

## Input/output linearization with a two-degree-of-freedom scheme for uncertain nonlinear processes

Pisit Sukkarnkha and Chanin Panjapornpon<sup>\*</sup>

Center of Excellence for Petroleum, Petrochemicals, and Advanced Materials, Department of Chemical Engineering,  
Faculty of Engineering, Kasetsart University, Bangkok 10900, Thailand  
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**Abstract**—This work presents a new method to control processes with unmeasured input disturbance and random noise parametric uncertainty. The developed method takes advantage of a two-degree-of-freedom control structure in which setpoint regulation and load disturbance rejection are integrated in the controller synthesis. Input/output linearization is selected to provide the setpoint tracking ability. For disturbance rejection, the high-gain technique is used to compensate for the effect of the uncertainty. The control performance of the method is evaluated through numerical simulation of continuous stirred tank reactors with uncertainty. The simulation results show that both unmeasured disturbance and parametric uncertainty can be effectively compensated for by the proposed control method.

**Key words:** High-gain Feedback, Input/Output Linearization, Nonlinear Control, Two-degree-of-freedom Control, Uncertain Processes

### INTRODUCTION

The influence of uncertainty is an important factor for controller design, as it may cause degradation of robustness and control performance, which could affect product quality and safety performance. Generally, the uncertainty can be classified into two categories: unmeasured disturbance and parametric uncertainty. Unmeasured disturbances during process operation are naturally from the fluctuation in the input or controlled output process streams, the friction of control valves, or the sensitivity of measuring devices. Parametric uncertainty is inherent in physical properties, kinetic parameters, and transport coefficients. The parameter values obtained from the laboratory-scale experiments might be inaccurate in commercial-scale reactors due to differences in mixing conditions, volume ratios, heat transfer rates, and/or mass transfer rates. Furthermore, the assumption that considers the values of process parameters to be constant is not realistic for processes with time-varying parameters.

To control uncertain processes, the controller should be able to compensate for uncertainty and maintain closed-loop stability. Two-degree-of-freedom (2DOF) control is an effective method for handling problems of uncertainty that has received considerable attention over the past decade [1-10], especially for input disturbances. The advantage of the 2DOF scheme is an ability to compensate for the uncertainty quickly because setpoint tracking and disturbance rejection can be handled independently [1]. The 2DOF control structure consists of two controllers, the setpoint tracking and disturbance rejection controllers. The 2DOF control methods have basically been developed in a Laplace transform domain [1-8].

The 2DOF control of nonlinear systems has not been much reported in the literature. Wright and Kravaris studied some related research work that developed model algorithm control with the 2DOF

structure in continuous-time [9] and discrete-time systems [10]. Furthermore, all of the work reviewed in 2DOF control has considered only unmeasured disturbances. Parametric uncertainty has not been mentioned in the studies.

Motivated by these previous works, this paper presents a new control method with the 2DOF control structure: input/output (I/O) linearization control and high gain technique are applied to handle nonlinear processes with unmeasured input disturbances and parametric uncertainties. The I/O linearizing controller with the disturbance-free model provides setpoint tracking ability, while the high-gain controller compensates for the offset caused by the mismatch between the actual and estimated disturbance-free outputs. As the information of disturbance-free states and outputs is required to calculate the control action, the disturbance-free state estimator is integrated into the control system. The advantage of the proposed method is its ability to handle a nonlinear system with uncertainty, which is important since only a few nonlinear control methods with 2DOF structure have been reported in the literature. The proposed method is also easy to implement, because it has a simple control structure and few tuning parameters which need to be adjusted. Furthermore, it provides a simple solution to apply the I/O linearizing controller for an uncertain nonlinear system compared to other formulation of the I/O linearizing controllers with 1DOF structure. To illustrate controller performance, the proposed control method is applied to chemical stirred tank reactors in the presences of unmeasured input disturbances and parametric uncertainties.

### CONTROL SYSTEM DESIGN

The schematic diagram of the proposed control system is shown in Fig. 1. The control system consists of a setpoint tracking controller, a disturbance rejection controller, and a disturbance-free state estimator. More details of the control system design are given as follows.

<sup>\*</sup>To whom correspondence should be addressed.  
E-mail: fengcnp@ku.ac.th

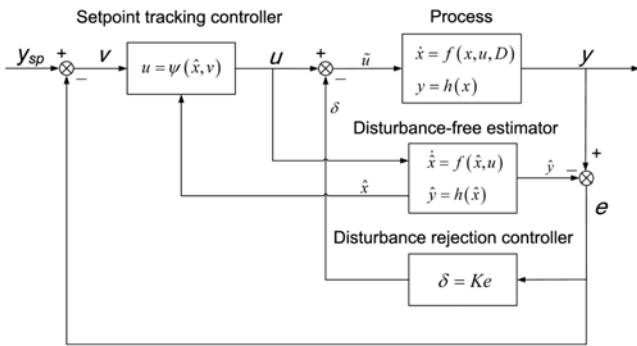


Fig. 1. Schematic diagram of the proposed control system.

### 1. Setpoint Tracking Controller

The setpoint tracking controller is formulated by using I/O linearization. The idea of I/O linearization is to find a direct relation between the output  $y$  and the input  $u$ . This is achieved by repeatedly differentiating the output  $y$  with respect to time, until it is explicitly related to the input  $u$ , which is called relative order. A review of the input/output linearization approach and definition of relative order can be found in Slotine and Li [11] and Henson and Seborg [12].

