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Modeling supply chain operations as multi-level programming problems and their parametric programming based computation methodology

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Abstract−This paper proposes a modeling methodology for supply chain operations with a focus on the relationships of supply chain entities. Supply chain operation problems are mathematically formulated into a multilevel programming problem. A multiparametric programming-based computation methodology is proposed to compute the solution of the problems. Numerical examples are presented to illustrate the proposed modeling and computational methodology with some remarks.

Key words: Supply Chain, Relationship, Modeling, Parallel, Sequential, Multilevel Programming, Parametric Programming

INTRODUCTION

Process systems engineering communities have been actively investigating supply chain and its decision-making problems. Most works approach by formulating supply chain operations as mathematical optimization problems in the context of scheduling [1,2], or planning [3,4], and simulation framework [5,6] including state-of-the-art reviews [7,8]. Other works expanded the area of interest by addressing supply chain in terms of asset and capital management [9].

In spite of such wide activities, process industry-based supply chains have many challenging issues. Consider the following typical issue: a minor problem initiated in one entity actually does affect the other and becomes a major trouble in the overall supply chains in the end. For example, when an order is to provide 5,000 tons of product A and B to a local warehouse, how can we can estimate the impact of the change like (i) delivery with 4,500 tons of A and 5,700 tons of B or (ii) delivery date is one day delayed or shortened, etc. If the transaction is between only two entities, they can conveniently negotiate such late/early or partial insufficiency by directly contacting each other. Supply chains in practice involve more than two entities that do not have explicit connections with each other in different places. That is, when an entity changes its specification without improving its responding performances, other entities including the entire supply chains are subject to the change. To prepare such a situation, it is of great importance to understand how individual entities are associated with each other. We should have the decision-supporting tool to estimate the impact of changes on the overall supply chain performance as well as others.

Little research has been done in analyzing the relationships of supply chain entities in the face of increasing attention. Bachx et al. [10] were interested in how multiple elements in process networks are operated. They analyzed the supply chain operation in terms of interaction and categorized them into coordination, cooperation and competition. Ryu and Pistikopoulos [11,12] extended the work by addressing how supply chain operation policy can be modeled based on their operating policy. Ryu et al. [13] provided a bilevel programming modeling on the supply chain planning problem in the context of the sequential relationship between manufacturing and transportation operations. In their work, a bilevel programming problem is constructed for the problem of the manufacturing and distribution problem. Their work was not generalized and expanded to address the generic feature of supply chains. From the above works, there is still room for further research on supply chain interconnections and relationships in spite of their impact on the overall supply chain performance.

The rest of this paper is organized as follows: supply chain relationships between participating entities are addressed. The relationships are employed into mathematical formulations for addressing supply chain operations. A computation methodology to compute the solution of the corresponding problem is presented with numerical examples and remarks.

SUPPLY CHAIN RELATIONSHIP

Supply chain entities are mainly categorized as two types, homogeneous or heterogeneous, in terms of their connections with others. Entities can be said to be *homogeneous* when they play the same role with each other, for example, manufacturing, or holding inventory, etc. Consider an assembly manufacturing plant in Asia and the same one in Southern Europe. They play the same role of manufacturing products. Their only difference is that they are located in a different geological region. For homogeneous entities, they share some resources: Same tools are purchased from the same vendor or the same kinds of raw materials are used. It is of course possible that they use different kinds of tools, but then it is difficult to manage their individual performances. Therefore, many companies use the same tools. This is also good for saving tool purchasing costs.

On the other hand, there are entities which play different roles. For instance, a warehouse and a manufacturing plant in the sample place play different roles, respectively: They can be said to be *hetero-*

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geneous. For a simple supply chain, most entities would be heterogeneous, for example, a raw material supplier, a manufacturer, a warehouse, and a retailer, etc. As the scope of the supply chain is enlarged, the number of homogeneous entities increases to meet the expanded demand in terms of capacity, geographical advantage, etc.

It will be shown that homogeneous entities are connected in parallel in a supply chain and heterogeneous entities are in a sequential mode. It can be said that a complex-looking supply chain may be constructed as a combination of parallel and sequential connections.

