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A new predictive PID controller for the processes with time delay

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Abstract−A new proportional-integral-derivative (PID) controller is proposed based upon a simplified generalized predictive control (GPC) control law. The tuning parameters of the proposed predictive PID controller are obtained from the simplified GPC control law for the 1^{st}-order and 2^{nd}-order processes with time delays of integer and non-integer multiples of the sampling time. The internal model technique is employed to compensate the effect of time delay of the target process. The predictive PID controller is equivalent to the PI controller when the target process is 1^{st}-order and to the PID controller when the target process is an integrating process. The performance of the proposed predictive PID controller is almost the same as that of the simplified GPC. The main advantage of the proposed control scheme over other control methods is the ease of tuning and operation.

Key words: Predictive PID, Simplified GPC, PID Tuning Method, Discrete PID Control Law

INTRORUCTION

Numerous advanced control techniques are developed and utilized in various control areas. But proportional-integral-derivative (PID) controllers are still the choice of most industrial companies because of the many advantages such as simplicity of implementation, robustness, wide applicability and familiarity of plant engineers [1]. It is obvious that the parameters of PID controllers should be tuned according to the process dynamics in order to maintain acceptable control performance. But most PID controllers implemented in many industries are tuned by experienced engineers using the trial-and-error method. This tuning method requires considerable time and cost. In particular, for the control of a process with time delay or showing nonminimum phase behavior, the PID controller should be retuned adaptively to maintain the stability of the control system. Therefore a tuning method is required which determines on-line optimal PID parameters based on the input and output operation data. Clarke et al. [2,3] presented the generalized predictive control (GPC) scheme which predicts the process outputs over the time range greater than the maximum time delay of the process. The GPC method, also known as a self-tuning controller, has the capability of controlling nonminimum phase or unstable processes by employing analytical solution procedures. But the large amount of cost and time as well as the non-familiarity of the process engineers to the complicated control method limit the application of the GPC scheme.

To overcome these difficulties and maintain typical advantageous characteristics of PID and GPC methods, many design methods for PID controllers by using the GPC strategy have been proposed so far. Miller et al. [4,5] presented a predictive PID controller based on the GPC with steady state weighting. Kwok et al. [6] suggested the use of the augmented PID control scheme with multiple PID blocks to apply the GPC method. Moradi et al. [7] defined a PID-type control structure which predicts the process outputs and recalculates new future set points. This type of the predictive PID controller maintains the basic structure of the PID scheme familiar to the process engineers and does not require additional software or hardware for implementation to be cost effective.

In the present work a new tuning method for the predictive PID controllers is proposed based on the simplified GPC scheme. Here the simplified GPC is defined as the precomputed GPC suggested by Bordons et al. [8]. CARIMA (controlled auto-regressive integrated moving average) process model is used so that the controller includes integral action in order to obtain offset-free closed-loop responses even for the random load disturbances. For the compensation of time delay we used the internal model G_{MP} which provides multistep long range predictions. The simplified GPC control law is derived and compared with the discrete PID control law for a 1^{st}-order process with time delay and an integrating process where the time delay may be integer and non-integer multiples of the sampling time. From this procedure we could extract tuning rules which represent the PID tuning parameters in terms of the coefficients of the simplified GPC scheme.

1. The Discrete PID Control Law

The conventional PID control law can be represented as

$$u(t_c)=K_{Pc}e(t_c)+K_{Ic}\int_0^{t_c}e(t_c)dt_c+K_{Dc}\frac{de(t_c)}{dt_c} \tag{1}$$

where $e(t_c)$ is the control error, K_{Pc} is the proportional gain, K_{Ic} is the integral constant and K_{Dc} is the derivative constant. Conversion of Eq. (1) into the discrete form with the sampling time T gives

$$u(t)=K_Pe(t)+K_I\sum_{i=0}^{t}e(i)+K_D[e(t)-e(t-1)] \tag{2}$$

where

$$t=nT \text{ (T: sampling time)}, K_P=K_{Pc}, K_I=K_{Ic}T, K_D=\frac{K_{Dc}}{T}, e(t)=w(t)-y(t)$$

The incremental control law is determined by applying the differ-

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encing operator to the control output.

$$\Delta u(t)=[(K_P+K_I+K_D)+(-K_P-2K_D)q^{-1}+(K_D)q^{-2}]e(t) \tag{3}$$

To prevent abrupt control actions due to the sudden change in the set point the proportional and derivative actions can be removed from Eq. (3) to give the SP-on-I (set point on integral) PID form:

$$\Delta u(t)=K_I w(t)-[(K_P+K_I+K_D)+(-K_P-2K_D)q^{-1}+(K_D)q^{-2}]y(t) \tag{4}$$

or

$$\Delta u(t)=G_{Cw}w(t)-G_{Cy}y(t) \tag{5}$$

where

$$G_{Cw}=K_I,\ G_{Cy}=(K_P+K_I+K_D)+(-K_P-2K_D)q^{-1}+(K_D)q^{-2}$$

2. The Multistep Long Range Predictor, G_{MP}

The PID control law given by Eq. (5) is not in the suitable form to perform PID tuning based on the simplified GPC for the processes with time delay. To compensate time delay and to achieve tuning of the predictive PID, the performance of which is equivalent to the simplified GPC, we introduced the internal model G_{MP}. G_{MP} is a multistep long-range predictor presented by Miller et al. [4,5]. Fig. 1 shows the block diagram of the stochastic predictive PID control loop. The stochastic predictive PID controller can be represented in the form of a model-based PID as shown in Fig. 2 where the set point filter has the effect of removing the proportional and derivative actions for the change in the set point.

