Korean J. Chem. Eng., **27**(5), 1384-1390 (2010)
DOI: 10.1007/s11814-010-0247-y

Dynamic modeling of the steam supply system for a paper drying cylinder

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(*Received 22 December 2009 • accepted 13 January 2010*)

Abstract−A dynamic model describing the principles of a steam supply system for a paper drying cylinder used in paper production plants was developed based on the mass and heat balances around the cylinder. The balance equations consist of sets of differential equations describing heat and mass transfer around the canvas, the web and the drying cylinder. The effects of the steam valve adjustment on steam pressure, temperature and moisture content were investigated based on the model developed. It was found that application of simple model predictive control to the operation of steam supply system is enough to achieve satisfactory drying performance in a single paper drying cylinder.

Key words: Drying Cylinder, Modeling, MPC, Steam Pressure, Moisture Contents

INTRODUCTION

A paper manufacturing process in general consists of wet and drying sections. The paper web is first dehydrated in the wet section followed by steam drying in the drying section. The paper web moving out from the pressing part of the wet section contains fibers and solids (40-45%) and the remaining water (55-60%), which is removed by heat supplied from drying cylinders in the drying section. The increasing interest in the dynamics of the paper drying process has heightened the need for more detailed descriptions of paper drying as well as relevant equipment. The primary reasons for the interest in the dynamics are the high frequency and duration of grade changes in paper production and machine start-ups. During a grade change operation, many parameters are adjusted in order to obtain a new paper quality. One parameter of primary concern in this matter is the steam pressure inside a drying cylinder.

There are different methods of modeling the paper drying process. Depoy [1] presented a simple analog model describing heat capacity, heat conductivity and heat transfer coefficients. The model employs mass and energy balances for the paper web structure. Another way of representing moisture content and bone dry mass is to employ steam pressure, velocity of drying cylinders and cylinder temperatures (Berrada [2]). Nilsson [3] adopted the heat mass transfer mechanism in cylinder groups to give a paper drying model. Gardner [4] provided basic principles of modeling of paper drying and showed the practical application of those principles in drying equipment. Management of steam pressure during a paper grade change operation is very important and Magnus [5] presented dynamic modeling simulation of steam supplying systems for a multiple drying cylinder during a grade change operation. A description of the overall paper drying process as well as relevant facilities can be found elsewhere (Karlsson [6]).

In this work a dynamic model for a paper drying cylinder is created to investigate the effects of major operation variables such as steam pressure on paper web drying performance. A simple model predictive control scheme is adopted to control moisture content by adjusting steam pressure based on the model developed.

1. Dynamic Model for Web Drying

Drying of a paper web can be considered as a combination of heat and mass transfer phenomena. The energy of steam is transferred to the paper web via drying cylinder wall followed by evaporation of water from the surface of the web. The model includes a mass balance for the cylinder and a description of the flow through the valve. The following assumptions were made:

- The steam inside the drying cylinder is saturated.
- Pressure drops caused by pipes are very small.
- Changes in condensate film thickness are small.
- The steam supply pressure is constant.
- Valve changes are instantaneous.
- Temperature and pressure gradients within the cylinder cavity are negligible.

The drying cylinder is heated by the steam supplied from steam distribution facilities. Obviously, the inner temperature of the cylinder is higher than that of paper web. Fig. 1 shows the temperature profile and energy flows for the cylinder. In Fig. 1, T_s represents steam temperature, T_m denotes the average cylinder temperature and T_p is

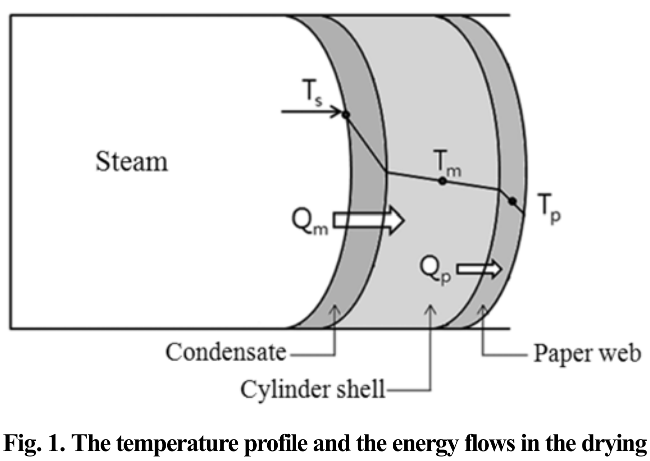

Fig. 1. The temperature profile and the energy flows in the drying cylinder.

