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Influence of particle distribution on filter coefficient in the initial stage of filtration

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Abstract−Under the condition that the size of suspended particles is nonhomogeneous, we studied how filter grain and suspended particles affect filter coefficient in the early stage. A stochastic method was used to study the variation of the initial filter coefficient. Through physical experiment, the collected data include the initial inflow and outflow concentration, the size distributions of particles in suspension and so on. By introducing the standard capture probability function P(d) and the characteristic length of filter bed L_c, this separated the grains' and particles' influence on filter coefficient. An example showed that we could use P(d), L_c to ascertain the result of effluent distribution, ratio of C_{out} to C_{in} in any depths and equivalent filter coefficient λ_m. We also studied the filter coefficient λ_l during the experiment and the first-order derivative of λ_l.

Key words: Variation of Filter Coefficient, Particle Size Distribution, The Characteristic Length of Filter Bed

INTRODUCTION

In water treatment systems, filtration is an important step in the solid-liquid separation process. A number of methods have been proposed in the literature [1-3] to calculate the removal of particles during the transient stage of deep bed filtration. Filter coefficient is an important parameter in the filtration study of those papers. It is a parameter reflecting the filtration efficiency of the filter bed that changes with time and space. During the filtration process, the filter coefficient varies due to the reducing of the filter's porosity as well as the increasing of the interfacial velocity caused by the accumulation of suspended particles [4,5]. It indicates that the filter coefficient is a function of time. Moreover, through the observation of initial stages of the filtration process, we found that the filter coefficient also is affected by the filter bed depth even in the same suspension and filter grain. This indicates that the filter coefficient is not only a function of time but also a function of space.

Up to now, few studies have been carried out on that how the size distribution of suspended particles affects the filtration coefficient during the filtering process. In general experimentsand theoretical analysis, the impact of size distribution of suspended particles on the filtration process is usually ignored, because they generally

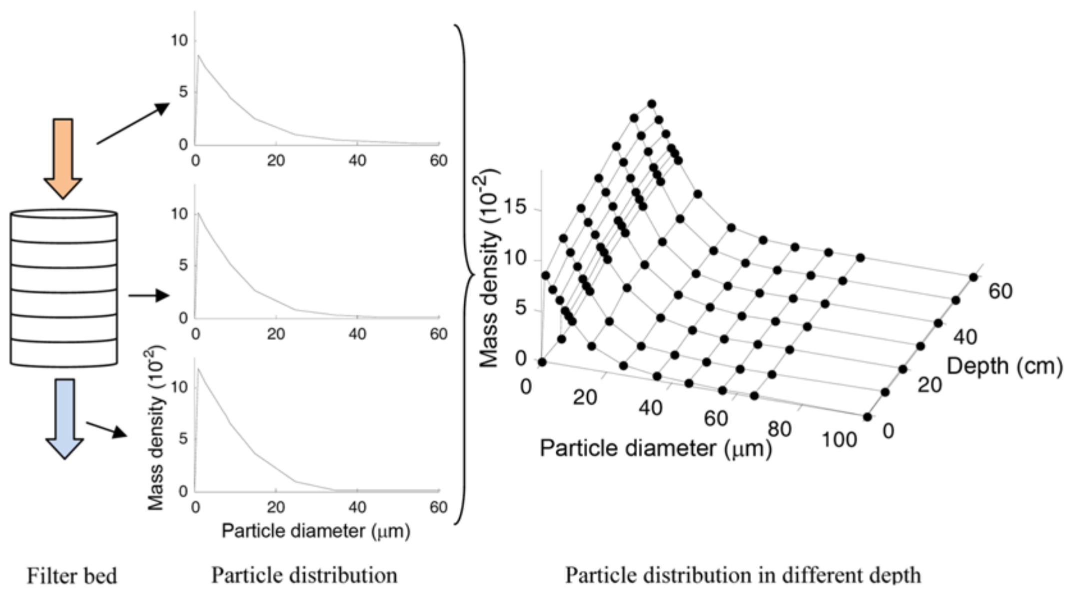

Fig. 1. Variation of size distribution of suspended particles at different depth of filter bed.

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assumed that the size of suspended particles is a fixed value. Several methods have been proposed in the literature for calculating the removal of particles during the transient stage of deep bed filtration. Also, many literatures state that particle size plays a major role in the removal of particles by a clean bed filter [6,7,9]. In fact, the size of the suspended particles and their size distribution are very important physical parameters that affect the filtration efficiency. With the suspension moving from the top to the bottom of the filter bed, the size distribution of suspended particleschanged constantly (Fig. 1), hence the filtration coefficient also varied.

In this study, the main object is to analyze the filtration coefficient function in the initial stage, which is a complex function. Although the impact of deposition on the filter coefficients is ignored in the early stages, the function is still subject to many factors, including physical and chemical properties of suspended particles and also these properties of filter bed. Among these factors, the size distribution of suspended particles would change during the filtration process, except the sizes of all the particles are absolutely the same (Fig. 1).

Hence, the objectives of this study include: 1) How the different size distribution of suspended particles affect the initial filter coefficient 2) How the different diameters of the filter affect the initial filter coefficient.

