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Improved fourier transform to estimate frequency responses

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Abstract−For the automatic tuning of PID controllers, a new identification method is proposed to estimate the frequency responses of the process from the activated process input and output. It can extract many more frequency responses as well as guarantee better accuracy compared with the previous describing function analysis algorithm. Also, the proposed method can be applied to the case that the initial part of the activated process input and output is periodic (cyclic-steady-state), whereas the previous method using the modified Fourier transform cannot incorporate the case.

Key words: Frequency Response, Identification, Autotuning, PID

INTRODUCTION

PID controllers are the majority of the process controllers used in industry because they give satisfactory control performance for usual processes and have good robustness to modeling errors. The tuning parameters of the PID controller have to be tuned with in-depth consideration of the process dynamics to guarantee acceptable control performance. Since the manual tuning depends on experience, it is inefficient and time-consuming. So, many studies have been focusing on the autotuning of the PID controller recently. On a demand from an operator or an external signal, the autotuner goes through the whole procedure of activating the process, identifying the model and tuning the PID controller in an automatic way.

Åström and Hägglund [1] proposed the original relay feedback identification method to obtain an approximated critical point from the relay oscillation and to tune the adjustable parameters of PID controllers automatically. Their idea has been applied in many areas [2-7].

The describing function analysis [1] has been widely used to identify the ultimate information from the relay feedback signal. It is derived on the basis of the Fourier series of the relay feedback signal, where only the fundamental term of the series is considered. In general, the obtained ultimate frequency and gain have good accuracy for usual processes [8]. However, since the square signal is approximated by one sinusoidal signal, it is always possible for high-order harmonic terms to be dominant. Sung et al. [9] proposed a modified relay feedback method to obtain the ultimate data set more accurately. Here, they used a two-level signal instead of the one-level signal of the original relay feedback to reduce the high-order harmonic terms. Also, Shen et al. [10] used a saturation-relay feedback method to reduce the high-order harmonic terms. Lee et al. [11] applied the describing function analysis to the integrals of the relay feedback signal, resulting in significant reduction of the harmonics. Though the above-mentioned approaches have contributed to improving accuracy of the estimates, they cannot still remove completely the estimation errors, originated from the describing function approximation.

Sung and Lee [12] proposed the Fourier analysis method to overcome the problems of the describing function analysis method. It estimates the exact frequency response data of the process without any approximations. But, it can provide only one or two frequency response data because it uses only the process data of the cyclic-steady-state.

Sung and Lee [13] and Ma and Zhu [14] proposed the most advanced nonparametric identification algorithm using a modified Fourier transform. It can provide many more frequency response data compared with the previous describing function analysis algorithm [1] and the Fourier analysis algorithm [12]. Also, the estimates are exact.

The previous method of the modified Fourier transform can be applied only to the cases that the initial part and the final part of the activated process data are steady-state and cyclic-steady-state, respectively, or both the initial part and the final part are steady-state. This means that it cannot incorporate the case that both the initial part and the final part are cyclic-steady-state. For this reason, we propose a new frequency model identification method to incorporate more various situations, applicable to all the three cases: the initial steady-state and the final cyclic-steady-state, the initial cyclic-steady-state and the final cyclic-steady-state, the initial steady-state and the final steady-state.

PROCESS ACTIVATION

The proportinal controller and relay feedback method have been widely used to activate the process. Consider the three types of the activated process input and output in Fig. 1. Both the initial part and the final part of Fig. 1(a) are steady-state. The process is activated by a proportional controller. Fig. 1(b) shows the activated process input and output of which the initial part is steady-state and the final part is cyclic-steady-state. A biased-relay feedback method is used to activate the process. Fig. 1(c) is the process input and output of which the initial part is cyclic-steady-state and the final part is also cyclic-steady-state. The process is activated by the relay feedback method proposed by Park et al. [15]. The periods of the initial part and the final part are the same.

