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RAPID COMMUNICATION

Double-command feedforward-feedback control of a nonlinear plant

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Abstract−A design approach is proposed for feedforward-feedback control systems. The basis of the proposed approach is a steady state control law which maintains the desired control output of the system and is employed as the feedforward controller. With this feedforward controller, for a wide class of systems, the stability of the control system is proved if the feedback controller is a gain with an arbitrarily high value; that is, the only limit for the feedback (transient) control command is the actuator's practical limit. Moreover, in continuous domain, there will be no overshoot. In this article, the proposed method has been applied to a catalytic stirred tank reactor (CSTR) to control the output concentration through adjusting the flow of two valves simultaneously and resulted in an excellent control response.

Key words: Process Control, Feedforward, Lyapunov, CSTR, Control Equilibrium Point

INTRODUCTION

Feedforward control commands are used extensively in control, sometimes together with feedback control commands. Some recent applications for feedforward control include power systems [1], medical engineering [2], aircraft/helicopter control [3,4], vibration and noise control [5], manufacturing [6] and robotics [7].

The most popular input to feedforward controllers is the reference or setpoint signal [1-4,6], although measurable disturbance signals may also play this role [5,8,9]. However, especially when disturbance rejection is not the main issue, there is no general methodology to determine whether a feedforward controller is useful, and if so, to find the feedforward control law. In this paper, these questions are addressed and a general methodology is offered to answer them. Based on the introduced methodology, in simulation environment, a double-command feedforward-feedback control system is designed for the outlet concentration of a non-thermic catalytic stirred tank reactor (CSTR).

DESIGN METHODOLOGY

Let us define the control equilibrium point of an under control system as below:

$$\frac{d^{(i)}e(t)}{dt^{(i)}}=0,\ i=0,\ \cdots,\ \text{system's order,} \tag{1}$$

where e(t) is the control error. In other words, at the control equilibrium point (CEP) the error is zero and remains zero.

The control input obtained by the solution of (1) *is called steady state control input.*

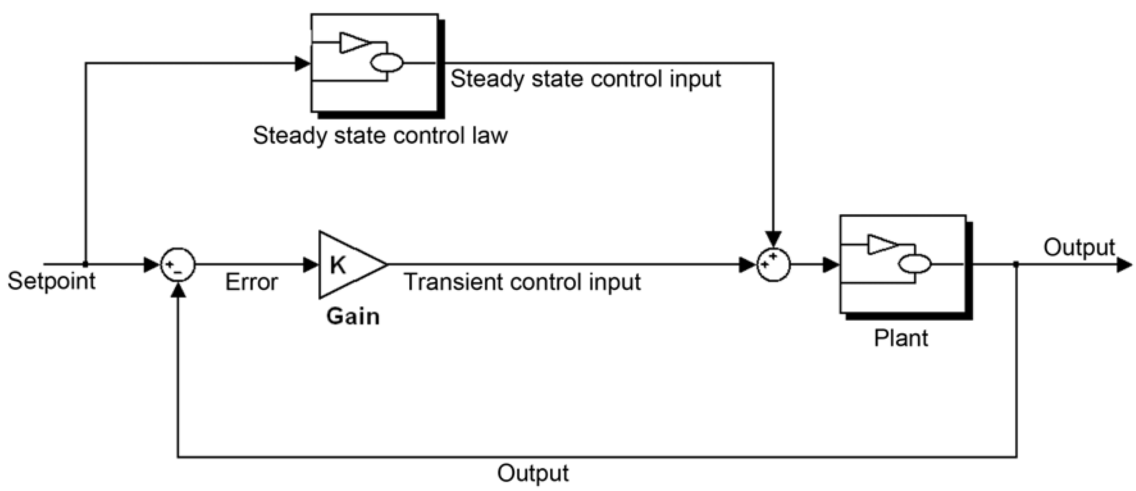

Fig. 1. Feedforward-feedback control system for a SISO system.