STOCHASTIC METHOD FOR INITIAL FILTRATION COEFFICIENT CALCULATION

1. Basic Theory

Particle removal in deep bed filter is expressed by Eq. (1) [8]:

$$\frac{\partial C}{\partial l} = -\lambda C, \tag{1}$$

where C is the particle concentration in the suspension l is the axial distance from the top of the filter bed and λ is the filter coefficient. As soon as the suspension reaches the depth l of filter bed, the corrected time is set and it can be expressed as Eq. (2):

$$\Theta = t - \int_0^L \frac{dl}{U/\theta}, \tag{2}$$

where Θ is the corrected time at different depths of filter bed L is the lengthof filter bed U is the filtration velocity and θ is the porosity of the filter bed.

In a definite filtration process, it is usually considered that the filter coefficient λ is influenced by the retention rate σ. Therefore, the initial filter coefficient λ_0 (σ=0 or Θ=0) can be deduced from Eq. (1) as:

$$C_{out}/C_{in} = \exp(-\lambda_0 L), \tag{3}$$

where C_{in} is the influent particle concentration C_{out} is the effluent particle concentration L is the depth of filter bed and λ_0 is the initial filter coefficient. When particles in the suspension are nonhomogeneous, the filter coefficient is not only a function of time but also of space. That is, the initial filter coefficient varies with depth of the filter bed. So, actually the initial equivalent filter coefficient calculated by Eq. (3) is the mean value of initial filter coefficient on the depth from 0 to L. In this paper, we consider it as the initial equivalent filter coefficient.

Since this study mainly worked at the initial stage of filtration, for convenience, the initial filter coefficient would be called filter coefficient and expressed as λ_l, while the initial equivalent filter coefficient wouldbe called equivalent filter coefficient and expressed as λ_m. The relationship between λ_m and λ_l is shown as the following:

$$\lambda_m(L) = \frac{1}{L}\int_0^L \lambda_l(l)dl, \quad l \in [0, L]. \tag{4}$$

We already know that λ_l varies with depth of the filter bed unless all suspended particles have the same size (Fig. 3). A reason for this phenomenon is that particles with different sizes have different captured probability. And the filter bed always preferentially captures the particles which have higher captured probability. Thus, with the depth increasing, particles with higher captured probability become less and less. In this case, the values of λ_m and λ_l decrease as the filter bed depth increases.

2. A Stochastic Method for Filter Coefficient

To analyze the decreasing process of the filter coefficient, a stochastic method was constructed which rests on the following assumptions:

1) Particles in suspensions cannot be polymerized.
2) Particles in suspensions cannot be cracked.
3) Eachlayer of the filter bed has the same capture ability to the same size of particles.
4) The captured probability of each particle in the suspension is independent.

Based on these assumptions, in this paper, the capture probability function p(d) can be defined as the captured probability of particles with diameter d, when they pass through a unit depth of the filter bed in the initial stage.

According to statistical theory, the captured proportion of a large number of particles with the same size is similar to the captured probability of a single particle. Therefore, when a particle with diameter d passes through a filter bed of L in depth, the probability of the particle cannot be captured can be expressed as:

$$q(d) = (1 - p(d))^L, \tag{5}$$

where q is the probability the particle was not captured. Furthermore, we introduced the function of mass density Q(d) to describe the size distribution of the particles in suspension. Q(d) is formulated as follows:

$$Q(d) = \lim_{\Delta d \to 0} \frac{m(d \to d + \Delta d)}{M\Delta d}, \tag{6}$$

where d is the diameter of particle, m (d→d+Δd) is the mass of particles that have a diameter in the range d to d+Δd, and M is the total mass of all particles. Based on Eq. (5) and Eq. (6), when the suspension passes through the depth (ΔL) of filter bed in the time (Δt), the decreased mass of particles can be expressed as:

$$\Delta m = \Delta t A U C_{in} \int_0^\infty Q(x)(1 - (1 - p(x))^{\Delta L})dx, \tag{7}$$

where A is the filtering surface area, p(d) is the capture probability function, and t is time. Combining Eq. (7) with Eq. (3), when the depth ΔL trends to infinitesimal, the relation between filter coefficient λ_l, capture probability function p(d) and mass density function Q(d) is shown as:

$$\lim_{\Delta L\to 0}\int_0^\infty Q(x)(1-(1-p(x))^{\Delta L})dx=\lim_{\Delta L\to 0}(1-\exp(-\lambda_l\Delta L)). \tag{8}$$

Eq. (8) can be transformed to:

$$\lambda_l=-\lim_{\Delta L\to 0}\frac{\ln\left(\int_0^\infty Q(x)(1-(1-p(x))^{\Delta L})dx\right)}{\Delta L} \tag{9}$$

Eq. (10) could be obtained by simplifying Eq. (9):

$$\lambda_l=-\int_0^\infty Q(x)\ln(1-p(x))dx. \tag{10}$$

Here, the capture probability function p(d) is invariant with depth of filter bed, but the mass density function Q(d) is not. Substituting Eq. (5) and Eq. (7) in Eq. (6), the result of the mass density function $Q_{\Delta L}(d)$ after the suspension has passed through ΔL depth of filter bed can be expressed as follows:

$$Q_{\Delta L}(d)=\frac{Q(d)(1-p(d))^{\Delta L}}{\int_0^\infty Q(x)(1-p(x))^{\Delta L}dx}, \tag{11}$$

where, functions of Q and $Q_{\Delta L}$ respectively, denote the mass density of particles before and after the particles pass through the filter bed.