For the ease of explanation, the following illustrative supply chain consisting of two entities A and B will be considered to explain how supply chain relationships are transformed into mathematical programming problems. It is assumed that decisions variables represent the conditions for entity A are x_1 and x_2 and for entity B, x_3 and x_4 respectively. If the decision-making is formulated as a linear form, the following two models can be formulated to represent individual operations:

Entity A

$$\text{Min } Z_A=c_1x_1+c_2x_2 \quad (1)$$

s.t.

$$a_{11}x_1+a_{12}x_2\le b_1 \quad (2)$$

$$a_{21}x_1+a_{22}x_2\le b_2 \quad (3)$$

Bntity B

$$\text{Min } Z_B=c_3x_3+c_4x_4 \quad (4)$$

s.t.

$$a_{31}x_3+a_{32}x_4\le b_3 \quad (5)$$

$$a_{41}x_3+a_{42}x_4\le b_4 \quad (6)$$

1. Connections for Homogeneous Entities

For two entities A and B playing the same role in a supply chain, they are connected in a homogeneous way. Since they do the same role, their resources may be shared with each other as can be seen in Fig. 1:

In modeling homogeneous entities doing the same role in a supply chain, a key issue is to minimize the consumption of a common resource while meeting the demand. The corresponding decision-making model can be formulated as follows:

Enterprise (Entity A, B)

$$\text{Min } Z_{TOT}=c_1x_1+c_2x_2+c_3x_3+c_4x_4 \quad (7)$$

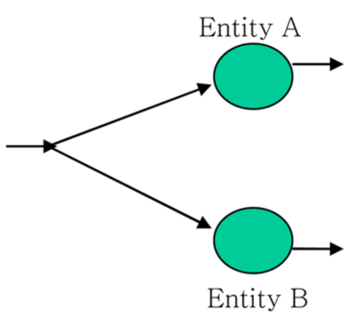

Fig. 1. An illustration of parallel relation.

s.t.

$$a_{11}x_1+a_{12}x_2+a_{31}x_3+a_{32}x_4\le(b_1+b_3) \quad (8)$$

$$a_{21}x_1+a_{22}x_2\le b_2$$

$$a_{41}x_3+a_{42}x_4\le b_4$$

where (8) denotes the resources shared by A and B.

At first, the connections between multiple entities doing the same role in different geological locations is said to be in parallel since these entities get the same resources from the same suppliers. If there is only one supplier, the entities receiving resources from the only supplier tend to compete with each other to increase their own profit.

Secondly, (8) is valid only if both entities share the entire information together. If both are under control of different companies, it is the maximum value both can achieve by sharing the entire information.

Thirdly, the overall model is subject to the constraints of the individual entity models as well as overall constraints representing the common resources shared by all entities. In terms of computation, the corresponding mathematical model can be computed by using conventional solvers without the help of novel methodologies.

2. Sequential Connection

When two heterogeneous entities A and B are connected, they take the form of a *sequential mode* as depicted in Fig. 2:

As shown in Fig. 2, a former entity A produces an intermediate product which is delivered to the latter entity, B for processing the final product. In the sequential connection, a process is completed only after the completion at the last entity.

If the performances of the entity A and B are different, the operation of the latter entity B are affected by that of A. If entity A does not provide enough output, entity B cannot make as much as it can. On the other hand, over-production at plant A is not profitable without the full consumption by entity B.

A sequential connection is used for entities whose operation is directly related to each other in exchange for material and information. For instance, a raw-material processing plant produces an intermediate product that is delivered to another plant for manufacturing end-products. A plant makes products for retailers which sells them to customers. Therefore, a two-entity sequential model can be mathematically formulated into the following bilevel programming problem:

Enterprise (Entity A+Entity B)

$$\min_{x_3,x_4} Z_B = c_3x_3 + c_4x_4 \quad (9)$$

s.t.

$$a_{31}x_3+a_{32}x_4\le b_3+x_1 \quad (5)$$

$$a_{41}x_3+a_{42}x_4\le b_4+x_2 \quad (6)$$

To determine the decision levels, x_1 and x_2 for the entity A, variables in entity B should be known. On the other hand, the model for entity B is in turn restricted by the entity A's decisions.

