}\right)\bigg|_{\omega=\omega_c}>0 \tag{10}$$

Where, $G(j\omega)=U(\omega)+jV(\omega)$ and $-1/N(a)=P(a)+jQ(a)$. ω_c is the frequency of the cycle. N(a) is the describing function of the proposed test signal generator. It is clear that $N(a)=(4d/\pi a)\exp(-j\phi)$ because it generates the output of $u(t)=(4d/\pi)\sin(\omega t-\phi)$ for the input of $-y(t)=a\sin(\omega t)$ and the effects of the additional signals are negligible for such a small perturbation. Then, $P(a)=-(\pi a/4d)\cos(\phi)$ and $Q(a)=-(\pi a/4d)\sin(\phi)$. Then, (10) becomes (11).

$$\left(-\frac{\partial U}{\partial\omega}\frac{\pi\sin(\phi)}{4d}+\frac{\partial V}{\partial\omega}\frac{\pi\cos(\phi)}{4d}\right)\bigg|_{\omega=\omega_c}>0 \tag{11}$$

We know that $\cos(\phi)$ and $\sin(\phi)$ are always positive because $0\le\phi\le\pi/2$. Note that most processes for $0\le\phi\le\pi/2$ (equivalently, $-\pi/2\le\angle G(j\omega)\le-\pi$) are one of the two cases of $\partial U/\partial\omega|_{\omega=\omega_c}\le 0$, $\partial V/\partial\omega|_{\omega=\omega_c}>0$ and $\partial U/\partial\omega|_{\omega=\omega_c}>0$, $\partial V/\partial\omega|_{\omega=\omega_c}>0$, because the imaginary part (V) of most processes increases as ω increases. It is clear that the first case of $\partial U/\partial\omega|_{\omega=\omega_c}\le 0$ and $\partial V/\partial\omega|_{\omega=\omega_c}>0$ satisfies (11).

(11) can be rewritten for the second case of $\partial U/\partial\omega|_{\omega=\omega_c}>0$ and $\partial V/\partial\omega|_{\omega=\omega_c}>0$.

$$\tan(\phi)<\frac{\partial V/\partial\omega|_{\omega=\omega_c}}{\partial U/\partial\omega|_{\omega=\omega_c}} \tag{12}$$

Let $\tan(\varphi)=(\partial V/\partial\omega|_{\omega=\omega_c})/(\partial U/\partial\omega|_{\omega=\omega_c})$ and $\tan(\phi)=U(\omega_c)/V(\omega_c)$. Most process processes satisfy $\phi<\varphi$ for a small ϕ, equivalently (12). Now, it is proven that the proposed test signal generator in Case 1 with a small ϕ provides a stable limit cycle for most processes.

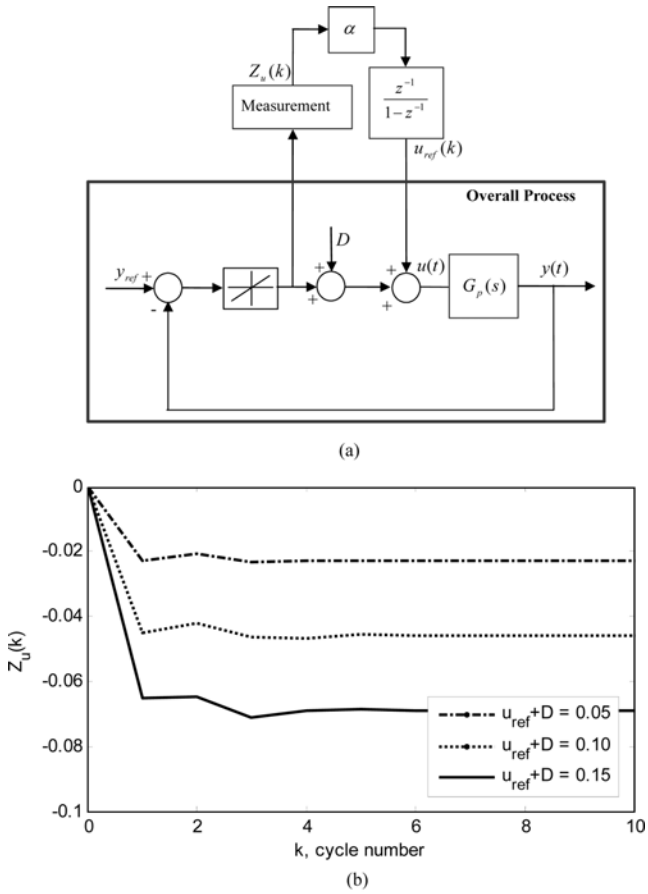

Fig. 3. Proposed u_{ref} update method (a) and a typical step response (b).