Consider the general class of nonlinear systems of the following form:

$$\begin{aligned}\dot{x} &= f(x, u) \\ y &= h(x)\end{aligned}\quad (1)$$

where  $x = [x_1, \dots, x_n]^T \in X$  is the vector of state variables,  $u = [u_1, \dots, u_m]^T \in U$  is the vector of manipulated inputs,  $y = [y_1, \dots, y_m]^T$  is the vector of controlled outputs, and  $h$  and  $f$  are smooth functions. For the nonlinear system in Eq. (1),  $r_1, \dots, r_m$  are the relative orders of the controlled output,  $y_1, \dots, y_m$ , with respect to the manipulated inputs.

We request a linear response of the following form for each output:

$$\begin{aligned}(\varepsilon_1 \mathcal{D} + 1)^{r_1} y_1 &= y_{sp,1} \\ &\vdots \\ (\varepsilon_m \mathcal{D} + 1)^{r_m} y_m &= y_{sp,m}\end{aligned}\quad (2)$$

where  $\mathcal{D}$  is the differential operator (i.e.  $\mathcal{D} = d/dt$ ),  $y_{sp,1}, \dots, y_{sp,m}$  are the desired setpoints and  $\varepsilon_1, \dots, \varepsilon_m$  are the tuning parameters that adjust the speed of the responses of the outputs,  $y_1, \dots, y_m$ , respectively. By substituting the time derivatives of the outputs into Eq. (2), one obtains:

$$\begin{aligned}h_1(x) + \binom{r_1}{1} \varepsilon_1 h_1^1(x) + \dots + \binom{r_1}{r_1} \varepsilon_1^{r_1} h_1^{r_1}(x, u) &= y_{sp,1} \\ &\vdots \\ h_m(x) + \binom{r_m}{1} \varepsilon_m h_m^1(x) + \dots + \binom{r_m}{r_m} \varepsilon_m^{r_m} h_m^{r_m}(x, u) &= y_{sp,m}\end{aligned}\quad (3)$$

where  $\binom{a}{b} = \frac{a!}{(b-a)!}$  and  $h_i^j$  is the  $j$ -th order time derivatives of the function of output  $i$

The closed-loop responses of the outputs in Eq. (3) can be pre-

sented in the compact form:

$$\begin{aligned}\Phi_1(x, u) &= y_{sp,1} \\ &\vdots \\ \Phi_m(x, u) &= y_{sp,m}\end{aligned}\quad (4)$$

By solving Eq. (4) for  $u$ , the static feedback controller can be obtained in following form:

$$u = \Psi(x, y_{sp})\quad (5)$$

Note that the feedback controller in Eq. (5) can be applied when the process under consideration is stable and minimum-phase system.

### 2. Disturbance Rejection Controller

The disturbance rejection controller is used to compensate for the mismatch between the process outputs and the estimated disturbance-free outputs, which arises from the uncertainty. The disturbance rejection controller is constructed based on a high-gain feedback technique as follows:

$$\begin{aligned}\delta_1 &= K_1(y_1 - \hat{y}_1) \\ &\vdots \\ \delta_m &= K_m(y_m - \hat{y}_m)\end{aligned}\quad (6)$$

where  $\delta_1, \dots, \delta_m$  are the estimated disturbances of the outputs  $y_1, \dots, y_m$ ,  $\hat{y}_1, \dots, \hat{y}_m$  are the estimated disturbance-free outputs, and  $K_1, \dots, K_m$  are the tuning parameters that should be selected for stabilizing the uncertain processes.

### 3. Disturbance-free State Estimator

The setpoint tracking controller and the disturbance rejection controller require information of the states and outputs of the disturbance-free process. To estimate that information, the open-loop observer is applied in the control scheme. The dynamics of the disturbance-free process are described by following equation:

$$\begin{aligned}\dot{\hat{x}} &= f(\hat{x}, u) \\ \hat{y} &= h(\hat{x})\end{aligned}\quad (7)$$

where  $\hat{x}$  is the vector of estimated states, and  $\hat{y}$  is the vector of estimated outputs.

### 4. Control System

To ensure offset-free response of the closed-loop system when there is process-model mismatch and/or load disturbance in the system, the integral action should be added in a control system. By combining the disturbance-free state estimator in Eq. (7), the state

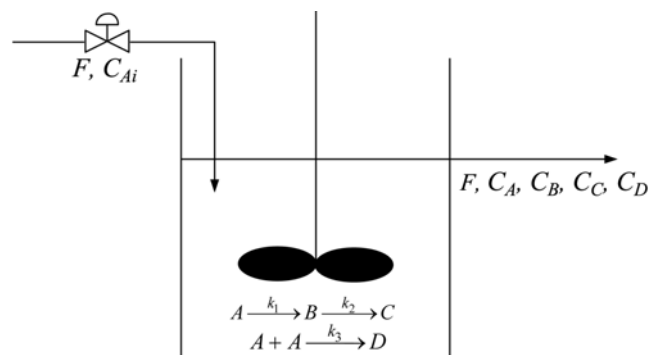


Fig. 2. Schematic of a continuous stirred tank reactor.