From the block diagram we can rewrite Eq. (5) as

$$\Delta u(t)=G_{Cw}w(t)-G_{Cy}y(t)-G_{MP}G_{Cy}\Delta u(t-1) \tag{6}$$

where

$$G_{MP}=\frac{1}{G_{Cy}}\sum_{k=0}^{d+1}(\text{coefficient of } \Delta u(t-1-k))z^{-k}$$

The parameters of the predictive PID controller can be determined from the comparison of the simplified GPC control law with the coefficients of Eq. (6) for each process.

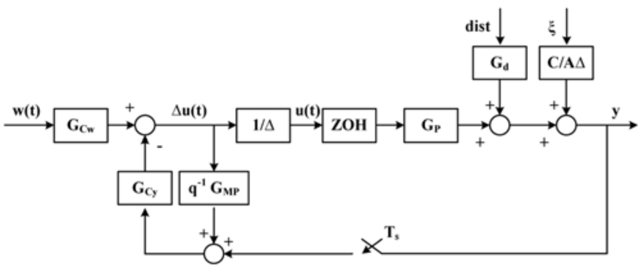

Fig. 1. Block diagram of the stochastic predictive PID control loop.

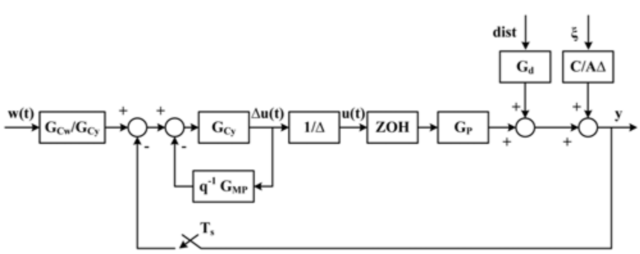

Fig. 2. Alternative diagram of the stochastic predictive PID control loop.

3. The PI/PID Form of Simplified GPC

For the 1st-order process with time delay (Eq. (7)) and the integrating process (Eq. (8)) we first derive the simplified GPC control law followed by determination of the parameters of the predictive PID controller.

$$G(s)=\frac{K}{\tau s+1}e^{-\tau_d s} \tag{7}$$

$$G(s)=\frac{K}{s(\tau s+1)}e^{-\tau_d s} \tag{8}$$

For the 1st-order and 2nd-order processes with time delay, we considered two cases:

Case I: the time delay is an integer multiple of the sampling time (i.e., $\tau d=dT$, d: integer).

Case II: the time delay is non-integer multiple of the sampling time (i.e., $\tau d=dT+\varepsilon T$, $0<\varepsilon<1$, d: integer).

3-1. 1st-Order Model with Time Delay

Case I:

When the time delay is an integer multiple of the sampling time the discrete form of Eq. (7) is simply given by

$$G(z^{-1})=\frac{bz^{-1}}{1-az^{-1}}z^{-d}=\frac{Y(z^{-1})}{U(z^{-1})} \tag{9}$$

where

$$a=e^{-T/\tau},\ b=K(1-a),\ d=\frac{\tau_d}{T}$$

By assuming random disturbances and applying CARIMA model, we have

$$\hat{y}(t+1)=(1+a)y(t)-ay(t-1)+b\Delta u(t-d)+\xi(t+1) \tag{10}$$

From Eq. (10) and for a given prediction horizon of N, we can see that future outputs are given by

$$\begin{bmatrix} \hat{y}(t+d+1|t) \\ \hat{y}(t+d+2|t) \\ \cdot \\ \cdot \\ \hat{y}(t+d+N|t) \end{bmatrix} = \begin{bmatrix} g_f^1 & 1-g_f^1 \\ g_f^2 & 1-g_f^2 \\ & \cdot \\ & \cdot \\ g_f^N & 1-g_f^N \end{bmatrix} \begin{bmatrix} y(t+d|t) \\ y(t+d-1|t) \end{bmatrix} + \begin{bmatrix} d_{fm}^1 & & & & 0 \\ d_{fm}^2 & d_{fm}^1 & & & \\ d_{fm}^3 & d_{fm}^2 & d_{fm}^1 & & \\ \cdot & \cdot & \cdot & \cdot & \\ d_{fm}^N & d_{fm}^{N-1} & \cdot & \cdot & d_{fm}^1 \end{bmatrix} \begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \cdot \\ \cdot \\ \Delta u(t+N-1) \end{bmatrix} \tag{11}$$

where

$$g_f^i=\left(\sum_{k=0}^{i}a^k\right),\ d_{fm}^i=b\left(\sum_{k=0}^{i-1}a^k\right)=bg_f^{i-1}$$

In Eq. (11), the subscript f means the 1st-order process and fm denotes the 1st-order process with the time delay of integer multiple sampling time. Using vector-matrix notations Eq. (11) can be written as

$$\hat{\underline{Y}} = G\underline{Y} + D\underline{U} \tag{12}$$

Based on Eq. (12), the cost function J is represented as

$$\begin{aligned} J(N_1, N_2) &= J(N) = (\hat{\underline{Y}} - \underline{W})^T \Lambda (\hat{\underline{Y}} - \underline{W}) + \underline{U}^T \Gamma \underline{U} \\ &= (G\underline{Y} + D\underline{U} - \underline{W})^T \Lambda (G\underline{Y} + D\underline{U} - \underline{W}) + \underline{U}^T \Gamma \underline{U} \end{aligned} \tag{13}$$

where

Λ=diag{$\delta(1)\ \delta(2) \ldots \delta(N)$}, Γ=diag{$\lambda(1)\ \lambda(2) \ldots \lambda(N)$}