Several algorithms such as the describing function analysis, Fourier analysis and modified Fourier transform have been used to ex-

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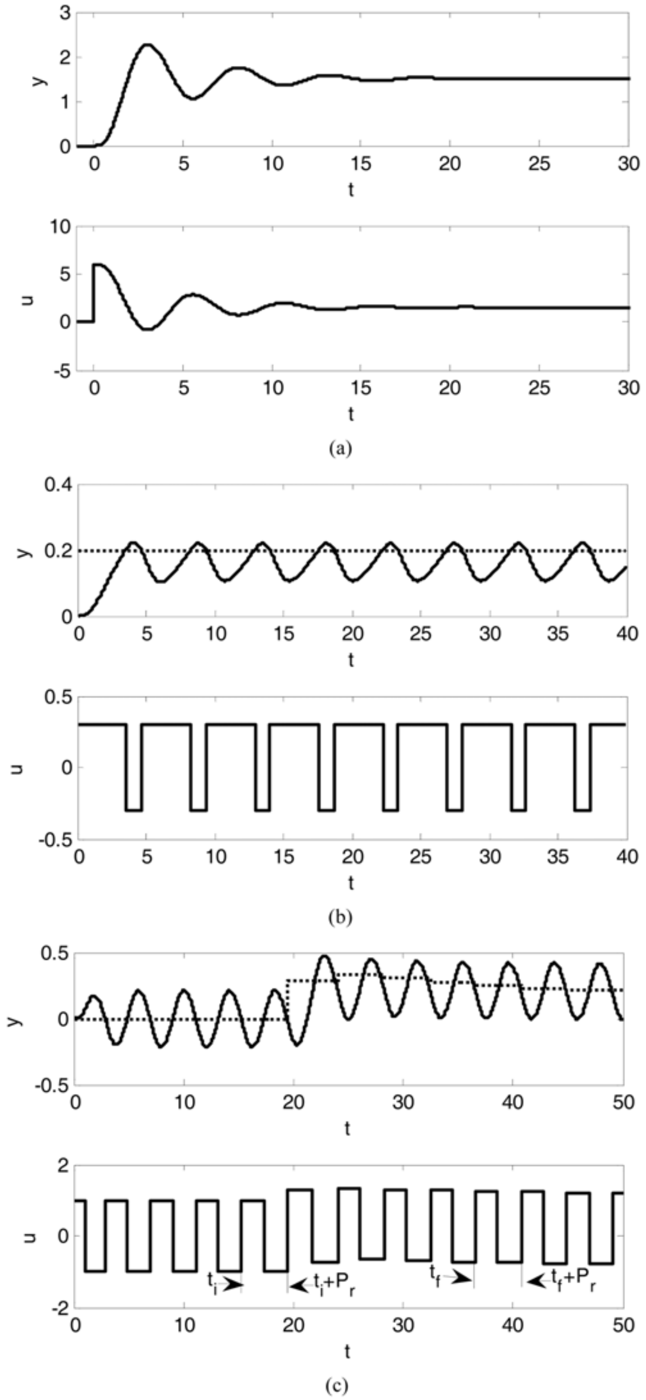

Fig. 1. Three types of process activation.

tract the frequency responses of the process from the activated process input and output. The describing function analysis can be applied to Fig. 1(b) and Fig. 1(c). But, only one or two frequency responses can be obtained by the describing function analysis. Also, the estimated frequency responses are not accurate due to the effects of the harmonics. A Fourier analysis can estimate exact frequency responses from Fig. 1(b) and Fig. 1(c), but it can extract only one or two data. The modified Fourier transforms can identify a wide range of frequency responses (theoretically, all frequency responses) from Fig. 1(a) and Fig. 1(b). The estimates are exact. But, it cannot be applied to Fig. 1(c). Table 1 summarizes the previous approaches. It is clear that no previous approaches can estimate a wide range of frequency responses for the activated process input and output of Fig. 1(c).

In this research, a new estimation method is developed to overcome the limitations of the previous approaches. The proposed method can estimate a wide range of frequency responses from any cases of Fig. 1(a), 1(b) and 1(c). Also, the estimates are exact.

PROPOSED IDENTIFICATION METHOD

Consider the activated process input and output of Fig. 1(c). The initial part from t_i to t_i+p_r is cyclic-steady-state, and the final part from t_f to t_f+p_r is also cyclic-steady-state of which the period is p_r. Let u(t) and y(t) denote the process input and the process output in Fig. 1(c). Meanwhile, we can repeat the initial part from t_i to t_i+p_r to obtain the process input and output as shown in Fig. 2. Let $u_{ref}(t)$ and $y_{ref}(t)$ denote the process input and the process output of Fig. 2.