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In some systems, the control equilibrium point is maintained only by continuous exertion of a control input. In other words, a (steady state) control input should be consistently applied on the system to maintain a desirable control output similar to type zero linear transfer functions. In this paper, such systems are called 'generalized type zero' (GTZ) systems. Some instances of so-called GTZ systems are level control of water tank with an outlet at the bottom, all temperature control problems with the reference different from environment temperature, and position control of mechanical systems influenced by the gravity. Other systems which can retain their desired output with no control input are called 'generalized non-type zero' (GNTZ) systems.

In summary:

In GNTZ control systems: steady state control input is zero ($u_{ss}=0$).

In GTZ control systems: steady state control input is not zero ($u_{ss}\neq 0$).

In the proposed method, steady state control command which satisfies (1) is employed as the feedforward control command which is used together with a feedback (transient) control command (u_{tr}).

$$u=u_{tr}+u_{ss}. \tag{2}$$

CASE STUDY: CONCENTRATION CONTROL OF A NON-THERMIC CATALYTIC STIRRED TANK REACTOR

A diagram of the studied CSTR is shown in Fig. 2:

Two flows of liquid enter the reactor with the concentration of $C_{b1}=24.9$ (kmol/m^3) and $C_{b2}=0.1$ (kmol/m^3). The flow rates of input flows are named u_1 and u_2. The reactor outlets another flow of liquid with the concentration of C_b and the flow rate of w. The level height of liquid in the reactor (h) represents w ($w=0.2\sqrt{h}$).

A simplified mathematical model of the system, derived by mass equilibrium equations, is:

$$\dot{h}(t)=u_1(t)+u_2(t)-0.2\sqrt{h(t)}, \tag{3}$$

$$\dot{C}_b(t)=[C_{b1}-C_b(t)]\frac{u_1(t)}{h(t)}+[C_{b2}-C_b(t)]\frac{u_2(t)}{h(t)}-\frac{k_1C_b(t)}{1+k_2C_b(t)^2}, \tag{4}$$

where k_1 and k_2 are parameters regarding the resistance valves located in the path of input and output flows and h is the height of liquid level. The concentration of the outlet flow and the height of liquid level are considered as the outputs.

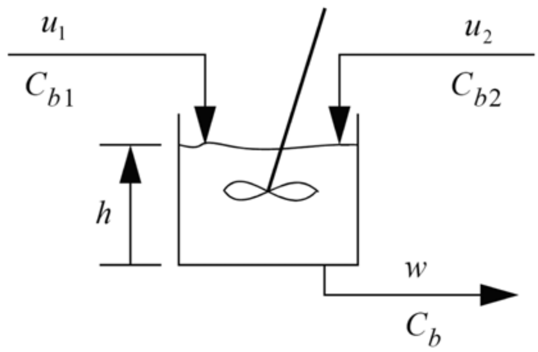

Fig. 2. A schematic of the studied CSTR [10].

CONTROL SYSTEM

The control of the outlet concentration is addressed in this paper. There are two control inputs available (u_1 and u_2) with the maximum mass flow rate of 4 liters/min.

First, the system is checked to determine if it is GTZ. To do so, the following equations are used, representing the error and its first derivative.

$$e=C_d-C_b(t), \tag{5}$$

$$\dot{e}(t)=[C_b(t)-C_{b1}]\frac{u_1(t)}{h(t)}+[C_b(t)-C_{b2}]\frac{u_2(t)}{h(t)}+\frac{k_1C_b(t)}{1+k_2C_b(t)^2}, \tag{6}$$

where C_b is the reference (desired outlet concentration). According to (4), if both control inputs equal zero, the outlet concentration still changes; as a result, a control input is needed to maintain the desired output, and the system is GTZ.

There are two variables (u_1 and u_2) in (6), so solving (5) and (6) would not output a unique couple of control commands.