Optimization of J gives

$$\underline{U} = Q_p \underline{Y} + Q_r \underline{W} \tag{14}$$

where

$Q_p = M^{-1}P$, $Q_r = M^{-1}R$, $M = D^T \Lambda D + \Gamma$, $P = -D^T \Lambda G$, $R = D^T \Lambda$

If we denote the first row of Q_p and Q_r as q_p and q_r respectively, we have

$$\Delta u(t) = q_p \underline{Y} + q_r \underline{W} \tag{15}$$

With constant reference sequence of $W = [1\ 1 \ldots 1]^T\ r(t)$, the simplified GPC control law can be written as

$$\Delta u(t) = l_{y1}\hat{y}(t+d|t) + l_{y2}\hat{y}(t+d-1|t) + l_{r1} r(t) \tag{16}$$

where

$$q_p = [l_{y1}\ \ l_{y2}],\ \ l_{r1} = \sum_{i=1}^{N} q_r(i),\ \ l_{y1} + l_{y2} + l_{r1} = 0$$

Fig. 3 shows the simplified GPC control scheme for the 1^{st}-order process with the time delay of integer multiple sampling time. To transform the simplified GPC control law (Eq. (16)) into the PID control law (Eq. (6)), we represent ŷ(t+d) and ŷ(t+d−1) in terms of y(t) and y(t−1). From Eq. (10), we have

$$\hat{y}(t+d|t) = g_f^d y(t) + (1 - g_f^d) y(t-1) + [d_{fm}^1\ \ d_{fm}^2\ \ldots\ d_{fm}^d] \begin{bmatrix} \Delta u(t-1) \\ \Delta u(t-2) \\ \cdot \\ \cdot \\ \Delta u(t-d) \end{bmatrix} \tag{17}$$

$$\hat{y}(t+d-1|t) = g_f^{d-1} y(t) + (1 - g_f^{d-1}) y(t-1) + [0\ \ d_{fm}^1\ \ d_{fm}^2\ \ldots\ d_{fm}^{d-1}] \begin{bmatrix} \Delta u(t-1) \\ \Delta u(t-2) \\ \cdot \\ \cdot \\ \Delta u(t-d) \end{bmatrix} \tag{18}$$

Substitution of Eq. (17) and (18) into Eq. (16) gives

$$\begin{aligned} \Delta u(t) &= l_{r1} r(t) + [l_{y1} g_f^d + l_{y2} g_f^{d-1}] y(t) \\ &\quad + [l_{y1}(1 - g_f^d) + l_{y2}(1 - g_f^{d-1})] y(t-1) \\ &\quad + [l_{y1} d_{fm}^1\ \ l_{y1} d_{fm}^2 + l_{y2} d_{fm}^1 \ldots\ l_{y1} d_{fm}^d + l_{y2} d_{fm}^{d-1}] \begin{bmatrix} \Delta u(t-1) \\ \Delta u(t-2) \\ \cdot \\ \cdot \\ \Delta u(t-d) \end{bmatrix} \end{aligned} \tag{19}$$

Application of the internal model G_{MP} into Eq. (19) gives

$$\begin{aligned} \Delta u(t) &= l_{r1} r(t) + [l_{y1} g_f^d + l_{y2} g_f^{d-1}] y(t) \\ &\quad + [l_{y1}(1 - g_f^d) + l_{y2}(1 - g_f^{d-1})] y(t-1) + G_{MP} G_{Cy} \Delta u(t-1) \end{aligned} \tag{20}$$

where

$$G_{MP} = \frac{1}{G_{Cy}} \begin{bmatrix} l_{y1} d_{fm}^1 + (l_{y1} d_{fm}^2 + l_{y2} d_{fm}^1) z^{-1} & \cdots \\ + (l_{y1} d_{fm}^d + l_{y2} d_{fm}^{d-1}) z^{-d+1} & \end{bmatrix}$$

From the comparison of Eq. (6) with Eq. (20) we have

$$\begin{aligned} K_P &= -[l_{y1} g_f^d + l_{y2} g_f^{d-1} + l_{r1}] \\ K_I &= l_{r1} \\ K_D &= 0 \end{aligned} \tag{21}$$

Case II:

When the time delay is not an integer multiple of the sampling time the discrete form of the 1^{st}-order model (Eq. (7)) is given by

$$G(z^{-1}) = \frac{b_0 z^{-1} + b_1 z^{-2}}{1 - az^{-1}} z^{-d} = \frac{Y(z^{-1})}{U(z^{-1})} \tag{22}$$