The key issue in the sequential connection is how to determine

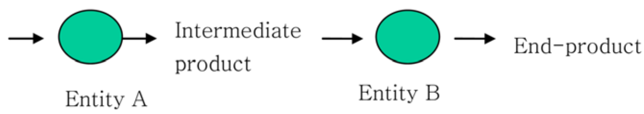

Fig. 2. An illustration of sequential relation.

individual capacities, structures which are affected by other entity's operation. For the case of two objective functions, Ryu et al. [13] proposed a bilevel programming structure by capturing a snapshot of industrial practices for a simple case of a plant and a distribution center without clarifying generic supply chain relationships. This paper expands their work and focuses on relationships between multiple entities and how they can be mathematically formulated.

3. A Mixture of Parallel and Sequential Connections

Since a supply chain involves numerous entities, it may be full of homogeneous and heterogeneous entities. Supply chain problems are thereafter posed as a combination of parallel and sequential connections. The proposed methodology will be illustrated by the following example.

4. Example 1: Three-level Supply Chain

Consider a company which has two manufacturing plants, B1 and B2, and a distribution center, C as illustrated in Fig. 3. To meet multiple demands spread in distinctive places, a distribution center is set up. Whereas, plant B2 has intermediate products from supplier A and plant B1 is outsourced externally. The sum of production from B1 and B2 in distribution center is used to meet the demands.

Based on the variables for entities, their decision-making problems can be formulated as the following problem (11):

Enterprise (A, B1, B2, C)

$$\min_{z} F_{A,B1,B2,C}(x, y_1, y_2, z) \tag{11-1}$$

s.t.

$f(x, y_1, y_2, z) \le 0$

$$y_1, y_2 \in \left\{ \begin{array}{ll} \min\limits_{y_1, y_2} G_{B_1,B_2}(x, y_1, y_2, z) & (11\text{-}2) \\ \text{s.t.} \quad g(x, y_1, y_2, z) \le 0 & \\ x \in \left\{ \begin{array}{l} \min\limits_{x} H_A(x, y_1, y_2, z) \\ \text{s.t.} \quad h(x, y_1, y_2, z) \le 0 \end{array} \right\} & (11\text{-}3) \end{array} \right\}$$

The problem (11) can be generalized into the following general multilevel programming problem:

$$\begin{aligned} &\min_{X} F(X, Y, Z, \cdots) \\ &\text{s.t. } f(X, Y, Z, \ldots) \le 0 \\ &\min_{Y} F(X, Y, Z, \cdots) \\ &\text{s.t. } g(X, Y, Z, \ldots) \le 0 \qquad (12) \\ &\min_{Z} H(X, Y, Z, \cdots) \\ &\text{s.t. } h(X, Y, Z, \ldots) \le 0 \\ &\quad \vdots \end{aligned}$$

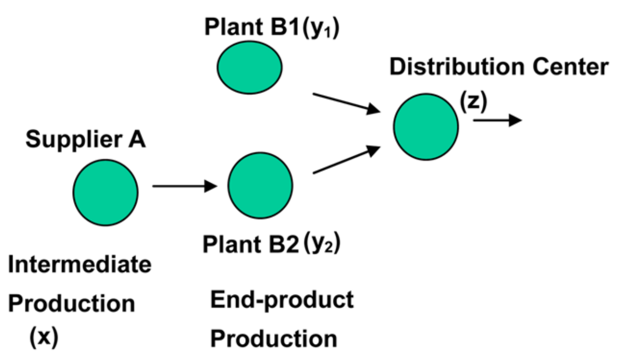

Fig. 3. An illustration of supply chain in example 1.

where X, Y, Z denote variables that should be determined at individual levels. How to compute the solution of (12) is another issue. The next section will provide a methodology to compute the solutions.