Case 2) Assume the situation that a static disturbance enters during the cyclic-steady-state operation and that u_{ref} is updated. The proposed update method can be expressed as in Fig. 3. D denotes the static input disturbance. The zero-frequency quantity of the process input ($Z_u(k-1)=(2d/\pi)(P_{on}(k-1)-P_{off}(k-1))/(P(k-1))$) can be measured from the previous cycle. Then, the discrete-time overall process can be defined, of which input and output are $u_{ref}(k)$ (or $u_{ref}(k)+D$) and $Z_u(k)$, respectively, as shown in Fig. 3(a). Here, assume that the perturbation by the static disturbance is so small that the internal feedback loop inside the overall process is internally stable as in Case 1. Now, note that $Z_u(k)$ is proportional to $-(u_{ref}(k)+D)$ around $u_{ref}(k)+D=0$ because the symmetricity of the signal continuously degrades as $u_{ref}(k)+D$ is increased. For example, Fig. 3(b) shows a typical step response of $Z_u(k)$ for the positive step input of $u_{ref}(k)+D$. As expected, it shows a linearity between $Z_u(k)$ and $u_{ref}(k)+D$. Now, we can infer that the proposed update method would be more stable as the gain α is decreased.

Let us derive a rough range of α to stabilize the update system. Note that the step response in Fig. 3(b) reaches the steady-state within almost one cycle. This is coincident with the fact that relay feedback methods can usually obtain a cyclic-steady-state within such a small number of cycles (2-3 cycles). Then, $Z_u(k)$ is approximately a function of $u_{ref}(k-1)+D$. Then, we can derive a qualitative condition for convergence as discussed below with the assumption that the approximation is valid.

$Z_y=k_p(D+u_{ref}+Z_u)$ is valid at the steady-state, where Z_y and k_p are the zero-frequency quantity of the process output and the static gain of the process, respectively. Assume $k_p>0$ without loss of generality. Then, we obtain $Z_u=Z_y/k_p-D-u_{ref}$. Here, Z_u and Z_y are negative and positive, respectively, for a positive $D+u_{ref}$ because $u_{ref}+D$ decreases the zero-frequency quantity of the process input, while it increases that of the process output. Then, $|Z_u(k+1)|$ is smaller than $|u_{ref}(k)+D|$ as shown in Fig. 3(b). So, the discrete-time overall process has the following transfer function.

$$Z_u(z)=-z^{-1}\beta\hat{u}_{ref}(z) \tag{13}$$

Where, $\hat{u}_{ref}(k)=u_{ref}(k)+D$ and $\hat{u}_{ref}(z)$ is the z-transform of $\hat{u}_{ref}(k)$. β is positive and less than 1. The proposed update method with a positive α has the following transfer function.

$$\hat{u}_{ref}(z)=\frac{\alpha z^{-1}Z_u(z)}{1-z^{-1}}+\frac{\hat{u}_{ref}(0)}{1-z^{-1}} \tag{14}$$

Then, we obtain the following closed-loop transfer function from (13) and (14).

$$Z_u(z)=\frac{-\beta\hat{u}_{ref}(0)}{1-z^{-1}+z^{-2}\alpha\beta} \tag{15}$$

It is straightforward to derive the condition of $0\le\alpha\le 1/\beta$ for a stable update from (15). This means that $\alpha=1$ will stabilize the closed-loop response because β is less than 1.

CASE STUDY

Two processes were simulated to confirm the performance of the proposed method.

Example 1) The following process is simulated to confirm the performance of the proposed approach.

$$G(s)=\frac{(zs+1)\exp(-0.3s)}{(s+1)^2} \tag{16}$$

Table 1 shows the estimated amplitude ratio and frequency corresponding to the pre-specified phase angle. As expected, the proposed method with $\tau_\alpha=3$ provides almost exact estimates for the range of $-\pi/2\le\angle G(j\omega)\le-\pi$. If a bigger τ_α is chosen and there are no numerical errors, the proposed method would provide exact estimates.

Example 2) The following process is simulated to compare the proposed approach with the previous methods of the conventional relay feedback method and Sung et al.'s method [4].

$$G(s)=\frac{\exp(-\theta s)}{s+1} \tag{17}$$

(17) is chosen because it usually shows bigger modeling errors than other high order processes for the harmonics included in the process input. Table 2 shows the estimated amplitude ratio and frequency corresponding to $\angle G(j\omega)=-\pi$. As expected, the proposed method provides the best estimates in most cases.