where

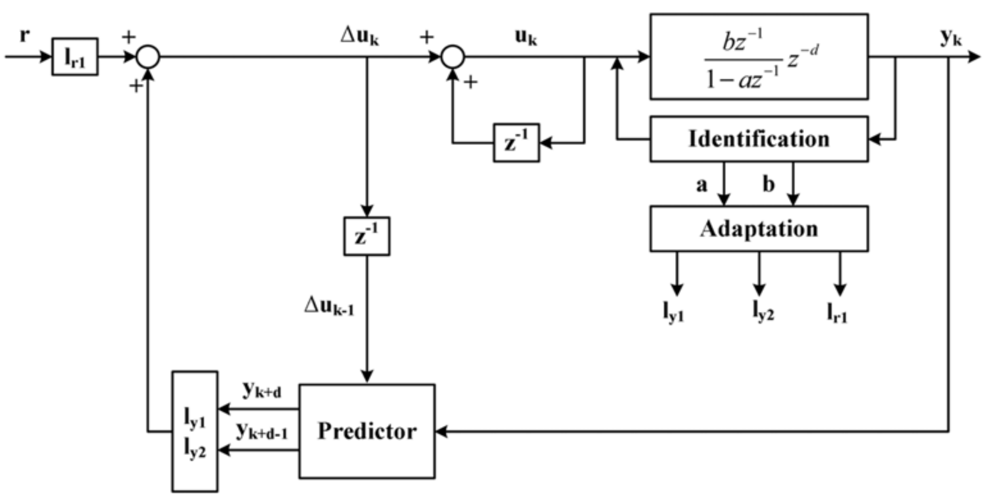

Fig. 3. The simplified GPC control scheme for the 1^{st}-order process: Case I.

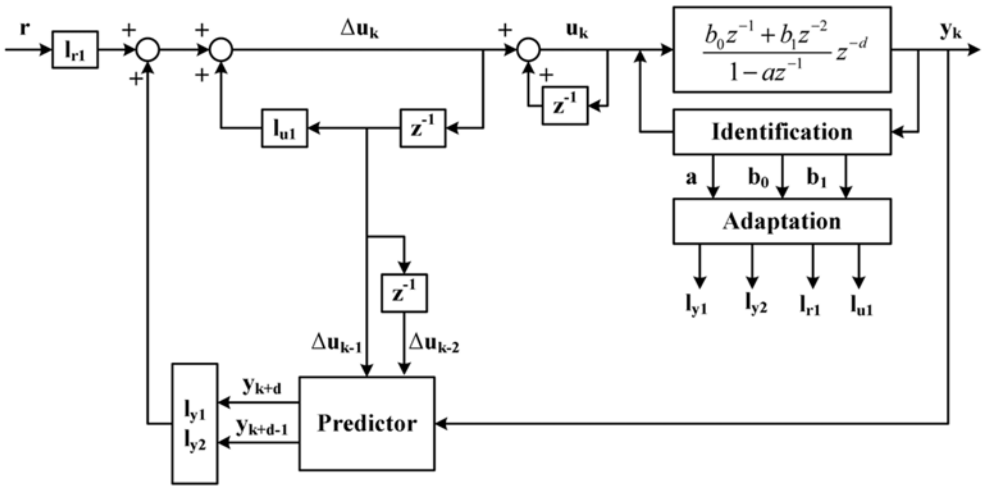

Fig. 4. The simplified GPC control scheme for the 1st-order process: Case II.

$$a = e^{-T/\tau},\ \ \alpha = \frac{a(a^{-\varepsilon} - 1)}{1 - a},\ \ b_0 = K(1-a)(1-\alpha),\ \ b_1 = K(1-a)\alpha$$

Following a similar procedure as above, the simplified GPC control law can be written as

$$\Delta u(t) = l_{y1}\hat{y}(t+d|t) + l_{y2}\hat{y}(t+d-1|t) + l_{r1}r(t) + l_{u1}\Delta u(t-1) \tag{23}$$

Fig. 4 shows the simplified GPC control scheme for the 1st-order process with the time delay of non-integer multiple sampling time. Again, we represent $\hat{y}(t+d)$ and $\hat{y}(t+d-1)$ in terms of $y(t)$ and $y(t-1)$ in order to transform the simplified GPC control law (Eq. (23)) into the PID control law (Eq. (6)):

$$\begin{aligned}\Delta u(t) = {} & l_{r1}r(t) + [l_{y1}g_f^d + l_{y2}g_f^{d-1}]y(t)\\ & + [l_{y1}(1-g_f^d) + l_{y2}(1-g_f^{d-1})]y(t-1) + G_{MP}G_{Cy}\Delta u(t-1)\end{aligned} \tag{24}$$

where

$$g_f^i = \left(\sum_{k=0}^{i} a^k\right)$$

$$d_{fn}^i = b_0\left(\sum_{k=0}^{i-1} a^k\right) + b_i\left(\sum_{k=0}^{i-2} a^k\right) = b_0 g_f^{i-1} + b_1 g_f^{i-2}\ (i \ge 2),\ \ d_{fn}^1 = b_0,$$

$$h_{fn}^i = b_1\left(\sum_{k=0}^{i-1} a^k\right) = b_1 g_f^{i-1}$$

$$G_{MP} = \frac{1}{G_{Cy}}\left[\begin{array}{l}(l_{y1}d_{fn}^1 + l_{u1}) + (l_{y1}d_{fn}^2 + l_{y2}d_{fn}^1)z^{-1} \quad \cdots\\ + (l_{y1}d_{fn}^d + l_{y2}d_{fn}^{d-1})z^{-d+1}\\ + (l_{y1}h_{fn}^d + l_{y2}h_{fn}^{d-1})z^{-d}\end{array}\right]$$

From the comparison of Eq. (6) with Eq. (24) we have

$$\begin{aligned} & K_P = -[l_{y1}g_f^d + l_{y2}g_f^{d-1} + l_{r1}]\\ & K_I = l_{r1}\\ & K_D = 0\end{aligned} \tag{25}$$

3-2. 2nd-Order Model with Time Delay

Case I:

The discrete form of the 2nd-order integrating process model (Eq. (8)) is given by

$$G(z^{-1}) = \frac{b_0 z^{-1} + b_1 z^{-2}}{(1-z^{-1})(1-az^{-1})} z^{-d} = \frac{Y(z^{-1})}{U(z^{-1})} \tag{26}$$

where

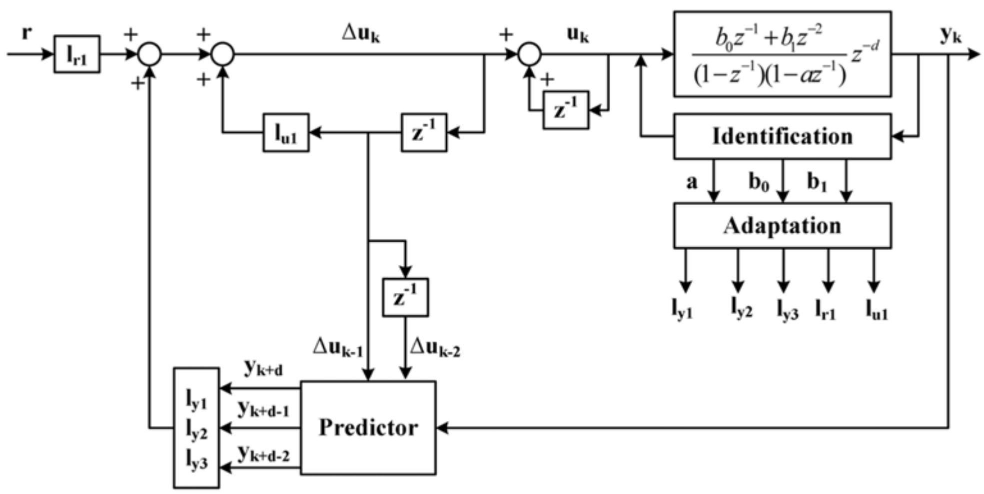

Fig. 5. The simplified GPC control scheme for the 2nd-order integrating process: Case I.

$$a=e^{-T/\tau},\ b_0=K[(a-1)\tau+T],\ b_1=K[\tau-a(T+\tau)]$$

Following a similar procedure as the 1st-order case, the simplified GPC control law is given by

$$\begin{aligned}\Delta u(t)&=l_{y1}\hat{y}(t+d|t)+l_{y2}\hat{y}(t+d-1|t)+l_{y3}\hat{y}(t+d-2|t)\\&+l_{r1}r(t)+l_{u1}\Delta u(t-1)\end{aligned}\tag{27}$$

Fig. 5 shows the simplified GPC control scheme for the 2nd-order integrating process with the time delay of integer multiple sampling time. As before, we represent $\hat{y}$(t+d), $\hat{y}$(t+d−1) and $\hat{y}$(t+d−2) in terms of y(t), y(t–1) and y(t–2) followed by application of the internal model G_{MP} in order to transform the simplified GPC control law (Eq. (27)) into the PID control law (Eq. (6)):

$$\begin{aligned}\Delta u(t)&=l_{r1}(t)+[l_{y1}g_s^{d}+l_{y2}g_s^{d-1}+l_{y3}g_s^{d-2}]y(t)\\&-\begin{bmatrix}l_{y1}(d+2ag_s^{d-1})+l_{y2}((d-1)+2ag_s^{d-2})\\+l_{y3}((d-2)+2ag_s^{d-3})\end{bmatrix}y(t-1)\\&+a[l_{y1}g_s^{d-1}+l_{y2}g_s^{d-2}+l_{y3}g_s^{d-3}]y(t-2)+G_{MP}G_{Cy}\Delta u(t-1)\end{aligned}\tag{28}$$

where

$$g_s^i=\left(\sum_{k=0}^{i}(i+1-k)a^k\right)$$

$$\begin{aligned}d_{sm}^i&=b_0\left(\sum_{k=0}^{i-1}(i-k)a^k\right)+b_i\left(\sum_{k=0}^{i-2}(i-1-k)a^k\right)\\&=b_0g_s^{i-1}+b_1g_s^{i-2}(i\geq2),\ d_{sm}^1=b_0\end{aligned}$$

$$h_{sm}^i=b_1\left(\sum_{k=0}^{i-1}(i-k)a^k\right)=b_1g_f^{i-1}$$

$$G_{MP}=\frac{1}{G_{Cy}}\begin{bmatrix}(l_{y1}d_{sm}^1+l_{u1})+(l_{y1}d_{sm}^2+l_{y2}d_{sm}^1)z^{-1}\\+(l_{y1}d_{sm}^3+l_{y2}d_{sm}^2+l_{y3}d_{sm}^1)z^{-2}\quad\cdots\\+(l_{y1}d_{sm}^d+l_{y2}d_{sm}^{d-1}+l_{y3}d_{sm}^{d-2})z^{-d+1}\\+(l_{y1}h_{sm}^d+l_{y2}h_{sm}^{d-1}+l_{y3}h_{sm}^{d-2})z^{-d}\end{bmatrix}$$

The subscript s denotes the 2nd-order process and sm denotes the 2nd-order integrating process with the time delay of integer multiple sampling time. From the comparison of Eq. (6) with Eq. (28) we have

$$\begin{aligned}K_P&=-\begin{bmatrix}l_{r1}+l_{y1}(g_s^d-ag_s^{d-1})\\+l_{y2}(g_s^{d-1}-ag_s^{d-2})+l_{y3}(g_s^{d-2}-ag_s^{d-3})\end{bmatrix}\\K_I&=l_{r1}\\K_D&=-a[l_{y1}g_s^{d-1}+l_{y2}g_s^{d-2}+l_{y3}g_s^{d-3}]\end{aligned}\tag{29}$$