feedback in Eq. (5), and the estimated disturbances in Eq. (6), the feedback control system can be described by

$$\begin{aligned}\dot{\hat{x}} &= f(\hat{x}, u) \\ \nu &= y_{sp} - (y - h(\hat{x})) \\ u &= \psi(\hat{x}, y_{sp} - (y - h(\hat{x}))) \\ \delta &= K(y - h(\hat{x})) \\ \tilde{u} &= u - \delta\end{aligned}\quad (8)$$

where  $\nu$  is the vector of compensated setpoints and  $\tilde{u}$  is the vector of compensated inputs. Note that the values of  $\varepsilon$  must be greater than zero ( $\varepsilon > 0$ ). A smaller value of  $\varepsilon$  gives a faster response of the output. The tuning parameters of the disturbance rejection control-

ler,  $K$ , need to be chosen to be positive when direct control action is required; likewise, a negative value needs to be chosen for indirect control action. The magnitude of tuning parameters is proportional to the error in the controlled outputs relative to the estimated outputs.

## ILLUSTRATIVE EXAMPLES

### 1. Single-input Single-output Isothermal Reactor with Van De Vusse Reaction

Consider a continuous stirred tank reactor (CSTR) as shown in Fig. 2. The reaction consists of two decomposition reactions of A occurring in parallel [13].



It is assumed that the reactor volume and physical parameters are constant, and the reactor is perfectly mixed. The mathematical model of the reactor can be expressed by:

$$\frac{dC_A}{dt} = -k_1 C_A - k_3 C_A^2 + (C_{Ai} - C_A) \frac{F}{V}$$

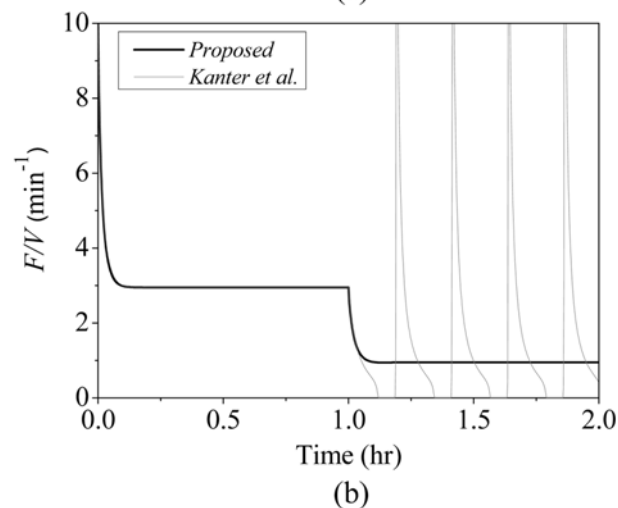
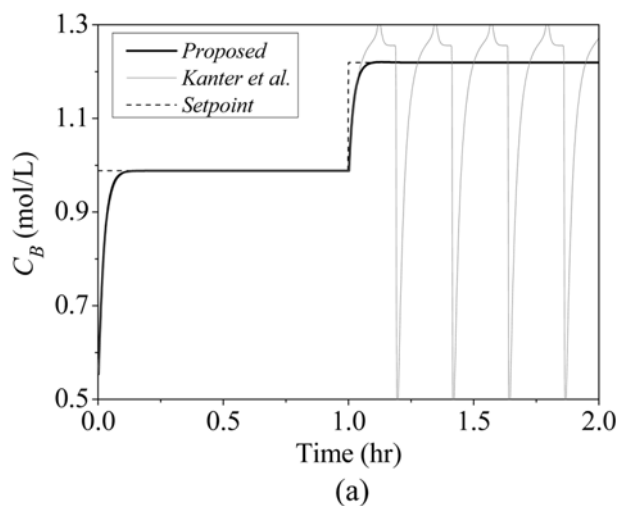


Fig. 3. Closed-loop responses of (a)  $C_B$ , (b)  $F/V$  for the case of unmeasured disturbance in  $F/V$ .

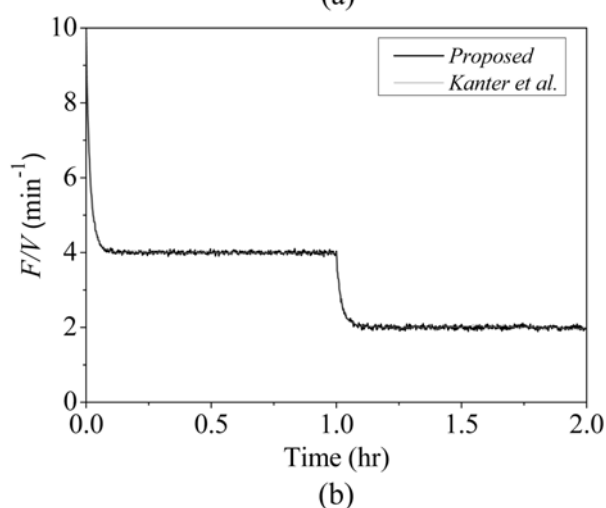
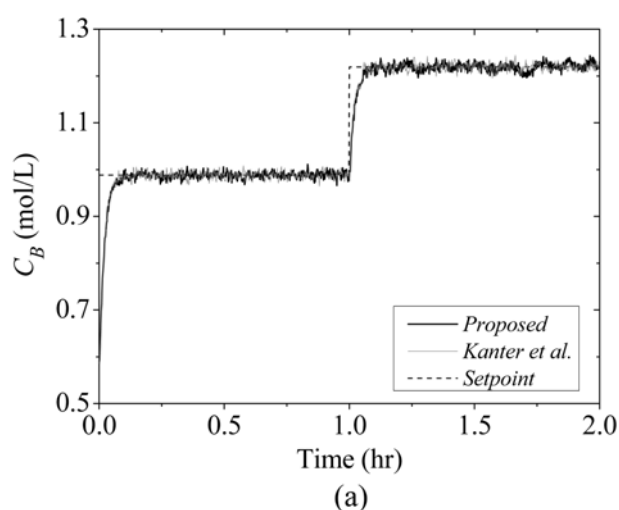