SOLUTION METHODOLOGY

The simple case of the multilevel programming problem is the case of a bilevel problem. The biLevel programming problem (BLPP) denotes an optimization problem that is constrained by another optimization problem. It is used to address industrial situations involving several indifferent groups which are inter-connected in a sequential structure. Each group may be an individual or an agency which has independent, perhaps conflicting, objectives. The general form of BLPPs can be formulated as follows:

$$\begin{aligned} &\min_{x} F(X, Y) \\ &\text{s.t. } G(X, Y) \le 0 \qquad (13) \\ &y \in \left\{ \begin{array}{l} \min\limits_{y} f(x, y) \\ \text{s.t. } \; g(x, y) \le 0 \end{array} \right\} \end{aligned}$$

where F, f: $R^{n1} \times R^{n2} \rightarrow R^1$, g: $R^{n1} \times R^{n2} \rightarrow R^m$, $X \subseteq R^{n1}$ and $Y \subseteq R^{n2}$. F(x, y) is called a higher level decision problem, namely *a leader's problem* or an outer problem, and f(x, y) is called a lower level decision problem, *a follower's problem* or an inner problem. The two problems are connected in a way that the leader's problem sets parameters influencing the follower's problem and the leader's problem in turn is affected by the outcome of the follower's problem. To compare the two problems in terms of scope of information, a follower makes decisions using only its local information, while a leader does by using the complete information including the follower's possible action. This feature was employed to address the sequentially interrelated industrial situations which may be difficult to be modeled realistically using other modeling formulations [13].

When f(x, y) and g(x, y) are twice continuously differentiable in x and y, and variables of the leader's problem x meet the basic sensitivity theorem in the follower's problem, which means that x and y are an affine function, the solution of the above problem can be obtained using employing parametric programming techniques by transforming the above problem into a bilevel programming problem [13].

Ryu et al. [13] proposed a solution methodology to compute the solution of the inner problem as a family of explicit functions which consist of variable x of the outer problem and uncertain parameters. In computing the solution of the inner optimization problem, the variables of the outer problems are regarded as uncertain parameters. The inner optimization problem is thus reformulated as a multi-parametric programming problem which can be computed by using existing available parametric programming techniques. For computing the solution of the multilevel programming problem, there is little research, to the author's knowledge.

Correspondingly, this paper will propose a computation methodology based on parametric techniques. The overall computation procedure of the multi-level programming problem is as follows:

[Step 1] Solve the most inner problem by transforming into a multi-parametric programming problem where variables of the remaining outer problems are treated as uncertain parameters.

[Step 2] Using the parametric solutions of the most inner problem, formulate the next inner problem into a family of the parametric programming problems. The solutions can be expressed as a family of explicit functions of the variables of the remaining outer problems and the uncertain parameters with the corresponding valid regions. The solutions of above parametric optimization problems have the following form:

$$y=\begin{cases}\xi_1(x)=l_1+m_1x+n_1 \ \text{ if } \ H_1x\le h_1\\ \xi_2(x)=l_2+m_2x+n_2 \ \text{ if } \ H_2x\le h_2\\ \vdots \qquad\qquad\qquad\qquad\qquad \vdots\\ \xi_k(x)=l_k+m_kx+n_k \ \text{ if } \ H_kx\le h_k\end{cases} \tag{14}$$

where k denotes the number of the computed parametric solutions, l_k is a constant parameter, m_k and n_k are constant vectors. H_k are constant matrices and h_k is constant vector. $H_kx\le h_k$ is called a critical region, CR^k which is the corresponding valid boundary condition of the *k*th parametric solution. By using the parametric solutions of the inner problem in the form of (14), the explicit solutions of the inner problem are obtained. Consequently, a multilevel programming problem is transformed into a number of optimization problems as follows:

$$\begin{aligned}
&z_1=\min_x c_1^Tx+d_1^T\xi_1(x)\\
&\text{s.t. } A_1x+B_1\xi_1(x)\le b_1\\
&\quad H_1x\le h_1\\
&z_2=\min_x c_2^Tx+d_2^T\xi_2(x)\\
&\text{s.t. } A_2x+B_2\xi_2(x)\le b_2\\
&\quad H_2x\le h_2\\
&\quad\vdots\\
&z_k=\min_x c_k^Tx+d_k^T\xi_k(x)\\
&\text{s.t. } A_kx+B_k\xi_k(x)\le b_k+K_k\\
&\quad H_kx\le h_k
\end{aligned} \tag{15}$$

Each of above problems corresponds to a multi-parametric linear programming problem.