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the web temperature. The amount of steam and water in the drying cylinder is given by:

$$\frac{d}{dt}(\rho_s V_s) = q_s - q_c - q_{bt} \quad (1)$$

$$\frac{d}{dt}(\rho_W V_w) = q_c - q_w \quad (2)$$

where q_s represents the steam rate supplied to the cylinder, and q_c and q_{bt} denote rates of condensing and blow-through steam respectively. q_w is the flow rate of condensate removed by siphon and V_s and V_w are volumes of steam and condensate, respectively. The volume occupied by steam and condensate is given by:

$$V = V_s + V_w \quad (3)$$

The steam rate supplied to the drying cylinder is a manipulated variable adjusted by the steam valve and is represented by:

$$q_s = c_v f_v(X)\sqrt{(p_{sh} - p)\rho_s} \quad (4)$$

where c_v is the valve coefficient and the valve is assumed to be equal percentage valve in which $f_v(X)=R_v^{X-1}$. Energy balances for steam, water content and drying cylinder can be written as:

$$\frac{d}{dt}(\rho_s u_s V_s) = q_s h_s - q_c h_s - q_{bt} h_s \quad (5)$$

$$\frac{d}{dt}(\rho_w u_w V_w) = q_c h_s - q_w h_w - Q_m \quad (6)$$

$$\frac{d}{dt}(m C_{p,m} T_m) = Q_m - Q_p \quad (7)$$

where u_s and u_w represent internal energy of steam and water, respectively, and m and $C_{p,m}$ denote mass and heat capacity of drying cylinder, respectively. The energy flow to the cylinder shell can be represented as:

$$Q_m = \alpha_{sc} A_{cyl}(T_s - T_m) \quad (8)$$

where A_{cyl} is the inner area of the cylinder and α_{sc} is the heat transfer coefficient between condensate and cylinder wall. The energy flow to the paper web can be written as:

$$Q_p = \alpha_{cp} A_{cyl} \eta (T_m - T_p) \quad (9)$$

where η is the fraction of cylinder surface covered by the paper web and α_{cp} is the heat transfer coefficient between cylinder surface and paper web. A linear correlation is used between the heat transfer coefficient at the cylinder/paper interface, α_{cp}, and the moisture content, ϕ, of the paper web at the surface (Wilhelmsson [7]):

$$\alpha_{cp}(u) = \alpha_{cp}(0) + 955\phi \quad (10)$$

Based on the assumptions described before, enthalpy, density and temperature of the steam within the drying cylinder can be represent as a function of pressure (Schmidt [8]):

$$\begin{aligned} &T_s = 0.1723(\ln p)^3 - 3.388(\ln p)^2 + 37.7\ln p + 124.5 \\ &h_s = [-0.07402(\ln p)^4 + 2.887(\ln p)^3 - 39.58(\ln p)^2 + 260\ln p + 1824]\cdot 10^3 \\ &h_w = [0.8842(\ln p)^3 - 18.77(\ln p)^2 + 200\ln p - 748.5]\cdot 10^3 \\ &\rho_s = [0.005048p + 64.26]\cdot 10^{-3} \\ &\rho_w = -0.3136(\ln p)^3 + 6.792(\ln p)^2 - 52.43\ln p + 1141 \end{aligned} \quad (11)$$

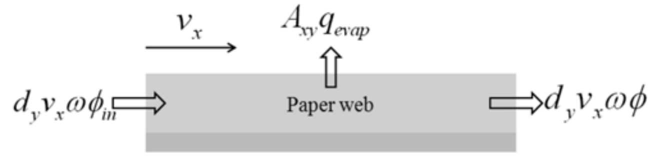

Fig. 2. The mass balance for moisture in the paper web.

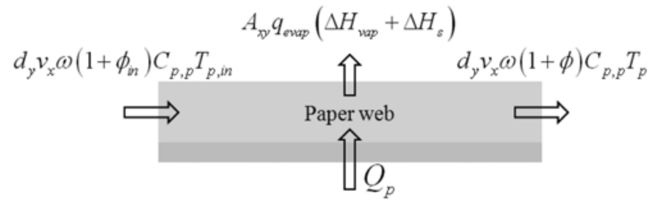

Fig. 3. The energy balance for moisture in the paper web.