If both control inputs are zero, error increases (according to (6)) and concentration decreases. Therefore, the role of the steady state control command will be to increase outlet concentration to some appropriate extent. Consequently, u_1 (with the concentration of 24.9 kmol/m^3) can play this role solely. In other words, in the steady state situation, the second flow is cut and u_1 is the only control input. In

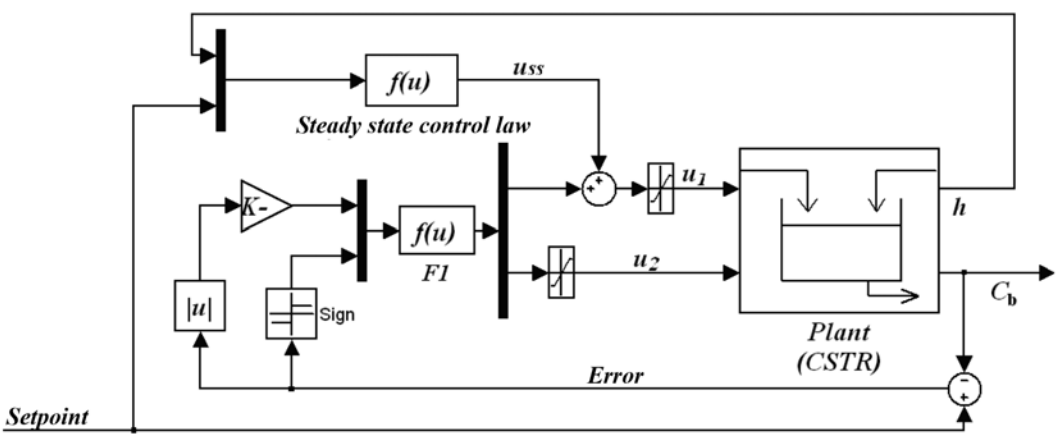

Fig. 3. Control system, with a model-based feedforward controller.

such a situation, the solution of (1), (5) and (6) results in

$$C_b=C_d, \text{ and } \begin{cases} u_{ss)1}=u_{ss}=\dfrac{h(t)}{C_{b1}-C_d}\left[\dfrac{k_1C_d}{1+k_2C_d^2}\right]. \\ u_{ss)2}=0 \end{cases} \tag{7}$$

It is observed that the steady state control command is a function of the setpoint/reference (C_d) and the height of liquid level.

Feedback controller is the modified form of an arbitrarily high gain (K) (transient control command is generated in F1, in Fig. 3). In total, (8) is the general control law:

$$\begin{cases} u_1=Ke(\text{if } e>0)+u_{ss} \\ u_2=-Ke(\text{if } e<0)+0 \end{cases} \tag{8}$$

where u_{ss} was defined in (7).

STABILITY

According to the Lyapunov theorem, for a system with the states vector of **x**, an equilibrium point and the system is globally stable if a scalar function (V(**x**)) with continuous first temporal derivative can be found so that [11]

Condition 1: V(**x**) is positive definite.

Condition 2: $\dot{V}(\mathbf{x})$ is negative definite.

Condition 3: $\|\mathbf{x}\|\to\infty \Rightarrow V(\mathbf{x})\to\infty$ [11].

For the first-order system shown in (3) and (6), x=[e h]. h is not subject to control, so it is not considered in stability analysis. The Lyapunov function is the error square or

$$V(\mathbf{x})=e^2. \tag{9}$$

With this Lyapunov function conditions 1 and 3 are evidently satisfied.

To prove condition 2, it should be proved that

$$\begin{cases} \mathbf{x}\neq 0\Rightarrow \dot{V}(\mathbf{x})<0 \\ \mathbf{x}=0\Rightarrow \dot{V}(\mathbf{x})=0 \end{cases}. \tag{10}$$

$\dot{V}(\mathbf{x})=2e\dot{e}$, so in this problem, (10) is written in the form of (11):

$$\begin{cases} e\neq 0\Rightarrow e\dot{e}<0 \\ e=0\Rightarrow e\dot{e}=0 \end{cases}. \tag{11}$$

Below equation of (11) is evident. As a result, if $e\neq 0\Rightarrow e\dot{e}<0$ is proved, the system is asymptotically stable (error square decreases continuously with no overshoot).

Two assumptions are used in this stability analysis:

A1: $\begin{cases} u_2\geq 0[\leq 0] \\ u_1\leq u_{ss}(C_b,h)[\geq u_{ss}(C_b,h)] \end{cases} \Rightarrow \dot{C}_b<0[>0]$ or $\dot{e}>0[<0]$ for the height of h and the concentration of C_b.