Assume that the dynamics of the process is described by the following linear time-invariant system.

$$G(s)=\frac{b_m s^m+b_{m-1}s^{m-1}+\ldots+b_1 s+b_0}{a_n s^n+a_{n-1}s^{n-1}+\ldots+a_1 s+1} \tag{1}$$

It is equivalent to the following differential equation.

$$\begin{aligned} a_n\frac{d^n y(t)}{dt^n}+a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}}+\ldots+a_1\frac{dy(t)}{at}+y(t) \\ =b_m\frac{d^m u(t)}{dt^m}+b_{m-1}\frac{d^{m-1}u(t)}{dt^{m-1}}+\ldots+b_1\frac{du(t)}{dt}+b_0 u(t) \end{aligned} \tag{2}$$

Here, u(t) and y(t) are the process input and the process output, respectively. It should be noted that (2) is also valid for u(t) and y(t) of Fig. 1(c) as well as $u_{ref}(t)$ and $y_{ref}(t)$ of Fig. 2. That is, (3) is valid also.

$$a_n\frac{d^n y_{ref}(t)}{dt^n}+a_{n-1}\frac{d^{n-1}y_{ref}(t)}{dt^{n-1}}+\ldots+a_1\frac{dy_{ref}(t)}{at}+y_{ref}(t)$$

Table 1. Summary of previous approaches

Algorithms / Applications	DFA[a]	FA[b]	MFT[c]
Initial steady-state and final steady-state	Not applicable	Not applicable	Applicable
Initial steady-state and final cyclic-steady-state	Applicable	Applicable	Applicable
Initial cyclic-steady-state and final cyclic-steady-state	Applicable	Applicable	Not applicable
Number of estimated frequency responses	One or two	One or two	Theoretically all
Accuracy	Approximated	Exact	Exact

[a]DFA: Describing function analysis, [b]FA: Fourier analysis, [c]MFT: Modified fourier transform

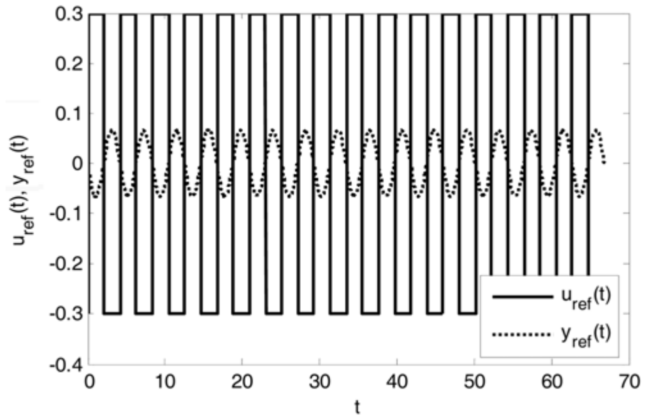

Fig. 2. The repeated signal of the initial part from t_i to t_i+p_r.

$$=b_m\frac{d^m u_{ref}(t)}{dt^m}+b_{m-1}\frac{d^{m-1}u_{ref}(t)}{dt^{m-1}}+\ldots+b_1\frac{du_{ref}(t)}{dt}+b_0u_{ref}(t) \quad (3)$$

Then, we obtain (4) by subtracting (3) from (2).

$$a_n\frac{d^n\bar{y}(t)}{dt^n}+a_{n-1}\frac{d^{n-1}\bar{y}(t)}{dt^{n-1}}+\ldots+a_1\frac{d\bar{y}(t)}{at}+\bar{y}(t)$$
$$=b_m\frac{d^m\bar{u}(t)}{dt^m}+b_{m-1}\frac{d^{m-1}\bar{u}(t)}{dt^{m-1}}+\ldots+b_1\frac{d\bar{u}(t)}{dt}+b_0\bar{u}(t) \quad (4)$$

Here, $\bar{u}(t)=u(t)-u_{ref}(t)$ and $\bar{y}(t)=y(t)-y_{ref}(t)$ are the deviated process input and the deviated process output, respectively. Then, the initial part of $\bar{u}(t)$ and $\bar{y}(t)$ from t_i to t_i+p_r are zero-steady-state and the final part of $\bar{u}(t)$ and $\bar{y}(t)$ from t_f to t_f+p_r are cyclic-steady-state of which the period is p_r.