Figs. 2 and 3 show schematics of mass and energy flow for moisture in the paper web. The mass balance is given by:

$$\frac{d(\phi\omega A_{xy})}{dt} = d_y y_y \omega\phi_{in} - A_{xy} q_{evap} - d_y v_x \omega\phi \quad (12)$$

where v_x is the velocity of the paper web and d_y is the width of the paper web. The evaporation rate can be represented as (Persson [9]):

$$q_{evap} = \frac{p_{tot} K M_w}{g T_p} \ln\left(\frac{p_{tot} - p_{va}}{p_{tot} - p_{vp}}\right) \quad (13)$$

where K is the mass transfer coefficient and M_w is the molecular weight of water. The partial pressure of steam in the air can be represented in terms of the moisture content θ and standard pressure p_{tot}:

$$p_{va} = \frac{\theta}{\theta + 0.62} p_{tot} \quad (14)$$

The partial pressure of steam at the web surface is given by:

$$p_{vp} = \varphi p_{v0} \quad (15)$$

The partial pressure of free water p_{v0} can be given by Antoine's equation as $p_{v0}=10^{(10127-(1690/(T_p-43.15)))}$. The correcting factor, or "sorption isotherm" φ for web drying can be found from experimental results (Heikkilä [10]). Usually φ takes a value between 0 and 1.

$$\varphi = 1 - \exp(-47.58\phi^{1.877} - 0.10085(T_p - 273)\phi^{1.0585}) \quad (16)$$

The heat loss due to evaporation at the paper web can be ignored because only conduction and radiation are significant. Moreover, changes in moisture volume due to the pressure can be ignored and we have an energy balance given by (see Fig. 3):

$$\begin{aligned} \frac{d(\omega(\phi+1)A_{xy}C_{p,p}T_p)}{dt} &= d_y y_x \omega(1+\phi_{in})C_{p,p}T_{p,in} + Q_p \\ &- A_{xy} q_{evap}(\Delta H_{vap} + \Delta H_s) - d_y v_x \omega(1+\phi)C_{p,p}T_p \end{aligned} \quad (17)$$

where ΔH_{vap} is the latent heat of evaporation and $C_{p,p}$, the heat capacity of paper web is given by $C_{p,p}=(C_{p,fiber}+\phi C_{p,w})/(1+\phi)$. The excess energy ΔH_s can be represented by Clausius-Clapeyron relation derived from the sorption isotherm:

$$\Delta H_s = -\frac{g}{M_w}\left[\frac{d(\ln\varphi)}{d(1/T_p)}\right] \quad (18)$$

Substitution of Eq. (16) into Eq. (18) gives

$$\Delta H_s = 0.10085\phi^{1.0585} T_p^2 g \frac{\varphi - 1}{M_w \varphi} \tag{19}$$

The steam rate manipulated by the steam valve alters the steam pressure within the drying cylinder followed by changes of temperature and moisture content in the paper web.

Based on the balance equations we can get the transfer function relating valve position and steam pressure change. We can choose p, V_w and T_m as state variables. By introducing $V_s=V-V_w$, $u_s=h_s-p/\rho_s$ and $u_w=h_w-p/\rho_w$, Eqs. (1)-(2) and (5)-(7) can be rearranged as:

$$A_1\frac{dV_w}{dt} + A_2\frac{dp}{dt} = f_1$$

$$B_1\frac{dV_w}{dt} + B_2\frac{dp}{dt} = f_2$$

$$C\frac{dT_m}{dt} = f_3 \tag{20}$$

where

$A_1=\rho_w-\rho_s$

$A_2=(V-V_w)\frac{d\rho_s}{dp}+V_w\frac{d\rho_w}{dp}$

$B_1=\rho_w h_w-\rho_s h_s$

$B_2=h_s(V-V_w)\frac{d\rho_s}{dp}+\rho_s\frac{dh_s}{dp}+h_w V_w\frac{d\rho_w}{dp}+\rho_w V_w\frac{dh_w}{dp}-V$