(C_b is used instead of C_d in (7), it means that $u_{ss}(C_b, h)$ is the control input which maintains the system at C_b)

A2: The higher (lower) setpoint, the higher (lower) u_{ss} (at the same level height)

Using these evident assumptions, the stability is proved (at any height).

Proof:

For current concentration of C_b and setpoint of C_d and the height of h:

$$e<0\Rightarrow \begin{cases} C_b>C_d[\text{see}(5)] \\ u_2>0[\text{see}(8)] \end{cases} \Rightarrow \begin{cases} u_{ss}(C_b,h)\overset{A_2}{>}u_{ss}(C_d,h)\overset{(8)}{=}u_1 \\ u_2>0 \end{cases}$$

$$\Rightarrow \begin{cases} u_1<u_{ss}(C_d,h) \\ u_2>0 \end{cases} \overset{A_1}{\Rightarrow} \dot{e}>0 \tag{12}$$

$$e>0\Rightarrow \begin{cases} C_b<C_d[\text{see}(5)] \\ u_2=0[\text{see}(8)] \end{cases} \Rightarrow \begin{cases} u_{ss}(C_b,h)\overset{A_2}{<}u_{ss}(C_d,h)\overset{(8)}{=}u_1 \\ u_2=0 \end{cases}$$

$$\Rightarrow \begin{cases} u_1>u_{ss}(C_d,h) \\ u_2=0 \end{cases} \overset{A_1}{\Rightarrow} \dot{e}<0 \tag{13}$$

Therefore $e\neq 0\Rightarrow e\dot{e}<0$.

End of proof

It was indicated that the system is stable and the squared error decreases continuously at any value of control gain (K). So, theoretically, control gain can be arbitrarily high to improve the performance. The performance is bounded only by the actuators' practical limitations (saturation functions in Fig. 3). That is, the control behavior can be described as *the quickest possible convergence to the reference (setpoint) without overshoot* (provided that assumption A1 and A2 are valid). Evident assumptions of A1 and A2 are the basis of the stability proof rather than a mathematical model, and the mentioned assumptions are valid for a wide variety of process plants, so this methodology can be extended to other process plants easily.

SIMULATION RESULTS

Having $k_1=k_2=1$, in (6), and a sampling time of 0.2 s, with the initial concentration of 20 kmol/m^3 and the initial height of 40 cm, the outlet concentration of CSTR with the proposed control system is shown in Figs. 4 and 5, with K=10 (see (8)) with two different sets of references. This response is compared to the response of a well-designed double command fuzzy controller whose merit has been well indicated [10]. Lyapunov stability was proved in continuous domain; however, in discrete domain it takes 0.2 seconds (sam-

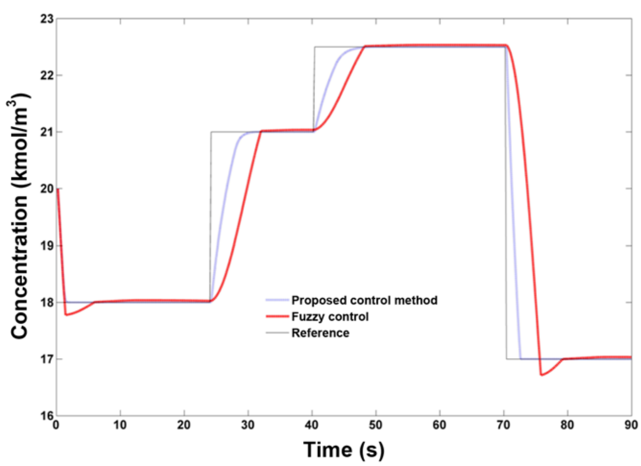

Fig. 4. The response of system with different control systems.