Fig. 4. Closed-loop responses of (a)  $C_B$ , (b)  $F/V$  for the case of parametric uncertainty in  $k_1$ .

$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B - C_B \frac{F}{V}$$

$$y = C_B \quad (10)$$

where  $C_A$  and  $C_B$  are the concentrations of A and B, respectively.  $C_B$  is the only measurable state in the process. The objective is to control  $C_B$  at the desired setpoints by manipulating the dilution rate ( $F/V$ ) within the operating range  $0 \leq F/V \leq 15 \text{ min}^{-1}$ . The values of the process parameters are given in Table 1.

To evaluate the control performance, the control system is tested by setpoint tracking of two given setpoints. The simulation studies are divided into two cases: the process under  $+1.05 \text{ min}^{-1}$  step disturbance in manipulated input ( $F/V$ ) and  $\pm 40\%$  random noise parametric uncertainty in rate constant  $k_1$ . In both cases, the uncertainties are introduced into the system at  $t=0$ . The error-feedback I/O lineariz-

ing method by Kanter et al. [14], in which the I/O linearizing controller uses a disturbance-free model to estimate the unmeasured states, is used for comparison purpose. The sets of parameter values,  $\{\varepsilon=0.8 \text{ min}, K=-6 \text{ L kmol}^{-1} \text{ min}^{-1}, r=1\}$  for the proposed control method and  $\{\varepsilon_1=0.8 \text{ min}, r=1\}$  for the Kanter et al. method, are applied in the test. The process starts with the initial condition:  $F/V=10 \text{ min}^{-1}$ ,  $C_A=8.20 \text{ mol/L}$ , and  $C_B=0.58 \text{ mol/L}$ . The first desired setpoint of the process is  $y_{sp}=0.99 \text{ mol/L}$  ( $(F/V)_{ss}=4 \text{ min}^{-1}$ ,  $C_{A,ss}=6.72 \text{ kmol/L}$ , and  $C_{B,ss}=0.99 \text{ kmol/L}$ ). Then, the setpoint is changed to  $y_{sp}=1.22 \text{ mol/L}$  ( $(F/V)_{ss}=2 \text{ min}^{-1}$ ,  $C_{A,ss}=5.37 \text{ kmol/L}$ , and  $C_{B,ss}=1.22 \text{ kmol/L}$ ) at  $t=1 \text{ hr}$ .

Fig. 3 shows the closed-loop responses of the process for a  $+1.05 \text{ min}^{-1}$  step disturbance in the manipulated input. It is clear that the proposed method successfully brings the reactor to the desired setpoints without offset, while the method of Kanter et al. shows an

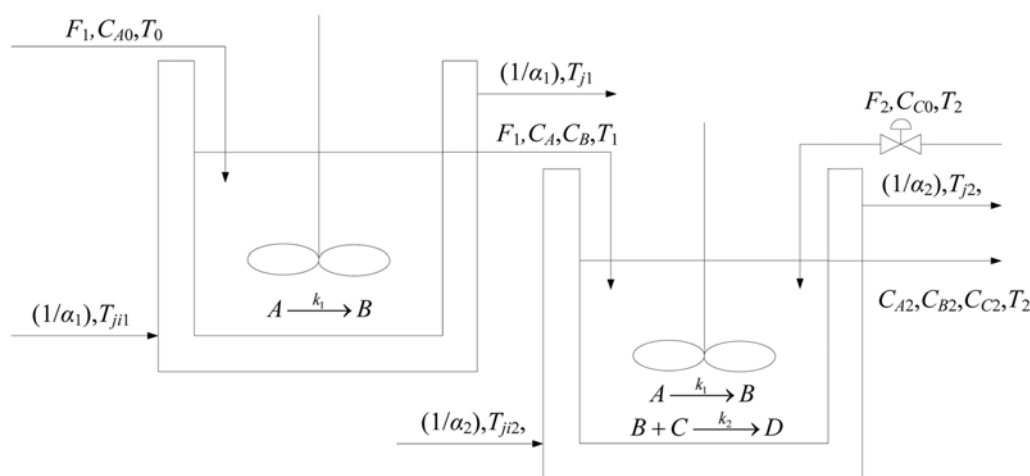


Fig. 5. Schematic of two continuous stirred tank reactors in series.