Let us define the following transform.

$$\bar{y}_t(s)=\int_0^t \exp(-s\tau)\bar{y}(\tau)d\tau \quad (5)$$

If the initial state of system is zero and steady-state, the following is easily proven.

$$\left\{\frac{d^n\bar{y}}{dt^n}\right\}_t(s)=\int_0^t\exp(-st)\frac{d^n\bar{y}(t)}{dt^n}dt$$
$$=s\left\{\frac{d^{n-1}\bar{y}}{dt^{n-1}}\right\}_t(s)+\exp(-st)\frac{d^{n-1}\bar{y}(t)}{dt^{n-1}} \quad (6)$$

By applying (5) and (6) to (4), we can obtain the following equation.

$$\mathrm{den}(s)\bar{y}_t(s)+\exp(-st)\left\{\frac{(\mathrm{den}(s)-1)}{s}\bar{y}(t)+A_1(s)\frac{d\bar{y}(t)}{dt}\right.$$
$$\left.+\ldots+A_{n-1}(s)\frac{d^{n-1}\bar{y}(t)}{dt^{n-1}}\right\}=\mathrm{num}(s)\bar{u}_t(s)$$
$$+\exp(-st)\left\{\frac{(\mathrm{num}(s)-b_0)}{s}\bar{u}(t)+B_1(s)\frac{d\bar{u}(t)}{dt}\right.$$
$$\left.+\ldots+B_{m-1}(s)\frac{d^{m-1}\bar{u}(t)}{dt^{m-1}}\right\} \quad (7)$$

$$\mathrm{den}(s)=a_ns^n+a_{n-1}s^{n-1}+\ldots+a_1s+1 \quad (8)$$

$$\mathrm{num}(s)=b_ms^m+b_{m-1}s^{m-1}+\ldots b_1s+b_0 \quad (9)$$

Here, A(s) and B(s) are time-independent constants. Then, (7)-(9) can be rewritten as follows.

$$s\times\mathrm{den}(s)\bar{y}_t(s)\exp(st)+\mathrm{den}(s)\bar{y}(t)-\bar{y}(t)+sA(s,t)$$
$$=s\times\mathrm{num}(s)\bar{u}_t(s)\exp(st)+\mathrm{num}(s)\bar{u}(t)-b_0\bar{u}(t)+sB(s,t) \quad (10)$$

$$A(s,t)=A_1(s)\frac{d\bar{y}(t)}{dt}+\ldots+A_{n-1}\frac{d^{n-1}\bar{y}(t)}{dt^{n-1}} \quad (11)$$

$$B(s,t)=B_1(s)\frac{d\bar{u}(t)}{dt}+\ldots+B_{m-1}\frac{d^{m-1}\bar{u}(t)}{dt^{m-1}} \quad (12)$$

Because $\bar{u}(t)$ and $\bar{y}(t)$ are periodic after t_f, the integrals of the derivatives ($d^i\bar{y}(t)/dt^i$, $d^i\bar{u}(t)/dt^i$, i=1, 2, 3, …) from t_f to t_f+p_r are zero. For the detailed proof, refer to [13]. Then, the following equations are obtained.

$$\int_{t_f}^{t_f+p_r}\left\{A(s,t)=A_1(s)\frac{d\bar{y}(t)}{dt}+\ldots+A_{n-1}\frac{d^{n-1}\bar{y}(t)}{dt^{n-1}}\right\}dt=0 \quad (13)$$

$$\int_{t_f}^{t_f+p_r}\left\{B(s,t)=B_1(s)\frac{d\bar{u}(t)}{dt}+\ldots+B_{m-1}\frac{d^{m-1}\bar{u}(t)}{dt^{m-1}}\right\}dt=0 \quad (14)$$

$$\int_{t_f}^{t_f+p_r}\bar{y}(t)dt=b_0\int_f^{t_f+p_r}\bar{u}(t)dt \quad (15)$$

So, we can obtain (16) by integrating (10) from t_f to t_f+p_r.

$$G(s)=\frac{b_ms^m+\ldots+b_0}{a_ns^n+\ldots+1}=\frac{s\int_{t_f}^{t_f+p_r}\bar{y}_t(s)\exp(st)dt+\int_{t_f}^{t_f+p_r}\bar{y}(t)dt}{s\int_{t_f}^{t_f+p_r}\bar{u}_t(s)\exp(st)dt+\int_{t_f}^{t_f+p_r}\bar{u}(t)dt} \quad (16)$$