$C=mC_p$

$f_1=q_s-q_w-q_{bt}$

$f_2=q_s-q_w h_w-Q_m$

$f_3=Q_m-Q_p$

Further rearrangement of above relations gives:

$$\frac{dV_w}{dt}=\frac{CB_2f_1-A_2Cf_2}{C(A_1B_2-A_2B_1)}$$

$$\frac{dp}{dt}=\frac{A_1Cf_2-CB_1f_1}{C(A_1B_2-A_2B_1)}$$

$$\frac{dT_m}{dt}=\frac{f_3}{C} \tag{21}$$

Thus the transfer function relating valve position and steam pressure change is given by:

$$G_p(s)=5.7\frac{215.3s+1}{(65.5s+1)(596.7s+1)}s^{-2.83s} \tag{22}$$

The heat transport through the drying cylinder shell by thermal conduction is given by:

$$\frac{\partial T_m}{\partial t}=\frac{k_c}{\rho_c C_{p,m}}\frac{\partial^2 T_m}{\partial z_c^2} \tag{23}$$

$$\frac{\partial T_p}{\partial t}=\frac{k_p}{\rho_p C_{p,p}}\frac{\partial^2 T_p}{\partial z_p^2}-v_x\frac{\partial T_p}{\partial x} \tag{24}$$

Change of temperature at the cylinder surface according to the steam pressure can be represented as:

$$G_{Tc}(s)=\frac{k(p)}{(65s+1)}e^{-22s} \tag{25}$$

The nonlinear gain is given by:

$$k(p)=\gamma\cdot T_s(p) \tag{26}$$

$$T_s(p)=\frac{a}{b-\log(p+p_{tot})}-c \tag{27}$$

where a=1668.21, b=7.092, and c=228.

Based on the evaporation model at the paper web, we can derive the transfer function relating changes of web temperature with those of steam pressure:

$$C_{Tp}(s)=\frac{0.08}{(65s+1)}e^{-33.2s} \tag{28}$$

We can see that the web temperature increases 0.08 °C for 1 kPa increase of steam pressure. The time delay originates from the heat transport through condensate layer.

We used operation data (Fig. 4 and 5) to derive the transfer function relating changes of moisture content with those of vapor pressure:

$$C_u(s)=\frac{-0.0698}{(47.4s+1)}e^{-16.4s} \tag{29}$$

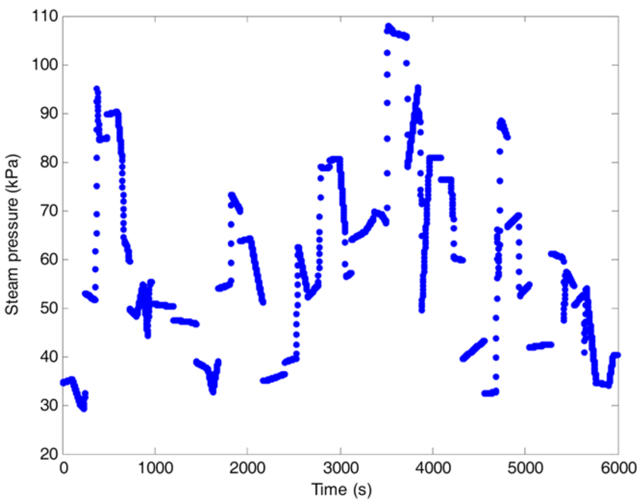

Fig. 4. Operating data: the steam pressure.

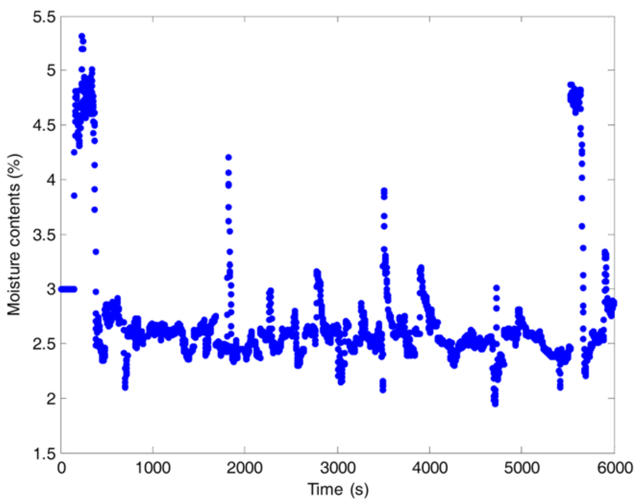

Fig. 5. Operating data: the moisture content.

RESULTS AND DISCUSSION

By using the proposed model we can investigate effects of steam pressure fed to the drying cylinder on the surface temperature, the web temperature and web moisture content. Table 1 shows data used in the numerical simulations. In this work, all the numerical simulations are performed for the paper with the basis weight of ω=77 g/m^2.