Table 2. Parameters for the jacketed stirred tank reactors in series

Symbol	Quantity	Value
$V_1$	Volume of reactor 1	$1 \text{ m}^3$
$V_2$	Volume of reactor 2	$1.5 \text{ m}^3$
$UA_1$	Heat transfer coefficient of reactor 1	$1.4 \text{ kJ/(s K)}$
$UA_2$	Heat transfer coefficient of reactor 2	$1.75 \text{ kJ/(s K)}$
$\alpha_1$	Feed jacket flowrate per volume of reactor 1	$300 \text{ s}$
$\alpha_2$	Feed jacket flowrate per volume of reactor 2	$300 \text{ s}$
$-\Delta H_1$	The change in enthalpy of reaction 1	$46,000 \text{ kJ/kmol}$
$-\Delta H_2$	The change in enthalpy of reaction 2	$20,000 \text{ kJ/kmol}$
$k_{01}$	Arrhenius factor of reaction 1	$1.8 \times 10^7 \text{ s}^{-1}$
$k_{02}$	Arrhenius factor of reaction 2	$2.0 \times 10^6 \text{ s}^{-1}$
$E_1$	Activation energy of reaction 1	$67,000 \text{ kJ/kmol}$
$E_2$	Activation energy of reaction 2	$60,000 \text{ kJ/kmol}$
$C_{A0}$	Concentration of A in feed stream 1	$7.5 \text{ kmol/m}^3$
$C_{c0}$	Concentration of C in feed stream 2	$20 \text{ kmol/m}^3$
$\rho$	Density of cooling water	$1,000 \text{ kg/m}^3$
$C_p$	Heat capacity of cooling water	$4.2 \text{ kJ/(kg K)}$
$T_0$	Temperature in feed stream 1	$300 \text{ K}$
$R$	Gas constant	$8.345 \text{ kJ/(kmol K)}$
$F_1$	Flowrate of feed stream 1	$0.00065 \text{ m}^3/\text{s}$

oscillation after changing to the second setpoint. Fig. 4 shows the closed-loop responses of the process for  $\pm 40\%$  random noise parametric uncertainty in rate constant  $k_1$ . The simulation results show that both methods can track the outputs around the desired setpoint despite of the occurrence of parametric uncertainty. The proposed

method provides a slight improvement in the reduction of the effect of uncertainty. The Kanter et al. method can compensate for the effect of parametric uncertainty due to the use of the disturbance-free model in the state estimation for I/O feedback controller; however, it cannot maintain closed-loop stability when an input distur-