In addition, substituting Eq. (10) and Eq. (11) in the definition of the derivative, the first derivative of filter coefficient λ_l with depth can be expressed as follows:

$$\frac{\partial\lambda_l}{\partial l}=\lim_{\Delta L\to 0}\frac{\lambda_{l+\Delta L}-\lambda_l}{\Delta L}$$
$$=\lim_{\Delta L\to 0}\frac{\int_0^\infty Q(x)\ln(1-p(x))dx-\int_0^\infty Q_{\Delta L}(x)\ln(1-p(x))dx}{\Delta L} \tag{12}$$

Simplified Eq. (12) is shown as the following:

$$\frac{\partial\lambda_l}{\partial l}=-\int_0^\infty Q(x)(\ln(1-p(x)))^2dx+\lambda_l^2. \tag{13}$$

Or another formulation:

$$\frac{\partial\lambda_l}{\partial l}=-\int_0^\infty Q(x)(\ln(1-p(x)))^2dx+\left(\int_0^\infty Q(x)(\ln(1-p(x))dx)^2.\right) \tag{14}$$

Here, if we regard the particle diameter X as a random variable and Q(x) as a probability density function of X, then there is Y as another random variable. The relationship of X and Y is:

$$Y=\ln(1-p(X)). \tag{15}$$

So, the first derivative of the filter coefficient could also be expressed as follows:

$$\frac{\partial\lambda_l}{\partial l}=-D(Y). \tag{16}$$

Here, D(Y) is the variance of Y. This is very interesting. If the particle sizes in the suspension could be represented in a discrete form, i.e., the type of particles size is limited, Eq. (16) applies equally here. If particles have definitely the same size, i.e., only one particle size, thus, D(Y) is equal to 0, and the initial filter coefficient does not change with the depth of the filter bed.

On the other hand, the capture probability function p(d) must be known to study the variation of filter coefficient. Based on the principle of mass conservation and Eq. (5), p(d) could be expressed as:

$$p(d)=1-\left(\frac{Q_{out}(d)C_{out}}{Q_{in}(d)C_{in}}\right)^{\frac{1}{L}}=1-\left(\frac{Q_{out}(d)\exp(-\lambda_m L)}{Q_{in}(d)}\right)^{\frac{1}{L}}, \tag{17}$$

where Q_{in} and Q_{out} denote the mass density of particles of influents and effluents, respectively, and L is the depth of filter bed.

THE EXPERIMENT

1. Physical Model Description

The experiments were performed in a cylindrical container filled with coarse sand as the filter grain (Fig. 2).

There are three kinds of filter grain (coarse sand) with different diameters, respectively: 1.6 mm, 1.8 mm, 2.0 mm. And four kinds of suspension the detailed physical properties of each filter grain and suspension are listed in Table 1. Therefore, 12 different groups of filtration experiments can be obtained by pairing together. In each group, the filter bed has six different depths of 10 cm, 20 cm, 30 cm, 40 cm, 50 cm and 60 cm.

2. Data Collection Process

To study how the initial filter coefficient changes with the depth

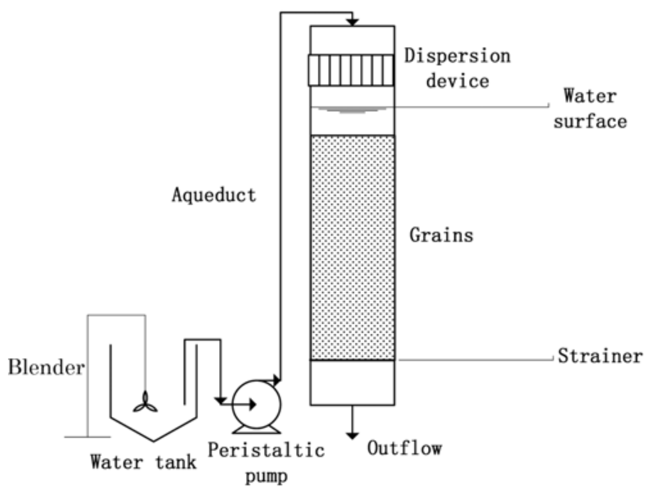

Fig. 2. Schematic diagram of the experimental apparatus.

Table 1. Details of experimental condition

Filter grain sizes, d, mm	Suspension serial number	Particle sizes, d_{50}, μm	Particle sizes, d-range, μm	Porosity
1.6	I	7.99	0.38-146.8	0.39
	II	7.82	0.38-161.2	
	III	6.94	0.38-234.1	
	IV	7.32	0.38-213.2	
1.8	I	7.99	0.38-146.8	0.40
	II	7.82	0.38-161.2	
	III	6.94	0.38-234.1	
	IV	7.32	0.38-213.2	
2.0	I	7.99	0.38-146.8	0.40
	II	7.82	0.38-161.2	
	III	6.94	0.38-234.1	
	IV	7.32	0.38-213.2	

of the filter bed, we need to measure the data, including the initial inflow concentration, initial out flow concentration, and the corresponding particle size distributions of suspension load. Assuming that the flow in the filter bed is plug flow, the time the suspension spends on reaching the bottom of the filter bed can be calculated according to Eq. (2). The suspension to reach the bottom of the filter bed takes about 2 minutes. So, the outflow measurement must be started 2 minutes after the beginning.

When the trial began, first, clean water accessed the container which was already equipped with filter grain. The flow velocity was controlled stable at 11.6 m/h by adjusting the given head height. The suspension was put into the container, and this moment was set as the start time (t=0).

In this test, within 15 minutes after the start of the filtration experiments was considered to be the initial stage. To improve accuracy, we measured a total of six times of the concentration of influents and effluents in each experiment. This is, from the first 5 minutes to 15 minutes after the start time, each measurement interval is 2 minutes. The value of concentration of the initial influents and effluents is the average of the six results.