Case II:

The discrete form of the 2nd-order integrating process model (Eq. (8)) is given by

$$G(z^{-1})=\frac{b_0z^{-1}+b_1z^{-2}+b_2z^{-3}}{(1-z^{-1})(1-az^{-1})}z^{-d}=\frac{Y(z^{-1})}{U(z^{-1})}\tag{30}$$

where

$$\begin{aligned}&a=e^{-T/\tau},\ b_0=K[(a^m-1)\tau+mT],\\&b_1=K[(a+1-2a^m)\tau+(1-m-am)T],\\&b_2=Ka[(a^{m-1}-1)\tau+(m-1)T],\ m=1-\varepsilon\end{aligned}$$

The simplified GPC control law is given by

$$\begin{aligned}\Delta u(t)&=l_{y1}\hat{y}(t+d|t)+l_{y2}\hat{y}(t+d-1|t)+l_{y3}\hat{y}(t+d-2|t)\\&+l_{r1}r(t)+l_{u1}\Delta u(t-1)+l_{u2}\Delta u(t-2)\end{aligned}\tag{31}$$

Fig. 6 shows the simplified GPC control scheme for the 2nd-order integrating process with the time delay of non-integer multiple sampling time. Again, we represent $\hat{y}$(t+d), $\hat{y}$(t+d−1) and $\hat{y}$(t+d−2) in terms of y(t), y(t−1) and y(t−2) followed by application of the internal model G_{MP} in order to transform the simplified GPC control law (Eq. (31)) into the PID control law (Eq. (6)):

$$\begin{aligned}\Delta u(t)&=l_{r1}(t)+[l_{y1}g_s^{d}+l_{y2}g_s^{d-1}+l_{y3}g_s^{d-2}]y(t)\\&-\begin{bmatrix}l_{y1}(d+2ag_s^{d-1})+l_{y2}((d-1)+2ag_s^{d-2})\\+l_{y3}((d-2)+2ag_s^{d-3})\end{bmatrix}y(t-1)\\&+a[l_{y1}g_s^{d-1}+l_{y2}g_s^{d-2}+l_{y3}g_s^{d-3}]y(t-2)+G_{MP}G_{Cy}\Delta u(t-1)\end{aligned}\tag{32}$$

where

$$g_s^i=\left(\sum_{k=0}^{i}(i+1-k)a^k\right)$$

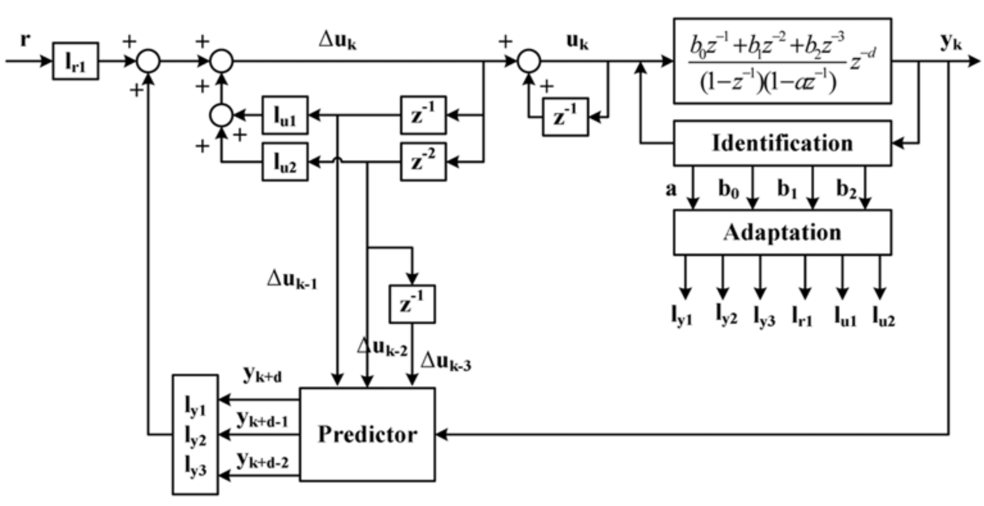

Fig. 6. The simplified GPC control scheme for the 2nd-order integrating process: Case II.

$$\mathrm{d}_{sn}^{i} = \mathrm{b}_0\left(\sum_{k=0}^{i-1}(\mathrm{i}-\mathrm{k})\mathrm{a}^k\right) + \mathrm{b}_1\left(\sum_{k=0}^{i-2}(\mathrm{i}-1-\mathrm{k})\mathrm{a}^k\right) + \mathrm{b}_2\left(\sum_{k=0}^{i-3}(\mathrm{i}-2-\mathrm{k})\mathrm{a}^k\right)$$

$$= \mathrm{b}_0\mathrm{g}_s^{i-1} + \mathrm{b}_1\mathrm{g}_s^{i-2} + \mathrm{b}_2\mathrm{g}_s^{i-3}(\mathrm{i}\geq 3),\ \ \mathrm{d}_{sn}^1 = \mathrm{b}_0,\ \ \mathrm{d}_{sn}^2 = \mathrm{b}_0(2+\mathrm{a}) + \mathrm{b}_1$$

$$l_{sn}^{i} = \mathrm{b}_1\left(\sum_{k=0}^{i-1}(\mathrm{i}-\mathrm{k})\mathrm{a}^k\right) + \mathrm{b}_2\left(\sum_{k=0}^{i-2}(\mathrm{i}-1-\mathrm{k})\mathrm{a}^k\right)$$