CONCLUSIONS

A new frequency response estimator has been proposed to solve the harmonics problem of previous approaches completely. It can remove all harmonics by using the sinusoidal signal directly, of which frequency is automatically updated to guarantee the desired phase angle. The proposed method also rejects the effects of static disturbances by adjusting the reference value of the sinusoidal signal.

ACKNOWLEDGEMENTS

This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD) (KRF-2005-041-D00178).

REFERENCES

1. K. J. Åström and T. Hägglund, *Automatica*, **20**, 645 (1984).
2. C. C. Hang, K. J. Åström and Q. G. Wang, *Journal of Process Control*, **12**, 143 (2002).
3. S. Shen, J. Wu and C. Yu, *AIChE J.*, **42**, 1174 (1996a).
4. S. W. Sung, J. H. Park and I. Lee, *Industrial & Engineering Chemistry Research*, **34**, 4133 (1995).
5. S. Shen, H. Yu and C. Yu, *Chemical Engineering Science*, **51**, 1187

Table 1. Estimated frequency responses for the second order plus time delay process of $G(s)=(zs+1)\exp(-0.3s)/(s+1)^2$

$\angle G(j\omega)$	Process (z=0.2)		Proposed (z=0.2)		Process (z=−0.2)		Proposed (z=−0.2)	
	$\lvert G(j\omega)\rvert$	ω	$\lvert G(j\omega)\rvert$	ω	$\lvert G(j\omega)\rvert$	ω	$\lvert G(j\omega)\rvert$	ω
$-\pi$	0.079	3.884	0.079	3.876	0.225	1.940	0.226	1.936
$-3\pi/4$	0.240	1.849	0.242	1.859	0.419	1.207	0.420	1.205
$-\pi/2$	0.555	0.911	0.558	0.908	0.678	0.700	0.679	0.700

Table 2. Comparisons of the proposed method with previous methods in estimating the ultimate data corresponding to $\angle G(j\omega)=-\pi$

$G(s)=\frac{\exp(-\theta)}{s+1}$	Process		Conventional [1]		Modified [4]		Proposed	
	$\lvert G(j\omega)\rvert$	ω	$\lvert G(j\omega)\rvert$	ω	$\lvert G(j\omega)\rvert$	ω	$\lvert G(j\omega)\rvert$	ω
$\theta=0.1$	0.061	16.32	0.075	16.36	0.066	16.32	0.061	16.31
$\theta=0.2$	0.118	8.444	0.143	8.537	0.125	8.468	0.118	8.440
$\theta=1.0$	0.442	2.029	0.486	2.107	0.437	2.040	0.444	2.024
$\theta=3.0$	0.774	0.819	0.746	0.856	0.792	0.821	0.776	0.818

(1996b).
6. K. K. Tan, T. H. Lee, S. Huang, K. Y. Chua and R. Ferdous, *Journal of Process Control*, **16**, 445 (2006).
7. T. H. Lee, Q. G. Wang and K. K. Tan, *IEEE Transactions on Control Systems Technology*, **3**, 330 (1995).
8. J. Crowe, M. A. Johnson and M. J. Grimble, *European Control Conference*, 1-4 September (2003).
9. J. Crowe and Johnson, *IEE Proceedings-Control Theory and Application*, **147**, 196 (2000).
10. Y. H. Kim, *J. Chem. Eng. Jap.*, **28**, 118 (1995).
11. K. K. Tan, T. H. Lee and Q. G. Wang, *AIChE J.*, **42**, 2555 (1996).
12. R. Chiang and C. Yu, *Industrial & Engineering Chemistry Research*, **32**, 90 (1993).
13. M. Friman and K. V. Waller, *Industrial & Engineering Chemistry Research*, **36**, 2662 (1997).
14. C. C. Hang, K. J. Åström and W. K. Ho, *Automatica*, **29**, 563 (1993).
15. J. H. Park, S. W. Sung and I. Lee, *Automatica*, **33**, 711 (1997).
16. J. H. Park, S. W. Sung and I. Lee, *Automatica*, **34**, 751 (1998).
17. S. Shen, J. Wu and C. Yu, *Industrial & Engineering Chemistry Research*, **35**, 1642 (1996c).
18. S. W. Sung and J. Lee, *Automatica*, **42**, 353 (2006a).
19. S. W. Sung, J. Lee, D. H. Lee, J. H. Han and Y. S. Park, *Industrial & Engineering Chemistry Research*, **45**, 4071 (2006).
20. S. W. Sung and J. Lee, *Industrial & Engineering Chemistry Research*, **45**, 4028 (2006b).
21. D. P. Atherton, *Nonlinear control engineering*, Van Nostrand Reinhold, London (1975).