Simultaneously, the size distribution of particles in each suspension samples was measured. So, for each group of the test, there are 12 suspended particle size distribution results that can be measured.

RESULTS AND DISCUSSION

In this study, there are 12 groups of filtration experiments data obtained. Each group contains 6 depths of filter bed. Data with each depth of filter bed contains one pair of concentration values of inflow and outflow and one pair of suspended particle size distribution of inflow and outflow.

1. Analysis of the Equivalent Filter Coefficients

By using Eq. (3), the equivalent filter coefficient can be obtained. Thus, each group of tests has six equivalent filter coefficients results corresponding to each depth of the filter bed. In this experiment, suspended particles size is nonhomogeneous. And as shown in Fig. 1, particle size distribution changes at different depths; therefore, the equivalent filter coefficients are not same at different depths. In other words, in the same homogeneous filter grain conditions, the size distribution of suspended particles is the reason causing the equivalent filter coefficient to vary with depth of filter bed.

In Fig. 3, wherethe horizontal axis is the depth of filter bed, the vertical axis is the ratio that the value of equivalent filter coefficient of each depth divided by the value at 10 cm depth, it shows the results of the 12 groups of experiments. We can directly find that the value of equivalent filter coefficient always decreases with increasing depth of the filter bed. And, at depth of 60 cm, the minimum value is less than 0.5. We can observe that when the size of suspended particles is nonhomogeneous, the loss of equivalent filter coefficient in the filtering direction cannotbe ignored.

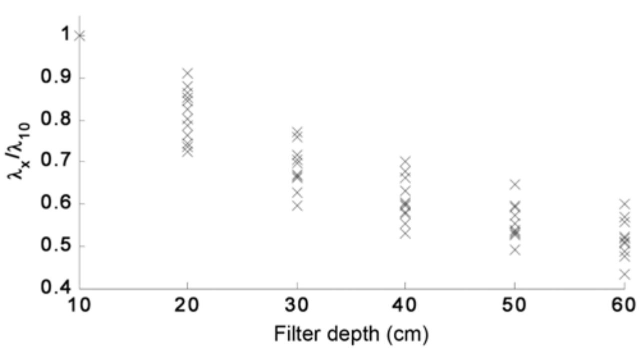

Fig. 3. Variation of the calculating value of initial filter coefficient with filter depth for every group experiment (λ_{10} denote the calculating value of initial filter coefficient by 10 cm depth filter bed and λ_x by x depth filter bed).

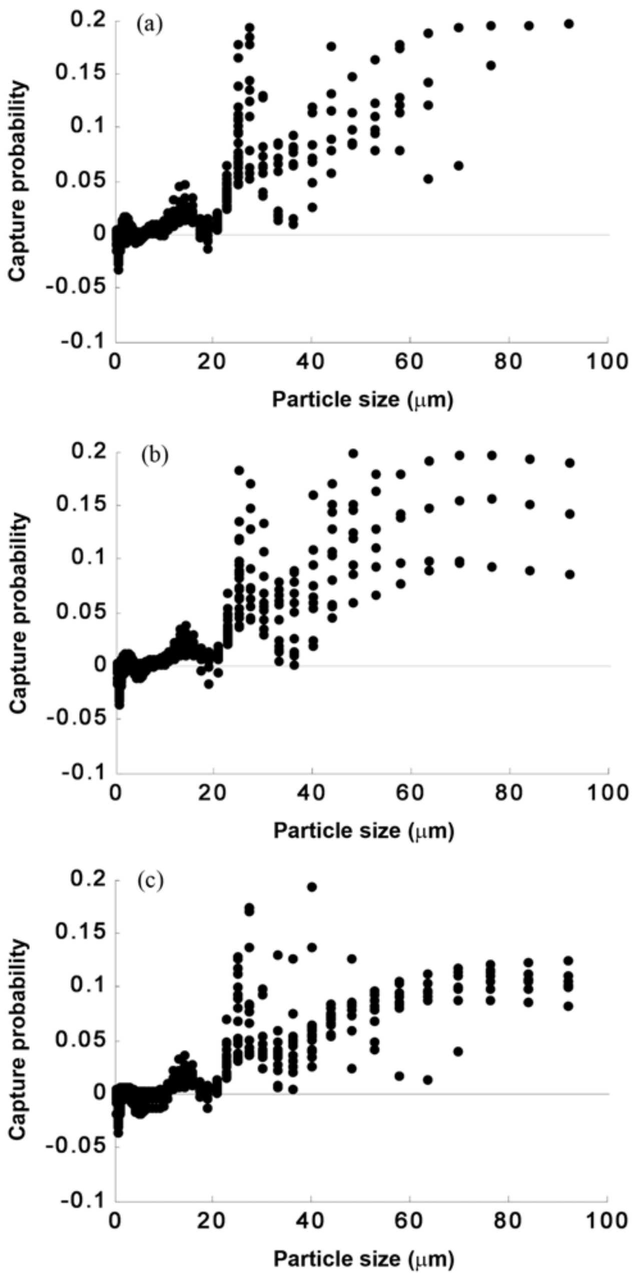

Fig. 4. Variation of the calculating value of capture probability with particle sizes for different filter grain sizes.
(a) The filter grain size with 1.6 mm
(b) The filter grain size with 1.8 mm
(c) The filter grain size with 2.0 mm

3. Analysis of the Capture Probability Function

To study the variation of the filter coefficient, the capture probability function p(d) must be estimated first. Eq. (17) can be used to obtain the values of p(d). Considering that the p(d) is affected by the character of the filter bed, Fig. 4(a) to (c) show the distribution of its value of the three sizes of filter grain, respectively.