Table 1. Data for simulation

Parameters	Values
Cylinder volume [m^3]	18.4
Cylinder mass [kg]	8300
Cylinder area [m^2]	45.5
Specific heat capacity of paper [J/(kg·K)]	1256
Latent heat of vaporization [J/kg]	2260
Speed of machine [m/min]	1100
Heat transfer coefficient steam-cylinder [W/(m^2·K)]	1100

Fig. 6 represents changes of the steam pressure with time for various valve positions. The pressure of the steam header is 510 kPa. To reach 450 kPa, we can see that it takes about 17 seconds for 30% valve opening, 9 seconds for 35% valve opening and 2.7 seconds for 40% valve opening. Fig. 7 shows heat conduction to the paper web with time during steam supply to the cylinder. Since the steam pressure is constant, the temperature at the interface between the web and the cylinder surface increases. Fig. 8 shows changes in the moisture content of the paper web as well as changes in the temperature of cylinder surface and the web. The moisture content decreases because of the continuous evaporation.

Appropriate adjustment of the steam pressure is imperative to improve drying efficiency and to enhance economics in the operation. To achieve these objectives we need maintenance of constant steam pressure during production of single paper product and rapid

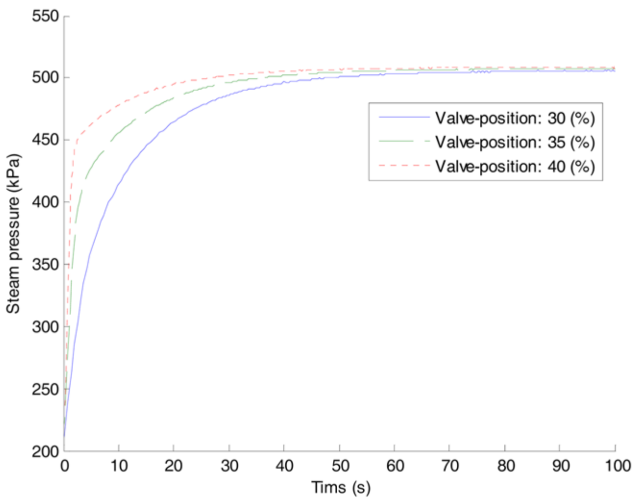

Fig. 6. Changes in the steam pressure of the drying cylinder.

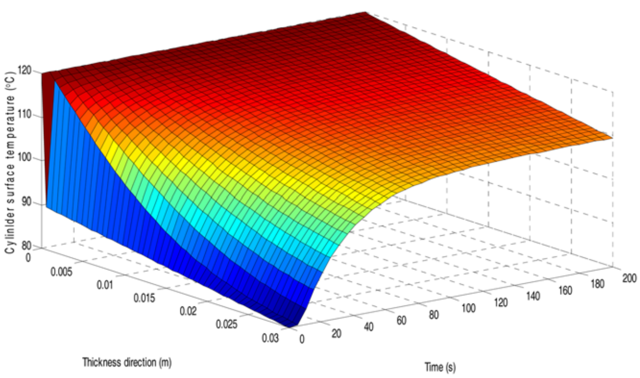

Fig. 7. Heat transport through the cylinder by thermal conduction.

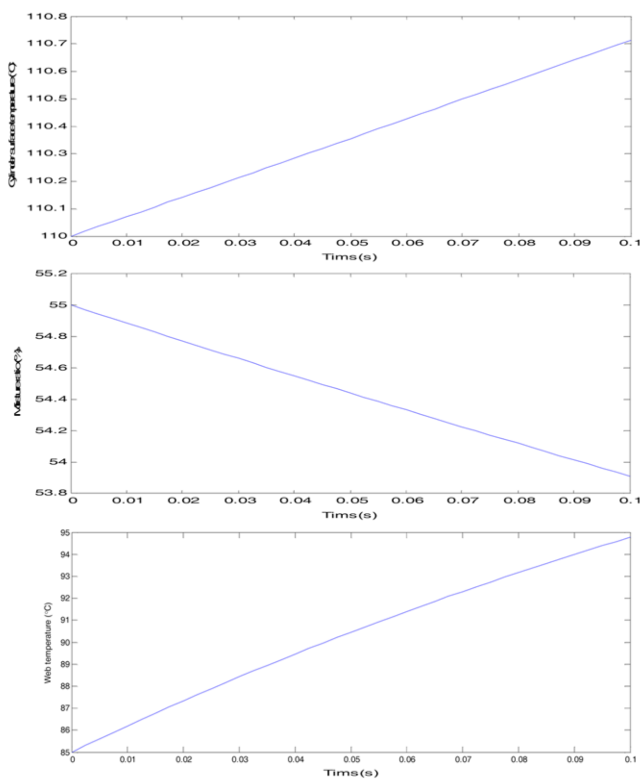

Fig. 8. Changes in major variables with time.