[Step 3] Continue to solve the next inner problem until the final outer problem. As a result, the final outer problem is formulated as a family of single optimization problems. By solving these problems, the solutions of the original problem are obtained. By comparing these local solutions for their feasibility in individual level of constraints, we can obtain the best solution.

NUMERICAL EXAMPLES

This section will present three numerical examples to illustrate the proposed methodology.

1. Example 2

Consider the following three-level linear programming problem:

$$\begin{aligned}
&\min_x 3x+y+4z\\
&\min_y -x+y+4z\\
&\text{s.t. } x+y+z\ge 1\\
&\quad\min_z 2x-4y+z\\
&\text{s.t. } 2x+y+z\ge 5\\
&\quad x+y-3z\le -3\\
&\quad 0\le x,\ y,\ z\le 10
\end{aligned} \tag{16}$$

According to the proposed methodology, the most inner level programming problem can be formulated as the following multi-parametric programming problem by treating the upper level variables as parameters:

$$\begin{aligned}
&\min_z z+2y-4y\\
&\text{s.t. } -z\le -5+2x+y\\
&\quad 3z\le 3+x+y
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\min_z z+[2\ \ -4]\begin{bmatrix}x\\ y\end{bmatrix}\\
&\text{s.t. } \begin{bmatrix}-1\\ 3\end{bmatrix}z\le\begin{bmatrix}-5\\ 3\end{bmatrix}+\begin{bmatrix}2 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}
\end{aligned} \tag{18}$$

The parametric solutions of (18) are as follows:

$$z=\begin{cases}0 & \text{if } -2x-y\le -5\\ -2x-y+5 & \text{if}\begin{cases}2x+y\le 5\\ -1.75x-y\le -3\end{cases}\end{cases} \tag{19}$$

Using the above result in (19), the second-level problem corresponds to the following two LP problems:

$$\begin{aligned}
&\min_y -x+2y\\
&\text{s.t. } -y\le -1+x\\
&\quad -y\le -5+2x
\end{aligned} \tag{20-1}$$

$$\begin{aligned}
&\min_y 5x+5y-15\\
&\text{s.t. } -y\le -5+2x\\
&\quad 0\le x\le 4
\end{aligned} \tag{20-2}$$

The solutions of (20-1) and (20-2) are as follows, respectively:

$$y=\begin{cases}-2x+5 & \text{if } 0\le x\le 2.5\\ 0 & \text{if } 2.5\le x\le 10\end{cases} \tag{21-1}$$

$$y=\begin{cases}x & \text{if } 1.7429\le x\le 4\\ -1.75x+3 & \text{if } 0\le x\le 1.7429\end{cases} \tag{21-2}$$

Consequently, the third-level problem corresponds to the following four LP problems, where the first two, (22-1) and (22-2) are formulated from (21-1) and the last two (22-3) and (22-4) are from (21-2), respectively:

$$\begin{aligned}
&\min_x 3x\\
&\text{s.t. } 2.5\le x\le 10
\end{aligned} \tag{22-1}$$

$$\begin{aligned}
&\min_x x+5\\
&\text{s.t. } 0\le x\le 2.5
\end{aligned} \tag{22-2}$$

$$\begin{aligned}
&\min_x -8x+20\\
&\text{s.t. } 1.71429\le x\le 4
\end{aligned} \tag{22-3}$$

Table 1. Result of Example 2

Variable	Subproblem			
	(22-1)	(22-2)	(22-3)	(22-4)
X	2.5	0	4	0
Y	0	5	4	3
Z	0	0	−7	2
Function	(22-1)	(22-2)	(22-3)	(22-4)
F	7.5	5	−12	11
	feasible	best	infeasible	feasible

$$\min_{x} 0.25x+11$$
$$\text{s.t. } 0\le x\le 1.71429 \tag{22-4}$$

The solutions of these problems are computed and summarized in Table 1. As a result, the optimal solution of the initial problem is 5 when (x, y, z) is (0, 5, 0). (4, 4, −7) is infeasible solution because (x, y, z) should be positive variables. The overall computational procedure is summarized in Table 2.