Thenegative values of capture probability caused by experimental errors should be neglected. In Fig. 4, the variations of capture probability increase with the size of particle getting larger. This may be due to the lower mass density in the larger size of particles causing the higher error.

According to the literature, a deep bed filtration is always used to clarify the suspension, which includes particles ranging in size from about 0.1 to 50 μm [9]. In this study, more than 90% of the particles have a diameter size between 0 μm to 30 μm that we should focus on, The corresponding distribution of p(d) is shown in Fig. 5(a) to Fig. 5(c), and their mean values are shown in Fig. 5(d).

In Fig. 5(d), the capture probability changes from high to low value as the filter grain size varies from small to large. This indicates that the smaller size of filter grain has the higher filtration efficiency.

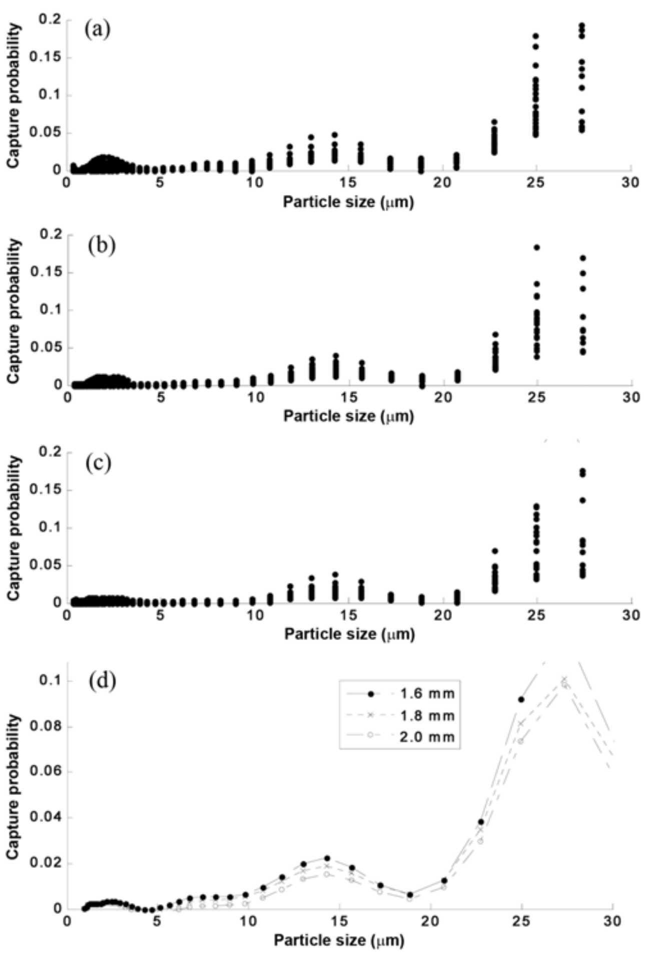

Fig. 5. Variation of capture probability with particle sizes.
(a) The filter grain size with 1.6 mm
(b) The filter grain size with 1.8 mm
(c) The filter grain size with 2.0 mm
(d) The mean values of different filter grain size

Furthermore, by Fig. 5(d) and Eq. (17), the capture probability function p(d) depends on the equivalent filter coefficient λ_m and the filter grain size. By the similarity of the three capture probability function curves in Fig. 5(d), we can make the following assumptions: The change of the capture probability function in the transverse direction was mainly determined by the size of suspended particles the difference in the longitudinal axis was mainly determined by the grain size of the filter bed.

4. Introduction of P(d) and L_c

In this paper, for the first time, we define the characteristic length of filter bed L_c and the standard capture probability function P(d).

Here, the characteristic length of filter bed, L_c, is defined as:if a suspended particle of diameter of 15 (μm) crosses through filter bed of certain depth, and then the particle capture probability equals to 0.01, this depth is the reciprocal of the characteristic length of this filter bed. Obviously, each filter bed has its own characteristic length.

If a filter bed already has its characteristic length, the standard capture probability function, P(d), means the captured probability when the particle with diameterd passes through the filter bed with the depth of $1/L_c$. So, according to the definition of characteristic length, we can know that P(15) is a fixed value that equals to 0.01.

Through the introduction of these two concepts, the effects on the capture probability function p(d) of the size of filter grain can be separated from those of the size distribution of suspended particles. And, it is assumed that L_c is independent of P(d). What should be emphasized is that the standard capture probability function P(d) has universal applicability for different filter beds.

In this way, the relationship between p(d), P(d) and L_c can be expressed as:

$$1-p(d)=(1-P(d))^{L_c}, \tag{18}$$

where P(d) is the standard function of capture probability and L_c is the characteristic depth.

For any homogeneous filter bed, if p(d) is known, then the characteristic length of the filter bed can be obtained by using Eq. (18), p(15) and P(15). In the present study, corresponding to Fig. 5(d), each filter bed has its own characteristic length, and the filter bed depth does not affect the characteristic length. The calculation results show that the characteristic lengths are 1.62, 1.34, and 0.98 corresponding to the three types of filter bed with diameter 1.6, 1.8 and 2.0 mm, respectively. And then, using Eq. (18), three standard capture probability functions can be calculated. The flow diagram for calculating is shown in Fig. 6, and the results are in Fig. 7 and Table 2.