Table 2. MPC parameters

	Steam pressure control	Web temperature control	Moisture content control
Sampling time	1	1	1
Prediction horizon	50	100	100
Control horizon	2	2	2
Constraints	0≤Valve position≤100 0≤Steam pressure≤510	0≤Valve position≤100 0≤Steam pressure≤510	0≤Valve position≤100

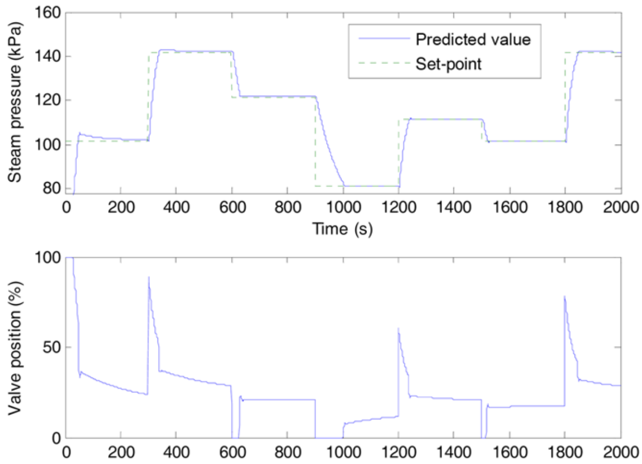

Fig. 9. Results of MPC simulations for the steam pressure.

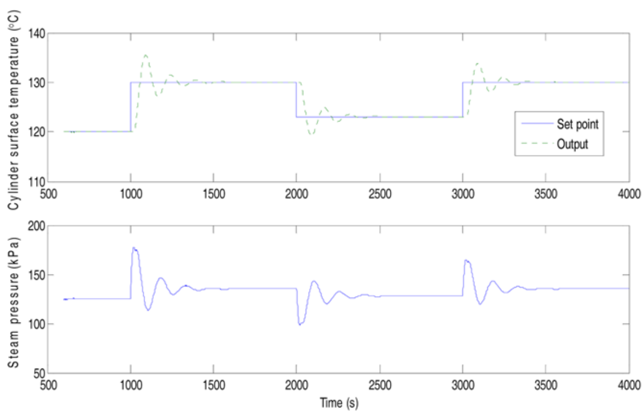

Fig. 10. Results of MPC simulations for the cylinder surface temperature.

tracking to a different set point during grade change operations. By using a simple model predictive control strategy, we can get future input changes for various outputs such as steam pressure, web temperature and moisture content. As we have examined before, a change in the steam pressure causes changes in the paper web surface temperature, which in turn changes the moisture content. Table 2 shows values of typical parameters used in the model predictive control simulations.

In numerical simulations valve positions were computed according to varying set points. Fig. 9 shows results of numerical simulations of model predictive control for the steam pressure. We can see satisfactory tracking performance for various step changes in the set point. Fig. 10 shows results of model predictive control for the surface temperature of the drying cylinder. We can see that nonlinearity in the gain causes some oscillations in the output. Fig. 11 shows results of model predictive control for the paper web temperature of the drying cylinder. Again, we can see good tracking performance for various step changes in the set point. Fig. 12 shows results of model predictive control for the moisture content during production of the flat art paper (Hi-Q mat N paper, Hansol Paper Co.). A step change in the moisture content means single grade change operation is performed. Operation data (Hansol Paper Co.) are shown