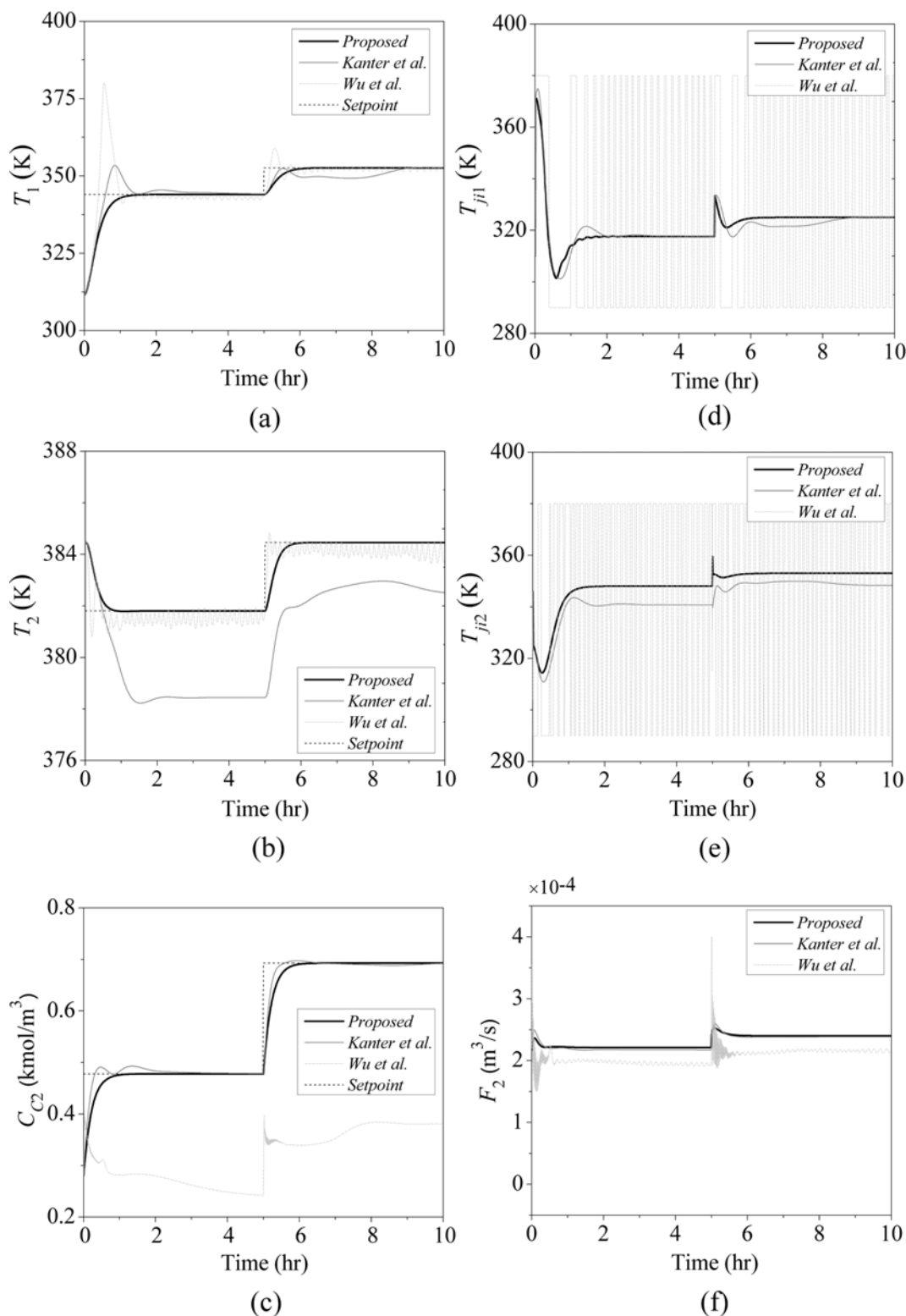


Fig. 6. Closed-loop responses of (a)  $T_1$ , (b)  $T_2$ , (c)  $C_{C2}$ , (d)  $T_{j11}$ , (e)  $T_{j12}$ , (f)  $F_2$  for the case of unmeasured disturbances in  $T_1$ ,  $T_2$ , and  $F_2$ .

bance is present in the process.

## 2. Two Jacketed Reactors in Series

The proposed control method is now applied to a more complex process that consists of two constant-volume jacketed reactors in series [15]. The simple process diagram is shown in Fig. 5. The exo-

thermic reaction  $A \rightarrow B$  takes place in the first reactor, while the exothermic reactions  $A \rightarrow B$  and  $B + C \rightarrow D$  take place in the second reactor. Assuming that all physical properties are constant, the reactor is operated in perfect mixing condition, and the state variables,  $T_1$ ,  $T_2$  and  $C_{C2}$ , are measurable, the mathematical model of

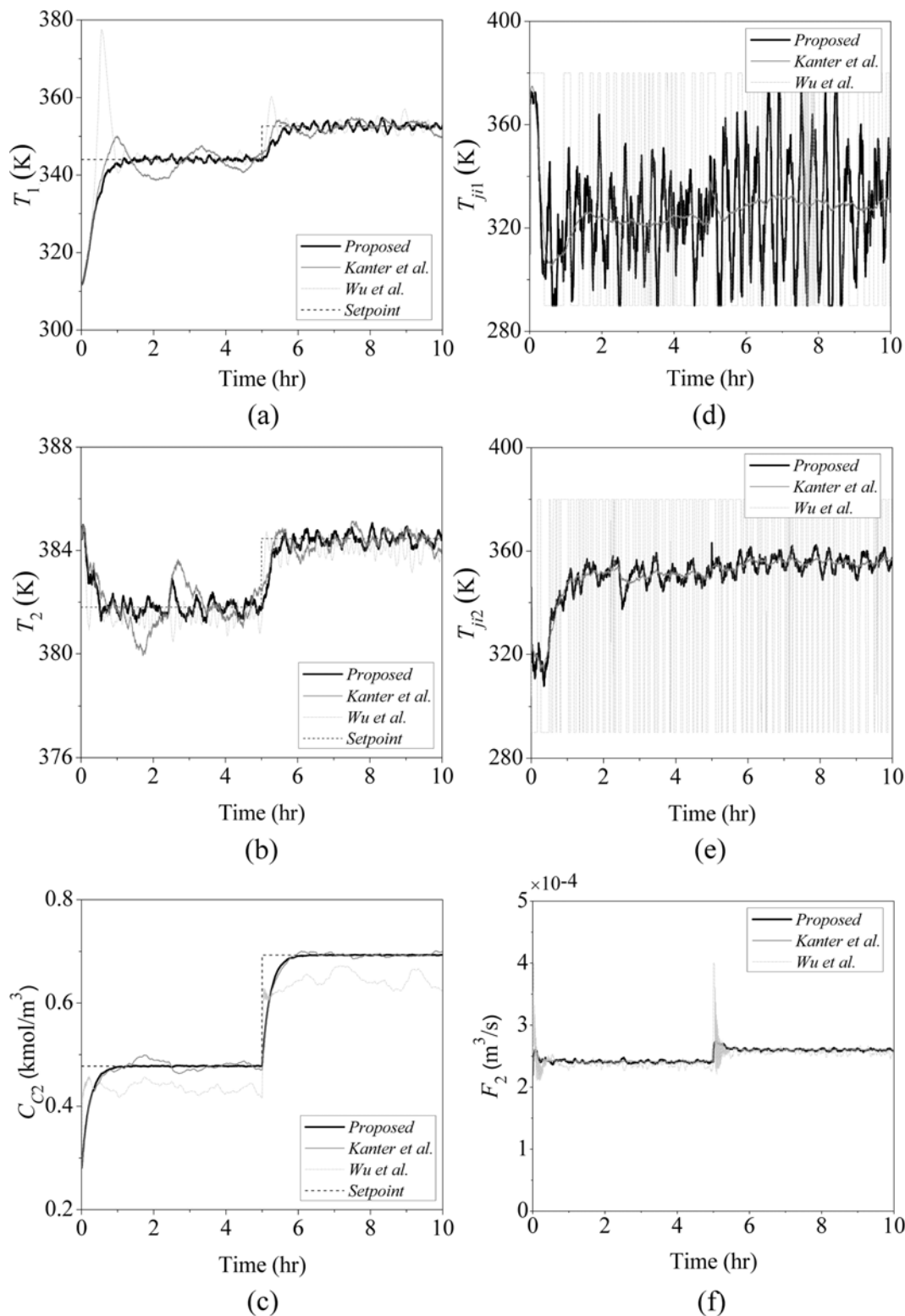


Fig. 7. Closed-loop responses of (a)  $T_1$ , (b)  $T_2$ , (c)  $C_{C2}$ , (d)  $T_{j1}$ , (e)  $T_{j2}$ , (f)  $F_2$  for parametric uncertainties in  $DH_1$  and  $\Delta H_2$ .