It can be seen from the Fig. 7 that the three standard capture rate function curves are very close. In this way, we can assume that P(d) does not vary with the different filter grain, but is only affected by the roughness of filter surface and by the texture of filter grain and suspended particles.

5. Theory Validation and Analysis

To validate that P(d) is independent of the size of filter grain, we need to assume that the result of capture probability function and the standard capture probability function is unknown when the diameter of filter grain is 2.0 mm. First, get the result of P(d), whichis the mean value of the results obtained when the grain size is 1.6 mm and 1.8 mm. And then, a series of test results corresponding to the diameter of filter grain which is 2.0 mm will be ascertained by using this P(d).

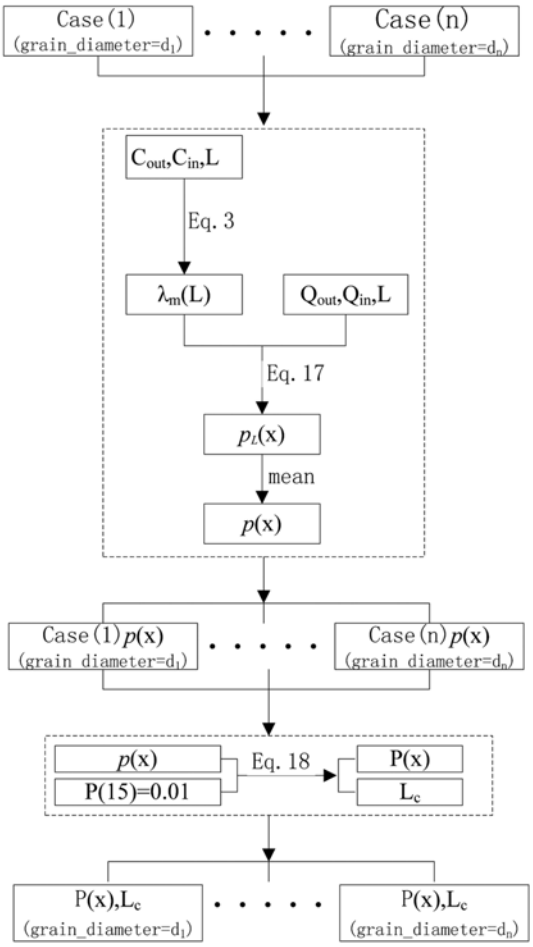

Fig. 6. The flow diagram for calculating P(d) and L_c.

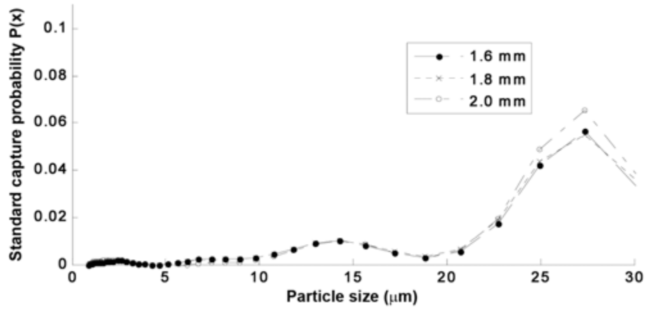

Fig. 7. Calculating values of standard capture probability with particle sizes for each grain size of filter bed.

According to Eq. (18) and Eq. (11), It is necessary to know the Q(d), P(d) and L_c for calculating the effluent particles distribution. Here, Q_{in}(d) is measured by the experiment, P(d) is substituted by the calculating result of the filter bed with the diameter of 1.6 mm and 1.8 mm. Based on the assumption, p(d) of the filter bed with diameter of 2.0 mm is unknown. And L_c of the filter bed with diameter of 2.0 mm can't be calculated by Eq. (18) directly.

In this section, L_c is calculated by the ratio of C_{out} to C_{in}. Here, assuming that the C_{out} and C_{in} is known, L_c can be obtained by Eq. (19) which produced by Eq. (7) and Eq. (18). Eq. (19) is shown as follow:

$$\int_0^{\infty} Q_{in}(x)(1-P(x))^{L_c * \Delta L}dx = \frac{C_{out}}{C_{in}}. \tag{19}$$

The left side of Eq. (19) is a monotone function with L_c hence L_c can be calculated by using the ratio of C_{out} to C_{in}. In addition, L_c can also be estimated by empirical formula which is summarized by

Table 2. The calculated value of P(d)