2. Example 3

Here the supply chain model presented in Example 1 will be revisited based upon the assumption that they are represented as linear constraints. The resulting supply chain problem can be mathematically formulated into the following three-level programming problem:

Enterprise (A, B1, B2, C)

$$\min_{z_1,z_2,z_3} COST_C=3z_1-z_2+z_3$$
$$\text{s.t. } z_1+z_2\le 3z_3$$
$$y_1+y_3\le z_1$$
$$y_2+y_4\le z_2$$

$$y_1, y_2, y_3, y_4\in\left\{\begin{array}{ll}\min\limits_{y_1,y_2,y_3,y_4} U_{B1,B2}=-2y_1-3y_2-4y_3-2y_4 & (23\text{-}2)\\ \text{s.t. } 2y_1+y_2+y_3+3y_4\le 500+z_1+2z_2 & \\ y_1+y_2\le 300+z_2+3z_3 & \\ 3y_1-2y_2\le 100+z_2 & \\ 4y_3+2y_4\le 600 & \\ y_3-y_4\le -300 & \\ x_1\le y_1+y_3 & \\ x_2\le y_2+y_4 & \\ x_1,x_2\in\left\{\begin{array}{l}\min\limits_{x_1,x_2} W_A=7x_1+3x_2\\ \text{s.t. } -x_1\le -200+y_1\\ -x_2\le -150+2y_1+y_2\\ x_1+2x_2\le 400-y_1+y_2\end{array}\right\} & (23\text{-}3)\end{array}\right\}$$

$$0\le x_1, x_2, 0\le y_1, y_2, y_3\le 100, 100\le z_1, z_2\le 500$$

The proposed methodology has been applied to this problem and its solutions are summarized in Table 3 and Table 4. As can be seen in Table 3, the lower level decision variables are obtained as a function of the upper level decision variables with their valid boundary condition. Therefore, the relationships of the associated entities are obtained in terms of mathematical formulation. This is an important advantage of the proposed methodology in addressing supply chain operations.

As can be seen in model (23), the original problem has the boundary condition for x, y and z, respectively. When all three-level problems are computed and their corresponding values are obtained, their solutions are checked for their feasibility against their initial

Table 2. Computation procedure of Example 2

$$\begin{array}{c}\min\limits_{x} 3x+4z\\ \min\limits_{y} -x+y+4z\\ \text{s.t. } x+y+z\ge 1\\ \min\limits_{z} 2x-4y+z\\ \text{s.t. } 2x+y+z\ge 5\\ x+y-3z\le -3\\ 0\le x, y, z\le 10\end{array}$$

$$\begin{array}{c}\min\limits_{z} z+2x-4y\\ \text{s.t. } -z\le -5+2x+y\\ 3z\le 3+x+y\end{array}\rightarrow\begin{array}{c}\min\limits_{z} z+[2\ \ -4]\begin{bmatrix}x\\y\end{bmatrix}\\ \text{s.t. } \begin{bmatrix}-1\\3\end{bmatrix}z\le\begin{bmatrix}-5\\3\end{bmatrix}+\begin{bmatrix}2&1\\1&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}\end{array}\rightarrow z=\begin{cases}0 & \text{if } -2x-y\le -5\\ -2x-y+5 & \text{if }\begin{cases}2x+y\le 5\\ -1.75x-y\le -3\end{cases}\end{cases}$$

$$\begin{array}{c}\min\limits_{y} -x+2y\\ \text{s.t. } -y\le -1+x\\ -y\le -5+2x\end{array}\rightarrow y=\begin{cases}-2x+5 & \text{if } 0\le x\le 2.5\\ 0 & \text{if } 2.5\le x\le 10\end{cases}$$

$$\begin{array}{c}\min\limits_{y} 5x+5y-15\\ \text{s.t. } -y\le -5+2x\\ 0\le x\le 4\end{array}\rightarrow y=\begin{cases}x & \text{if } 1.7429\le x\le 4\\ -1.75x+3 & \text{if } 0\le x\le 1.7429\end{cases}$$