the process is given by

$$\begin{aligned}
 \frac{dT_1}{dt} &= \frac{F_1}{V_1}(T_0 - T_1) - \frac{UA_1}{V_1\rho C_p}(T_1 - T_{j1}) + \frac{(-\Delta H_1)}{\rho C_p}k_{01}C_{A1}\exp\left(-\frac{E_1}{RT_1}\right) \\
 \frac{dC_{A1}}{dt} &= \frac{F_1}{V_1}(C_{A0} - C_{A1}) - k_{01}C_{A1}\exp\left(-\frac{E_1}{RT_1}\right) \\
 \frac{dC_{B1}}{dt} &= -\frac{F_1}{V_1}C_{B1} + k_{01}C_{A1}\exp\left(-\frac{E_1}{RT_1}\right) \\
 \frac{dT_{j1}}{dt} &= \frac{1}{\alpha_1}(T_{j1l} - T_{j1}) \\
 \frac{dT_2}{dt} &= \frac{F_2}{V_2}(T_1 - T_2) - \frac{UA_2}{V_2\rho C_p}(T_2 - T_{j2}) + \frac{(-\Delta H_1)}{\rho C_p}k_{01}C_{A2}\exp\left(-\frac{E_1}{RT_2}\right) \\
 &\quad + \frac{(-\Delta H_2)}{\rho C_p}k_{02}C_{B2}C_{C2}\exp\left(-\frac{E_2}{RT_2}\right) \\
 \frac{dC_{A2}}{dt} &= \frac{F_1}{V_2}C_{A1} - \frac{F_1+F_2}{V_2}C_{A2} - k_{01}C_{A2}\exp\left(-\frac{E_1}{RT_2}\right) \\
 \frac{dC_{B2}}{dt} &= \frac{F_1}{V_2}C_{B1} - \frac{F_1+F_2}{V_2}C_{B2} + k_{01}C_{A2}\exp\left(-\frac{E_1}{RT_2}\right) \\
 &\quad - k_{02}C_{B2}C_{C2}\exp\left(-\frac{E_2}{RT_2}\right) \\
 \frac{dC_{C2}}{dt} &= \frac{F_2}{V_2}C_{C0} - \frac{F_1+F_2}{V_2}C_{C2} - k_{02}C_{B2}C_{C2}\exp\left(-\frac{E_2}{RT_2}\right) \\
 \frac{dT_{j2}}{dt} &= \frac{1}{\alpha_2}(T_{j2l} - T_{j2}) \\
 y &= [T_1 \ T_2 \ C_{C2}]^T, \quad u = [T_{j1} \ T_{j2} \ F_2]^T
 \end{aligned} \quad (11)$$

The process parameters and their values are given in Table 2. The objective is to control the temperature of both reactors ( $T_1$ ,  $T_2$ ) and the outlet concentration of C in the second reactor ( $C_{C2}$ ) at the desired setpoints by adjusting the inlet coolant temperature of both reactors ( $T_{j1}$ ,  $T_{j2}$ ) and the feed flow rate of C in the second reactor ( $F_2$ ). The operating ranges of the manipulated inputs are  $290 \leq T_{j1}$ ,  $T_{j2} \leq 380$  K and  $0 \leq F_2 \leq 0.004$  m<sup>3</sup>/s, respectively.

To evaluate the control performance, the control system is tested by setpoint tracking of two given sets of setpoints. In the simulation studies, +5 K, +2 K, and  $+2 \times 10^{-5}$  m<sup>3</sup>/s step disturbances in  $T_{j1}$ ,  $T_{j2}$ , and  $F_2$  and  $\pm 20\%$  random noise parametric uncertainty in  $\Delta H_1$  and  $\Delta H_2$  are taken into consideration. In both cases, uncertainties are introduced into the system at  $t=0$ . The control method by Kanter et al. [14] mentioned in the previous example and the linear controller with 2DOF scheme by Wu et al. [2] are used for comparison purposes. The Wu et al. method uses the proportional (P)-controller for the tracking controller, while the high-gain technique is applied for disturbance rejection controller. The sets of parameter values,  $\{\varepsilon_1=700$  s,  $\varepsilon_2=500$  s,  $\varepsilon_3=800$  s,  $K_1=50$ ,  $K_2=10$ ,  $K_3=0.003$  m<sup>6</sup> kmol<sup>-1</sup> s<sup>-1</sup>,  $r_1=2$ ,  $r_2=2$ ,  $r_3=1\}$  for the proposed control method,  $\{\varepsilon_1=700$  s,  $\varepsilon_2=500$  s,  $\varepsilon_3=800$  s,  $r_1=2$ ,  $r_2=2$ ,  $r_3=1\}$  for Kanter et al. method, and  $\{K_{C1}=1000$ ,  $K_{C2}=2000$ ,  $K_{C3}=0.005$  m<sup>6</sup> kmol<sup>-1</sup> s<sup>-1</sup>,  $K_{F1}=50$ ,  $K_{F2}=50$ , and  $K_{F3}=0.01$  m<sup>6</sup> kmol<sup>-1</sup> s<sup>-1</sup> $\}$  for the Wu et al. method, are applied in the test. The process initially starts from  $T_1=311.7$  K,  $C_{A1}=6.36$  kmol/m<sup>3</sup>,  $C_{B1}=1.14$  kmol/m<sup>3</sup>,  $T_{j1}=310$  K,  $T_2=384.4$  K,  $C_{A2}=0.17$  kmol/m<sup>3</sup>,  $C_{B2}=0.65$  kmol/m<sup>3</sup>,  $C_{C2}=0.28$  kmol/m<sup>3</sup>, and  $T_{j2}=345.8$  K. The first set of desired setpoints of the process is  $y_{sp}=[344, 381.8, 0.5]$ . Then, the set of setpoints is changed to  $y_{sp}=[352.6, 384.5, 0.7]$ .