d (μm) (Particlesize)	P(d) (G.d=1.6 mm)	P(d) (G.d=1.8 mm)	P(d) (G.d=2.0 mm)	Mean of P(d)	d (μm) (Particlesize)	P(d) (G.d=1.6 mm)	P(d) (G.d=1.8 mm)	P(d) (G.d=2.0 mm)	Mean of P(d)
0.9536	0.0000	0.0001	0.0001	0.0001	5.6105	0.0009	0.0004	0.0000	0.0004
1.0468	0.0004	0.0005	0.0006	0.0005	6.1590	0.0015	0.0011	0.0000	0.0009
1.1491	0.0008	0.0008	0.0011	0.0009	6.7611	0.0021	0.0016	0.0005	0.0014
1.2615	0.0009	0.0010	0.0014	0.0011	7.4221	0.0024	0.0020	0.0007	0.0017
1.3848	0.0010	0.0011	0.0015	0.0012	8.1477	0.0024	0.0021	0.0009	0.0018
1.5202	0.0010	0.0011	0.0015	0.0012	8.9443	0.0024	0.0023	0.0010	0.0019
1.6688	0.0010	0.0011	0.0015	0.0012	9.8187	0.0029	0.0029	0.0017	0.0025
1.8319	0.0011	0.0012	0.0016	0.0013	10.7786	0.0041	0.0042	0.0032	0.0038
2.0110	0.0013	0.0014	0.0017	0.0015	11.8323	0.0063	0.0064	0.0057	0.0062
2.2076	0.0015	0.0015	0.0017	0.0016	12.9891	0.0089	0.0089	0.0086	0.0088
2.4234	0.0016	0.0016	0.0017	0.0016	14.2589	0.0100	0.0100	0.0100	0.0100
2.6603	0.0016	0.0015	0.0015	0.0015	15.6529	0.0082	0.0084	0.0084	0.0083
2.9204	0.0013	0.0012	0.0010	0.0012	17.1832	0.0047	0.0052	0.0049	0.0049
3.2059	0.0009	0.0008	0.0004	0.0007	18.8630	0.0028	0.0035	0.0029	0.0031
3.5193	0.0005	0.0003	0.0001	0.0003	20.7071	0.0057	0.0067	0.0062	0.0062
3.8634	0.0000	0.0000	0.0001	0.0001	22.7315	0.0173	0.0187	0.0196	0.0185
4.2411	0.0000	0.0000	0.0001	0.0000	24.9538	0.0420	0.0439	0.0486	0.0448
4.6557	0.0000	0.0000	0.0001	0.0000	27.3934	0.0563	0.0548	0.0656	0.0589
5.1109	0.0003	0.0000	0.0000	0.0001					

*G.d=Grain diameter

many experiments.

The comparison between calculated values and observed values will be shown below, including the effluent distribution, the ratio of C_{out} to C_{in} in any depths, and the equivalent filter coefficient λ_m. For the filter bed with the diameter of 2.0 mm, 24 groups of experimental data can be used in verification, including six different filter bed depth and four types of suspension.

The mean absolute percentage error, i.e., Eq. (20) was used to judge the coincidence degree of calculated values to observed values.

$$R_s = \frac{\sum_{i=1}^{n} |y_i - y_i'|}{n(y_{max} - y_{min})}, \tag{20}$$

where R_s is the mean absolute percentage error, y is the observed value, y′ is the calculated value and y_{max} and y_{min} are the maximum and the minimum of observed values, respectively.

6. The effluent particle distribution

In this section,the particle distribution of the effluent is obtained by putting Q_{in}(d), P(d) and L_c into Eq. (18) and Eq. (11). Fig. 8 shows the flow diagram for calculating Q_{out}(d). Furthermore, based on Eq. (20), the mean absolute percentage errors of the 24 effluent distributions are presented in Table 3.

It has been indicated that all these 24 distributions of effluents are the results of experiments with the filter grain size 2.0 mm. And the average of all these 24 values is 0.041. There are three pairs of results presented in Fig. 9, to show the contrast of the effluent mass density between calculation results and the observations clearly.

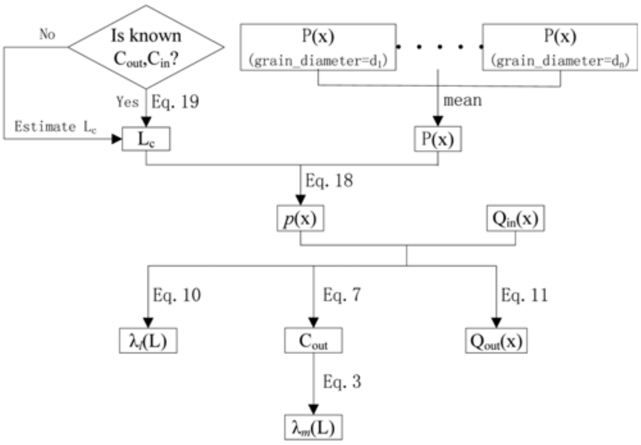

Fig. 8. The flow diagram for calculating Q_{out}(d).

Table 3. Mean absolute percentage error of the effluent distribution

Depth (cm)	Mean absolute percentage error			
	Suspension serial number			
	I	II	III	IV
10	0.014	0.055	0.060	0.016
20	0.034	0.068	0.057	0.009
30	0.018	0.066	0.051	0.041
40	0.022	0.054	0.051	0.041
50	0.043	0.067	0.048	0.035
60	0.013	0.036	0.037	0.039

7. The Ratio of C_{out} to C_{in}

On the other hand, if the ratio of C_{out} to C_{in} at certain depth of filter bed is already known, the ratio at other depths can be calcu-