$\min_{x} 3x$	$\min_{x} x+5$	$\min_{x} -8x+20$	$\min_{x} 0.25x+11$
s.t. $2.5\le x\le 10$	s.t. $0\le x\le 2.5$	s.t. $1.71429\le x\le 4$	s.t. $0\le x\le 1.71429$

Table 3. Parametric intermediate solution of Example 3

	3^{rd} Level solution	2^{nd} Level solution	2^{nd} Level critical region
CR1	$x_1=200-y_1$ $x_2=150-y_2$ $y_1-7y_2\leq-100$ $0\leq y_2\leq 75$	$y_1=\frac{1}{2}z_1-z_2+145.833$ $y_2=75$ $y_3=133.33$ $y_4=33.333$	$0.375z_1-z_2\leq-46.875$ $-0.5z_1+z_2\leq 83.333$
CR2		$y_1=0.5z_1-z_2+133.333$ $y_2=100$ $y_3=133.333$ $y_4=133.333$	$0.375z_1-z_2\leq-25$ $-0.5z_1+z_2\leq 100$
CR3		$y_1=62.5$ $y_2=75$ $y_3=z_1-2z_2+400$ $y_4=0$	$0.5z_1-z_2\leq-150$ $-0.5z_1+z_2\leq 200$
CR4	$x_1=200-y_1$ $x_2=0$ $75\leq y_2\leq 100$	$y_1=0.5z_1-z_2+133.333$ $y_2=100$ $y_3=133.333$ $y_4=133.333$	$0.375z_1-z_2\leq-25$ $-0.5z_1+z_2\leq 100$
CR5		$y_1=0.167z_1+0.333z_2+133.333$ $y_2=100$ $y_3=0.333z_1-0.667z_2+200$ $y_4=0.333z_1-0.667z_2+100$	$0.5z_1-z_2\leq-100$ $-0.5z_1+z_2\leq 150$
CR6		$y_1=50$ $y_2=z_1-2z_2+400$ $y_3=100$ $y_4=0$	$-0.5z_1+z_2\leq 162.5$ $0.5z_1-z_2\leq-150$

Table 4. Result of Example 3

	CR1	CR2	CR3	CR4	CR5	CR6
X_1	137.5	216.667	137.5	166.6665	−33.308	150
X_2	75	50	75	0	0	0
Y_1	62.5	−16.667	62.5	33.3335	233.308	50
Y_2	75	100	75	100	100	75
Y_3	133.333	133.333	0	133.333	99.875	100
Y_4	33.333	133.333	0	133.333	-0.125	0
Z_1	95.83	62.5	62.5	266.667	150	150
Z_2	131.248	181.25	231.25	233.333	225	237.5
Z_3	75.693	81.25	97.917	166.667	125	129.167
Objective	231.935	87.5	54.167	733.333	350	341.667
feasibility	infeasible	infeasible	infeasible	feasible	infeasible	Best

given condition.

DISCUSSION

There are some issues worthy of comments regarding the proposed framework. The first is to clarify the key contribution of the proposed multilevel programming framework for modeling supply chains. It would be clearer to address the difference of the proposed methodology against the conventional one-big-model in which all constraints and objective functions are aggregated into a single set. For instance, example 2 can be formulated by aggregating all the constraints and objective functions simultaneously as follows:

$$
\begin{aligned}
&\min\ \alpha(3x+y+4z)+\beta(-x+y+4z)+\gamma(2x-4y+z)\\
&\text{s.t.}\\
&x+y+z\geq 1\\
&2x+y+z\geq 5\\
&-x-y+3z\leq 3\\
&0\leq x,\ y,\ z\leq 10
\end{aligned}
\tag{24}
$$