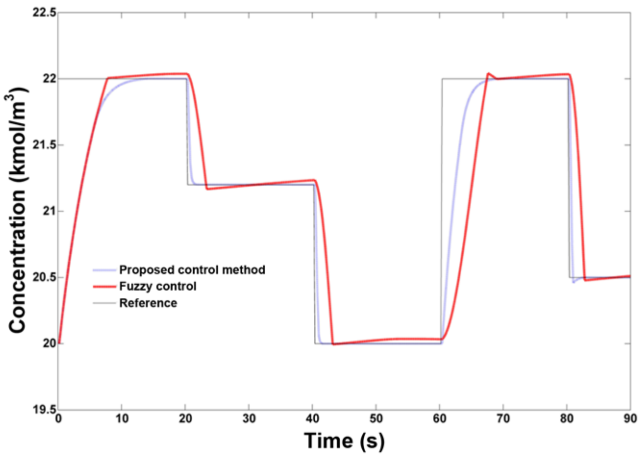

Fig. 5. Control command of different control systems.

Table 1. Results of simulation

MAE (kmol/m^3)	Fig. 4	Fig. 5
Proposed control	0.1357	0.2468
Fuzzy control	0.1849	0.4796

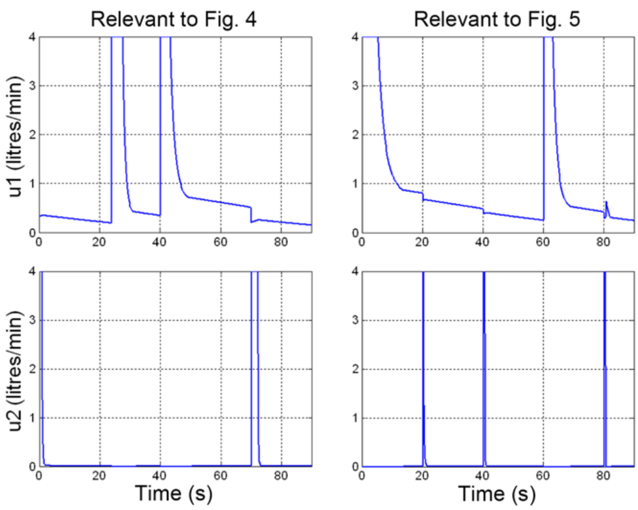

Fig. 6. Control inputs during simulation shown in Figs. 4 and 5.

pling time) for the control system to change the control command, and within this time, the setpoint might be passed. This is the reason for the tiny overshoot seen in Fig. 5.

Table 1 shows the mean of absolute error (MAE) for both offered figures.

Fig. 6 shows the control input during the operation that leads to the response shown in Figs. 4 and 5. Apart from occasions of significant setpoint change, which rarely happen in reality, the change of control input is smooth and non-oscillating.

CONCLUSION

In this article, based on the concept of 'control equilibrium point' a steady state control law is derived from the system's mathematical model. The aforementioned control law is used as a feedforward controller while the feedback controller is a proportional (P-action) controller. The proposed control system is applied on a CSTR to command both control valves simultaneously. Lyapunov asymptotic stability is proved based on two evident assumptions (not mathematical model), so the proof can be easily extended to a wide range of plants. In stability proof, there is no limit on the amplitude of control gain and it can be arbitrarily high to accelerate control response in the area of actuator's capabilities. The simulation results are outstanding, showing very quick convergence to the reference with almost no overshoot.

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NOMENCLATURE

A : assumption
C_b : outlet concentration of CSTR [kmol/m^3]
C_{b1} : the concentration of the first inlet flow to CSTR [kmol/m^3]
C_{b2} : the concentration of the second inlet flow to CSTR [kmol/m^3]
C_d : the desired outlet concentration of CSTR [kmol/m^3]
CEP : control equilibrium point
CSTR : catalytic stirred tank reactor
e : error [kmol/m^3]
F, **f** : function
GNTZ : generalized non-type-zero
GTZ : generalized type-zero
h : level height [cm]
$k_{1,2}$: CSTR parameters relevant to resistance of valves
K : feedback controller gain
MAE : mean of absolute error
t : time [s]
u : control input [liters/min]
u_1 : volume rate of high concentration input to CSTR [liters/min]
u_2 : volume rate of low concentration input to CSTR [liters/min]
V : lyapunov function
w : volume rate of CSTR output [liters/min]
x : states' vector

Subscripts

d : desired
i : numerator
ss : steady state
tr : transient

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