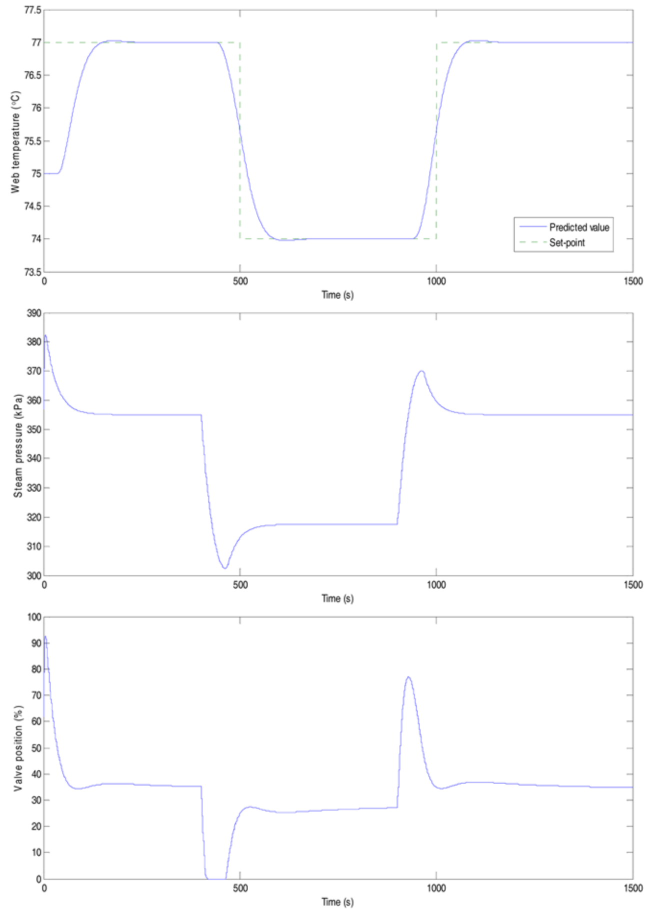

Fig. 11. Results of MPC simulations for the web temperature.

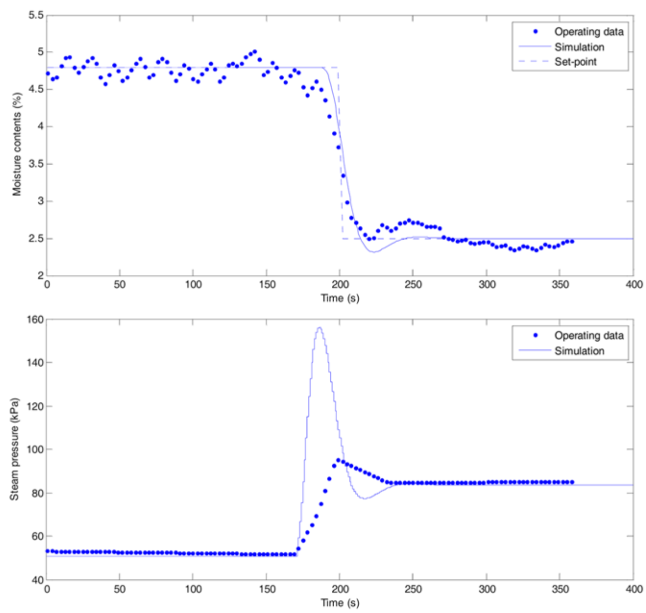

Fig. 12. Comparison between numerical simulations and operating data.

for the purpose of comparison. Good agreement demonstrates the effectiveness of the models developed in this work. Fig. 13 shows