In (16), $s\int_f^{t_f+p_r}\bar{y}_t(s)\exp(st)dt$ can be rewritten to (17) by integration by parts.

$$s\int_f^{t_f+p_r}\bar{y}_t(s)\exp(st)dt=\exp(st)\int_o^t\exp(-s\tau)\bar{y}(\tau)d\tau\Big|_{t_f}^{t_f+p_r}-\int_{t_f}^{t_f+p_r}\bar{y}(t)dt$$
$$=\exp(st_f)\Big((\exp(sp_r)-1)\int_o^{t_f}\exp(-s\tau)\bar{y}(\tau)d\tau$$
$$+\exp(sp_r)\int_{t_f}^{t_f+p_r}\exp(-s\tau)\bar{y}(\tau)d\tau\Big)$$
$$-\int_{t_f}^{t_f+p_r}\bar{y}(t)dt \quad (17)$$

Then, (16) is equal to (18).

$$G(s)=\frac{(1-\exp(-sp_r))\int_o^{t_f}\exp(-s\tau)\bar{y}(\tau)d\tau+\int_{t_f}^{t_f+p_r}\exp(-s\tau)\bar{y}(\tau)d\tau}{(1-\exp(-sp_r))\int_o^{t_f}\exp(-s\tau)\bar{u}(\tau)d\tau+\int_{t_f}^{t_f+p_r}\exp(-s\tau)\bar{u}(\tau)d\tau} \quad (18)$$

By substituting $i\omega_j$, j=1, 2, …, n for s in (18), (19) is obtained.

$$G(i\omega_j)=\frac{(1-\exp(-i\omega_jp_r))\int_o^{t_f}\exp(-i\omega_j\tau)\bar{y}(\tau)d\tau+\int_{t_f}^{t_f+p_r}\exp(-i\omega_j\tau)\bar{y}(\tau)d\tau}{(1-\exp(-i\omega_jp_r))\int_o^{t_f}\exp(-i\omega_j\tau)\bar{u}(\tau)d\tau+\int_{t_f}^{t_f+p_r}\exp(-i\omega_j\tau)\bar{u}(\tau)d\tau},\quad j=1,2,\ldots,n \quad (19)$$

Here, $\bar{u}(t)=u(t)-u_{ref}(t)$ and $\bar{y}(t)=y(t)-y_{ref}(t)$. $u_{ref}(t)$ and $y_{ref}(t)$ are the repeated signals of the initial part of Fig. 1(c) as shown in Fig. 2. Now, we can calculate a wide range of frequency response data for the specified n frequencies of $i\omega_j$, j=1, 2, …, n from the activated process input and output of Fig. 1(c).

It should be noted that (19) is also applicable to Fig. 1(a) and Fig. 1(b). For the activated process input and output of Fig. 1(b), the period of the initial part of Fig. 1(b) can be assumed to be p_r because the initial part is constant. Here, p_r is the period of the final part of Fig. 1(b). And, $u_{ref}(t)$ and $y_{ref}(t)$ are constant because the initial part is steady-state. So, (19) is applicable to Fig. 1(b), where, p_r is the period of the final part and $u_{ref}(t)$ and $y_{ref}(t)$ are the process input and output of the initial part.

Without loss of generality, the period p_r of (19) can be assumed to be any small value for the activated process input and output of Fig. 1(a) because the initial and final parts are constant. Then, (19) is applicable to Fig. 1(a), where, p_r is any small value and $u_{ref}(t)$ and $y_{ref}(t)$ are the input and output of the initial part.

SIMULATIONS

Consider the following third-order plus time delay process:

$$G(s) = \frac{\exp(-0.1s)}{(s+1)^3} \tag{20}$$

We activated the process using the relay feedback method proposed by Park et al. [15] as shown in Fig. 3(a). Fig. 3(b) shows the estimated frequency responses by the proposed method. It demonstrates that the proposed method provides the exact frequency response data of the process. Fig. 4(a) corresponds to the activated process input and output when the measured process output data are contaminated by uniformly distributed random noises between –0.01 and 0.01. As shown in Fig. 4(b), the proposed method shows the acceptable robustness of the proposed method to the measurement noises.