$$= \mathrm{b}_1\mathrm{g}_s^{i-1} + \mathrm{b}_2\mathrm{g}_s^{i-2}(\mathrm{i}\geq 2),\ \ l_{sn}^1 = \mathrm{b}_1$$

$$\mathrm{h}_{sn}^{i} = \mathrm{b}_2\left(\sum_{k=0}^{i-1}(\mathrm{i}-\mathrm{k})\mathrm{a}^k\right) = \mathrm{b}_1\mathrm{g}_f^{i-1}$$

$$\mathrm{G}_{MP} = \frac{1}{\mathrm{G}_{Cy}}\left[\begin{array}{l} (l_{y1}\mathrm{d}_{sn}^1 + l_{u1}) + (l_{y1}\mathrm{d}_{sn}^2 + l_{y2}\mathrm{d}_{sn}^1 + l_{u2})\mathrm{z}^{-1} \\ +(l_{y1}\mathrm{d}_{sn}^3 + l_{y2}\mathrm{d}_{sn}^2 + l_{y3}\mathrm{d}_{sn}^1)\mathrm{z}^{-2}\ \cdots \\ +(l_{y1}\mathrm{d}_{sn}^d + l_{y2}\mathrm{d}_{sn}^{d-1} + l_{y3}\mathrm{d}_{sn}^{d-2})\mathrm{z}^{-d+1} \\ +(l_{y1}l_{sn}^d + l_{y2}l_{sn}^{d-1} + l_{y3}l_{sn}^{d-2})\mathrm{z}^{-d} \\ +(l_{y1}\mathrm{h}_{sn}^d + l_{y2}\mathrm{h}_{sn}^{d-1} + l_{y3}\mathrm{h}_{sn}^{d-2})\mathrm{z}^{-d-1} \end{array}\right]$$

From the comparison of Eq. (6) with Eq. (32) we have

$$\mathrm{K}_P = -\left[\begin{array}{l} l_{r1} + l_{y1}(\mathrm{g}_s^d - \mathrm{a}\mathrm{g}_s^{d-1}) \\ +l_{y2}(\mathrm{g}_s^{d-1} - \mathrm{a}\mathrm{g}_s^{d-2}) + l_{y3}(\mathrm{g}_s^{d-2} - \mathrm{a}\mathrm{g}_s^{d-3}) \end{array}\right]$$
$$\mathrm{K}_I = l_{r1} \qquad (33)$$
$$\mathrm{K}_D = -\mathrm{a}[l_{y1}\mathrm{g}_s^{d-1} + l_{y2}\mathrm{g}_s^{d-2} + l_{y3}\mathrm{g}_s^{d-3}]$$

4. Simulation Results

For each case (i.e., Case I and Case II) we compared the control performance of the proposed predictive PID with that of the simplified GPC.

Fig. 7 and Fig. 8 show results of numerical simulations for the 1^{st}-order process with time delay.

For the processes given by

$$\mathrm{G}(\mathrm{s}) = \frac{2}{3\mathrm{s}+1}\mathrm{e}^{-3s} \qquad (34)$$

$$\mathrm{G}(\mathrm{s}) = \frac{2}{3\mathrm{s}+1}\mathrm{e}^{-3.05s} \qquad (35)$$

responses for the set point changes were computed by using sampling time T=1 and control horizon N=2 with the weighting parameters of λ=0.8 and δ=0.9. The proposed predictive PID (P-PID) has of the form of the PI controller and shows the control perfor-

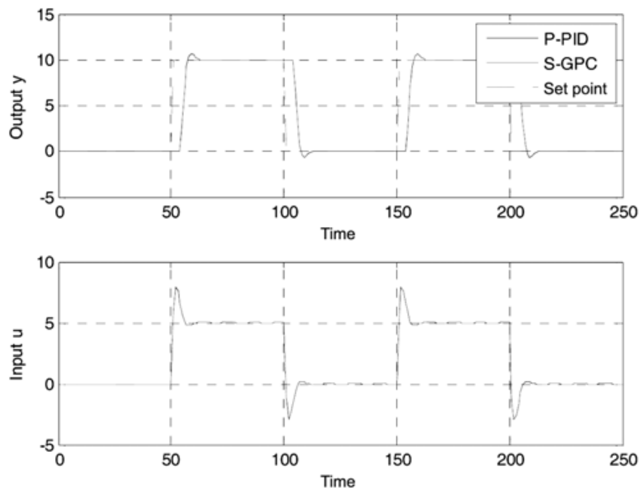

Fig. 7. Simulation results for the 1^{st}-order process: Case I.

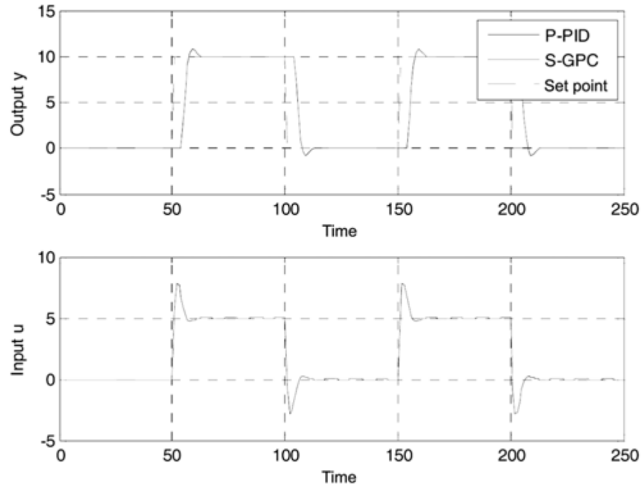

Fig. 8. Simulation results for the 1^{st}-order process: Case II.