When the weight factor (α, β, γ) is (1, 1, 1), the solution is −20 when

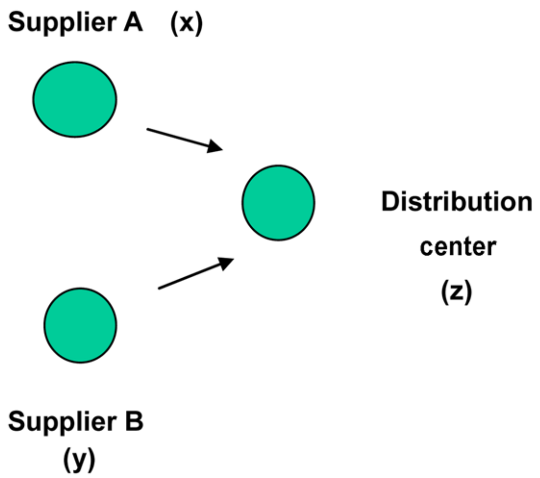

Fig. 4. An illustration of supply chain.

(x, y, z) is (0, 10, 0). In this case the 3rd level solution is 10. Since the solution of the 3rd level programming problem is 5 according to the proposed methodology, it is better than 10 in minimizing the problem. But it cannot be assumed that this will always happen. The key advantage of the proposed method is that the local level of decision-making for its interaction with the other entities is reflected in the proposed framework since the process of computing the solution resembles the actual practices. That has not been explicitly addressed in other previous works.

The next advantage of the proposed is that it can model a case where independent entities are present in a supply chain. For instance, when supplier A and supplier B provide distinctive products to a distribution center, their operation only affects the distribution center, its design can be represented by using the proposed methodology as (25).

Enterprise (A, B, C)

$$\underset{z}{\mathrm{Min}}\, F_{A,B,C}(x,y,z) \tag{25-1}$$

s.t.

$f(x, y, z)\leq 0$

$$x\in\begin{Bmatrix}\underset{x}{\mathrm{Min}}\, H_A(x,y,z)\\ \text{s.t. } h(x,y,z)\leq 0\end{Bmatrix} \tag{25-2}$$

$$y\in\begin{Bmatrix}\underset{y}{\mathrm{Min}}\, G_B(x,y,z)\\ \text{s.t. } g(x,y,z)\leq 0\end{Bmatrix} \tag{25-3}$$

Next, the proposed methodology can be applied to address the other type of complex decision-making problems. In this paper, it is assumed that a decision-making problem for an entity corresponds to a one-level problem. It is possible to apply the proposed decision-making framework for the problems inside an entity as well.

As a further research topic, an interesting point is to associate the supply chain relationships and supply chain dynamics. Actual material flows in processes are divided into *push* and *pull* type depending on which has more emphasis between supplier and demand. When a key driver of a supply chain is on demand, the supply chain is subject to be processed from the request from demands. On the other hand, when the supply is not enough, its supply chains are dominated by the supply, the dynamic is initiated from the supply side. While any industries have both features of push and pull, the distinction can be made by placing where to put more emphasis. In this paper, the design is mainly based on the pull approach.

Another issue would be whether the proposed methodology can directly handle problems of large scale in industry. Parametric programming techniques have been employed to address industrial problems of large scale in the context of a commercial software package [14,15]. In this respect, it would be fair to assume that the proposed methodology can handle them as well.

It would be also important to provide a suitable proof for optimality of the proposed methodology for multilevel problems. In the literature, there are not many previous works on multilevel programming problems. For the case of a two-level, namely bilevel, case, Ryu et al. [13] already showed that global optimum solutions are obtained. For the multilevel case of more than three, it can be said that the proposed method provides a fairly good solution. It would be further interesting to provide a proper proof for the optimality of the proposed methodology.

CONCLUDING REMARKS

A modeling formulation is presented in this paper for supply chain operations with a focus on the relationship of the participating entities. Individual entities are connected in two ways, parallel for competing entities and sequential for entities sharing resources. The corresponding supply chain problems have been transformed into multilevel programming problems. Numerical examples illustrate the applicability of the computational methodology for the problems.

For the success of the companies, an essential issue is to figure out how to incorporate separate competences of individual entities for generating the overall value. That is, the competitiveness of a company is dependent not only on the individual excellences but also by establishing their harmony. How to connect geographically distributed entities in the most timely and cost-effective manner to raise overall efficiencies can help to improve their competitiveness. The proposed methodology can contribute to the improvement, and it is thought that it can be further utilized to that end.

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