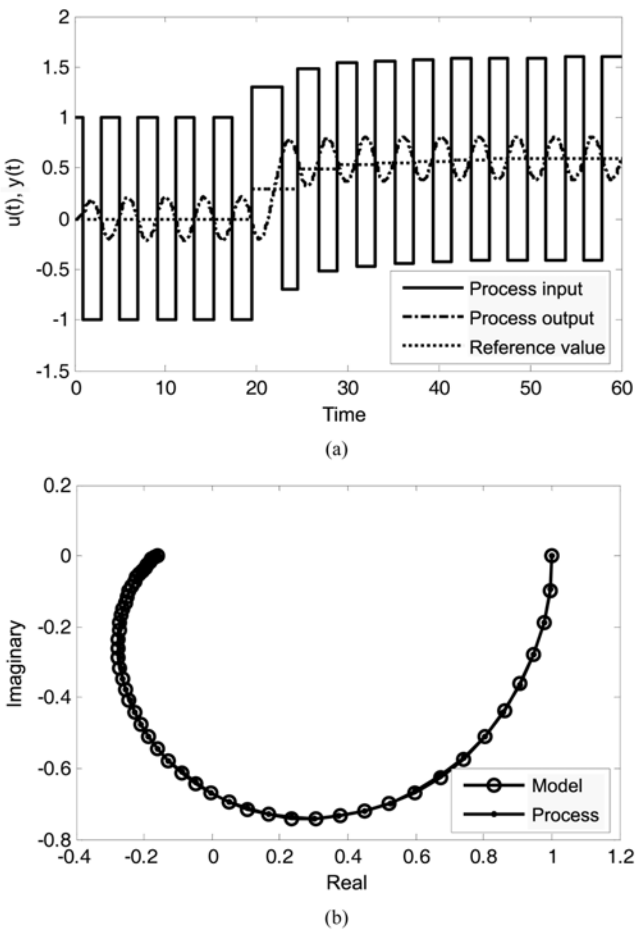

Fig. 3. (a) Process activation, (b) identified frequency response.

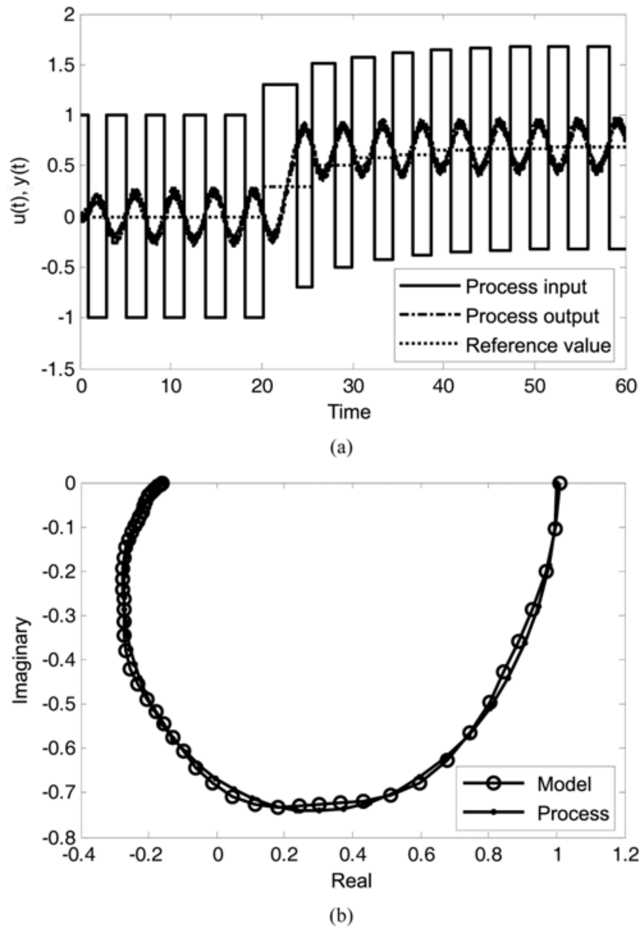

Fig. 4. (a) Process activation in the case of measurement noises, (b) identified frequency response.

CONCLUSIONS

A new method is proposed to estimate a wide range of frequency responses of the process. The proposed method provides exact estimates and it can incorporate all the cases of the initial steady-state and the final cyclic-steady-state, the initial cyclic-steady-state and the final cyclic-steady-state, the initial steady-state and the final steady-state. Simulation study confirms that the proposed method is applicable to the case that the previous approaches cannot incorporate and it shows acceptable robustness to measurement noises.

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