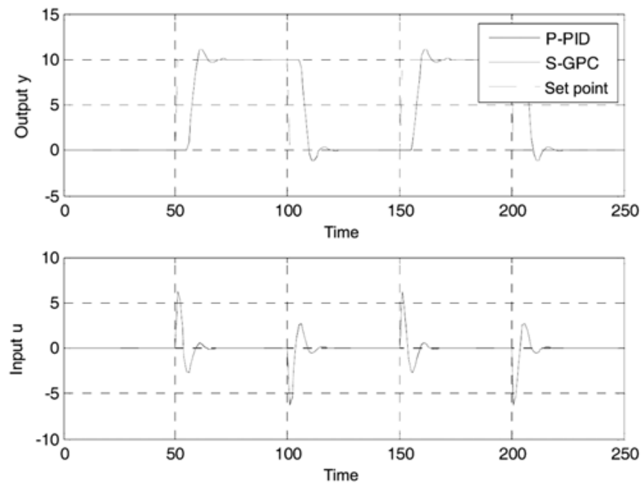

Fig. 9. Simulation results for the 2^{nd}-order process: Case I.

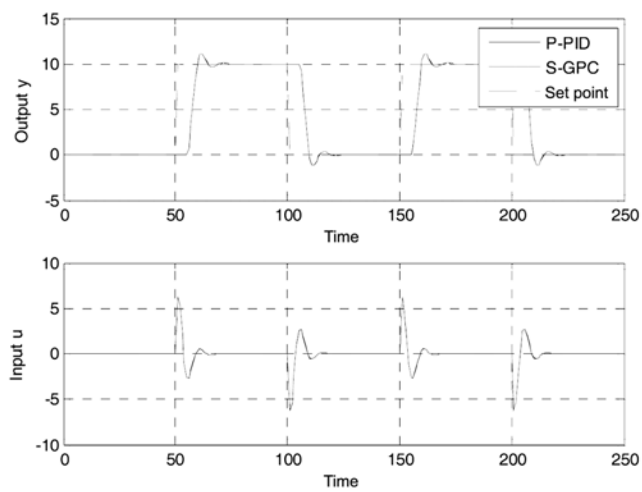

Fig. 10. Simulation results for the 2^{nd}-order process: Case II.

Table 1. Parameters of the simplified GPC control law and the proposed predictive PID

	Eq. (34)	Eq. (35)	Eq. (36)	Eq. (37)
a	0.7165	0.7165	0.8351	0.8351
b	0.5669	·	·	·
b_0	·	0.5429	0.1398	0.1265
b_1	·	0.0241	0.1316	0.1446
b_2	·	·	·	0.0003
l_{y1}	−1.2190	−1.1503	−5.0206	−5.1219
l_{y2}	0.6125	0.5723	7.0403	7.2121
l_{y3}	·	·	−2.6379	−2.7085
l_{r1}	0.6064	0.5779	0.6183	0.6183
l_{u1}	·	−0.0192	−0.4158	−0.4694
l_{u2}	·	·	·	−0.0005
K_P	1.1943	1.1340	4.2375	4.2682
K_I	0.6064	0.5779	0.6183	0.6183
K_D	0	0	10.2529	10.3670
$G_{MP} \cdot G_{Cy}$	$[-0.6911-0.8390z^{-1}$ $-0.9450z^{-2}]$	$[-0.6437-0.7888z^{-1}$ $-0.8929z^{-2}-0.0381z^{-3}]$	$[-1.1175-1.6663z^{-1}$ $-2.2924z^{-2}-2.9831z^{-3}$ $-1.6160z^{-4}]$	$[-1.1174-1.6656z^{-1}$ $-2.2922z^{-2}-2.9829z^{3}$ $-1.7977z^{-4}-0.0039z^{-5}]$

mance as good as that of the simplified GPC (S-GPC).

Fig. 9 and Fig. 10 show results of numerical simulations for the 2^{nd}-order process with time delay. For the processes given by Eq. (36) and (37), responses for the set point changes were computed by using sampling time T=1 and control horizon N=10 with the weighting parameters of λ=0.8 and δ=1.

$$G(s)=\frac{1.646}{s(5.55s+1)}e^{-4s} \tag{36}$$

$$G(s)=\frac{1.646}{s(5.55s+1)}e^{-4.05s} \tag{37}$$

The proposed predictive PID (P-PID) for the 2^{nd}-order process with time delay has the form of the PID controller and shows control performance as good as that of the simplified GPC (S-GPC). The parameters for the simplified GPC control law and the proposed predictive PID scheme are summarized in Table 1.

CONCLUSIONS

For the 1^{st}-order and 2^{nd}-order processes with time delay, a new predictive PID tuning method is proposed based upon the simplified GPC strategy. The proposed tuning method employs the internal model G_{MP} to provide multistep long-range predictions in order to compensate the time delay and to achieve the control performance equivalent with the simplified GPC scheme. The present predictive PID method has the advantage of tuning PID parameters simply by using the process transfer function without the help of additional control software or hardware. For the 1^{st}-order process with time delay, the proposed predictive PID scheme has the form of PI controller. But for the 2^{nd}-order process with time delay, the proposed predictive PID scheme has the form of PID controller. The present predictive PID scheme exhibited equivalent control performance with that of the simplified GPC strategy.

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