The control performance is tested with step disturbances in each manipulated input, as shown in Fig. 6. Under the proposed method,

both setpoint tracking and disturbance rejection controllers successfully force the controlled output to move to the desired setpoints without offsets and efficiently eliminate the disturbances, while the Kanter et al. method and Wu et al. method cannot compensate for the effect of input disturbance. The control actions of  $T_{j1}$  and  $T_{j2}$  under the Wu et al. method show high oscillation because the control structure with the P-controller is not proper for a highly nonlinear process. The comparison results in the presence of parametric uncertainties in  $\Delta H_1$  and  $\Delta H_2$  are shown in Fig. 7. The results indicate that the proposed controller effectively compensates for the effect of uncertainty to maintain the outputs at the desired setpoints, while the Wu et al. method cannot. The proposed method provides slight improvement compared to the Kanter et al. method because the proposed method is a generalization of the Kanter et al. method.

## CONCLUSIONS

An input-output linearization controller with 2DOF control structure is proposed to handle nonlinear processes with input and parameter uncertainties. With the 2DOF structure, the setpoint tracking and disturbance rejection responses can be independently designed. The I/O linearizing controller provides tracking ability with the use of few tuning parameters. The robustness of disturbance rejection is achieved by adjusting the gain of the disturbance rejection controller. Using the chemical processes with step disturbances and random noise in the process parameters, the performance of the proposed control method is illustrated. Results of simulations show that the proposed control scheme can successfully operate the reactors at the desired setpoints; the effects of unmeasured disturbance and parametric uncertainties were compensated for by the disturbance rejection controller to maintain closed-loop stability.

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## NOMENCLATURE

K	: tuning parameters of disturbance rejection controller
m	: number of manipulated inputs
n	: number of state variables
r	: relative order of the output
t	: time
u	: vector of manipulated inputs
x	: vector of state variables
y	: vector of controlled outputs

## Greek Letters

$\delta$	: vector of estimated disturbances
$\varepsilon$	: tuning parameters of setpoint tracking controller

$\nu$  : compensated setpoint  
 $\mathcal{D}$  : differential operator

### Subscripts

ss : steady state  
sp : setpoint

### Superscripts

$\hat{\phantom{x}}$  : estimated value

## REFERENCES

1. M. H. Tsai and P. C. Tung, *J. Process. Contr.*, **20**, 777 (2010).
2. K. L. Wu, C. C. Yu and Y. C. Cheng, *J. Process. Contr.*, **11**, 311 (2001).
3. W. Tan, *ISA T.*, **49**, 311 (2010).
4. T. Liu, W. Zhang and D. Gu, *J. Process. Contr.*, **15**, 559 (2005).
5. T. Liu, W. Zhang and F. Gao, *Ind. Eng. Chem. Res.*, **46**, 6546 (2007).
6. T. Liu, D. Gu and W. Zhang, *J. Process. Contr.*, **15**, 159 (2005).
7. M. Shamsuzzoha and M. Lee, *Korean J. Chem. Eng.*, **25**, 637 (2008).
8. J. Chen and K.-T. Chou, *Korean J. Chem. Eng.*, **26**, 1512 (2009).
9. R. A. Wright and C. Kravaris, *Chem. Eng. Sci.*, **60**, 4323 (2005).
10. R. A. Wright and C. Kravaris, *Chem. Eng. Sci.*, **61**, 4676 (2006).
11. J.-J. E. Slotine and W. Li, *Applied nonlinear control*, Prentice Hall, Englewood Cliffs, NJ (1991).
12. M. A. Henson and W. Seborg, *Nonlinear process control*, Prentice Hall, Upper Saddle River, NJ (1996).
13. B. Aufderheide and B. W. Bequette, *Comput. Chem. Eng.*, **27**, 1079 (2003).
14. J. M. Kanter, M. Soroush and W. D. Seider, *Ind. Eng. Chem. Res.*, **40**, 2069 (2001).
15. R. A. Wright and C. Kravaris, *Chem. Eng. Sci.*, **58**, 3243 (2003).