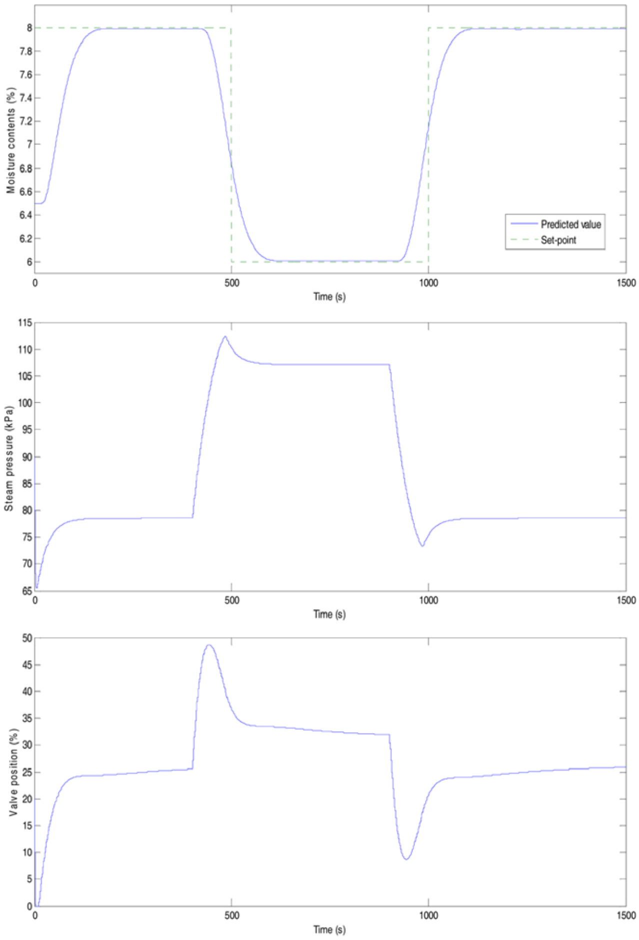

Fig. 13. Results of MPC simulations for the moisture content.

results of numerical simulations for multiple grade change operations. Even for multiple and frequent grade changes, stable and satisfactory tracking performance is achieved.

CONCLUSIONS

The present dynamic model describing the principles of a steam supply system for a paper drying cylinder used in paper production plants is based on the mass and heat balances around the cylinder. The balance equations consist of sets of differential equations describing heat and mass transfer around the canvas, the web and the drying cylinder. The effects of the steam valve adjustment on steam pressure, temperature and moisture content were investigated based on the model developed. The present model of the steam pressure dynamics in a paper drying cylinder can be represented in the form of a transfer function which can be used for simple model predictive control operation. It was found that the present model can be effectively used even in the control of multiple grade change operations.

ACKNOWLEDGEMENT

This work was supported by Korea Research Foundation Grant funded by the Korean Government (KRF-2008-313-D00177).

NOMENCLATURE

A_{cyl} : inner cylinder area [m^2]
A_{xy} : area of paper covering the cylinder [m^2]
c_v : valve conductance [m^2]
$C_{p,fiber}$: specific heat capacity of fiber (=1256) [J/(kg·K)]
$C_{p,m}$: specific heat capacity of cylinder shell [J/(kg·K)]
$C_{p,p}$: specific heat capacity of the paper [J/(kg·K)]
$C_{p,w}$: specific heat capacity of water [J/(kg·K)]
d_y : width of paper sheet [m]
g : gas constant [J/(mole·K)]
h_s : enthalpy of steam [J/kg]
h_w : enthalpy of condensate [J/kg]
k_c : thermal conductivity of cylinder [J/m·K]
k_p : thermal conductivity of paper [J/m·K]
K : mass transfer coefficient for paper sheet [m/s]
m : mass of cylinder shell [kg]
M_w : molecular weight of water [kg/mole]
p : steam pressure inside cylinder [kPa]
p_{sh} : steam pressure in header [kPa]
p_{tot} : standard pressure (=101.325) [kPa]
p_{v0} : partial vapor pressure for free water [kPa]
p_{va} : partial pressure for water vapor in the air [kPa]
p_{vp} : partial pressure for water vapor at paper surface [kPa]
q_{bt} : blow through steam [kg/s]
q_c : condensation rate [kg/s]
q_{evap} : evaporation rate [kg/(m^2·s)]
q_s : inflow of steam to cylinder [kg/s]
q_w : outflow of condensate [kg/s]
Q_m : energy flow to cylinder shell [W]
Q_p : energy flow to paper [W]
R_v : valve rangeability [-]
T_m : temperature of cylinder shell [K]
T_p : temperature of paper [K]
T_s : temperature of steam [K]
u_s : internal energy of steam [J/kg]
u_w : internal energy of condensate [J/kg]
v_x : speed of paper sheet [m/s]
V : volume inside cylinder [m^3]
V_s : volume of steam [m^3]
V_w : volume of condensate [m^3]
x : the coordinate in the machine direction [m]
X : valve opening [-]
z : thickness coordinate [m]

Greek Letters

α_{sc} : heat transfer coefficient steam-cylinder [W/(m^2·K)]
α_{cp} : heat transfer coefficient cylinder-paper [W/(m^2·K)]
ΔH_s : heat of sorption [J/kg]
ΔH_{vap} : latent heat of vaporization [J/kg]
ϕ : moisture ratio [kg/kg]
φ : sorption isotherm [-]
η : fraction of dryer surface covered by paper [-]
θ : water content in air [kg/kg]
ρ_s : density of steam [kg/m^3]
ρ_w : density of condensate [kg/m^3]
ω : dry basis weight of paper [kg/m^2]

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