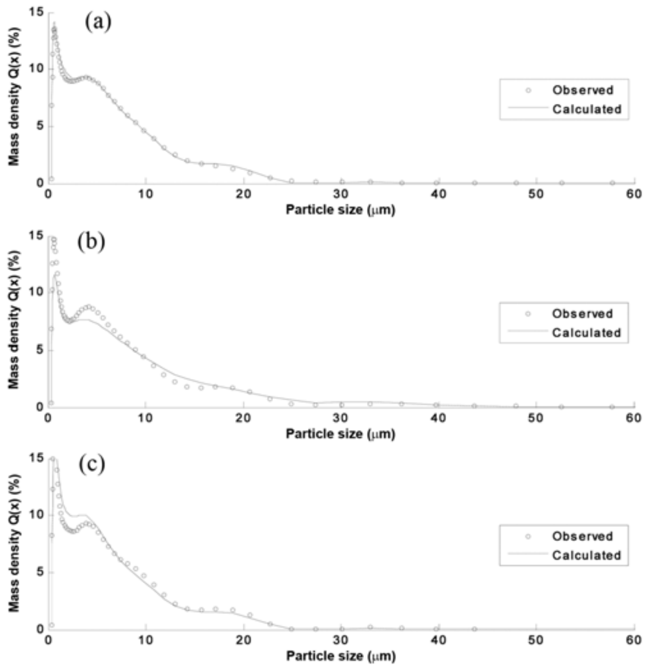

Fig. 9. The mass density from the experiment with the 2.0 mm grain size.
(a) The effluent mass density from the experiment with the first type of suspension in 60 cm depth (R_s=0.013)
(b) The effluent mass density from the experiment with the second type of suspension in 10 cm depth (R_s=0.055)
(c) The effluent mass density from the experiment with the fourth type of suspension in 60 cm depth (R_s=0.039)

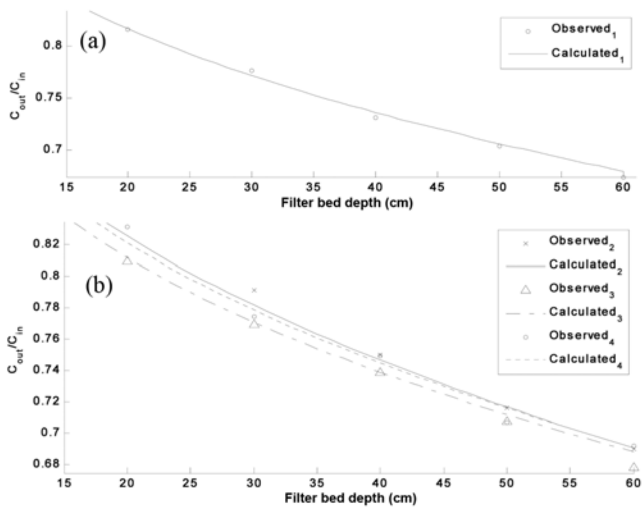

Fig. 10. The effluent quality profile of calculated and observed from the experiment with 2.0 mm grain size.
(a) The effluent quality profile from the experiment with the type I of suspension
(b) The effluent quality profile from the experiment with type II, III, IV of suspensions

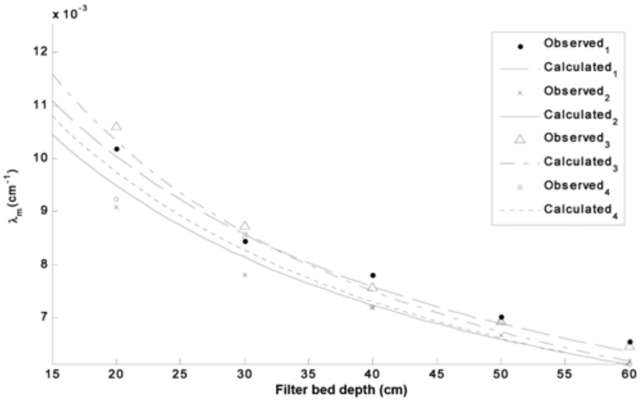

Fig. 11. The filter coefficient of calculated and observed from the experiment with 2.0 mm grain size.

lated. That is, if L_c is calculated by the observed concentration at 10 cm depth of filter bed, then the ratio of C_{out} to C_{in} at other depths can be obtained by putting $Q_{in}(d)$, P(d) and L_c into Eq. (18) and Eq. 7. The results are shown in Fig. 10(a).

The same L_c in Fig. 10(a) was used to calculate the variation of effluent quality with type II, III, IV of suspension in the experiment with 2.0 mm grain size (shown in Fig. 10(b)).

8. The Equivalent Filter Coefficient λ_m

Using Eq. (3), the calculated and observed results of equivalent filter coefficient λ_m are shown in Fig. 11.

9. Filter Coefficient λ_l and the First Order Derivative of λ_l

Every filter bed has its own characteristic depth L_c. And, L_c is only related with the attribute of filter grain, and has no bearing upon the depth of filter bed. Thus, inthis section, the mean value of L_c obtained by different depths of experiments wasused to study the variation of filter coefficient λ_l.

Filtercoefficient λ_l can't be calculated by the observed data directly. As shown in Eq. (4), if we want to obtain the experimental value of λ_l, the function of equivalent filter coefficient must be known. There are only six discrete values of equivalent filter coefficient in each depth. Therefore, a function was used to fit these observed values.

First, there are some basic characteristics of filter coefficient and equivalent filter coefficient shown in Eq. (21):

$$\lambda_m \geq 0$$
$$\lambda'_m \leq 0$$
$$\lambda_l \geq 0$$
$$\lambda'_l \leq 0 \tag{21}$$

Then, combining Eq. (4) with Eq. (21), the fitting function must satisfy the following criteria shown in Eq. (22):

$$\lambda'_m \leq 0$$
$$L\lambda'_m + \lambda_m \geq 0$$
$$L\lambda''_m + 2\lambda'_m \leq 0 \tag{22}$$

So, as shown in Eq. (23), a simple logarithmic functionwas used to fit the observed values of equivalent filter coefficient.

$$f(x)=a\ln(x)+b. \tag{23}$$

The fitting results about the first type of suspension are shown in Fig. 12. The mean value of all the coefficients of determination is 0.98.

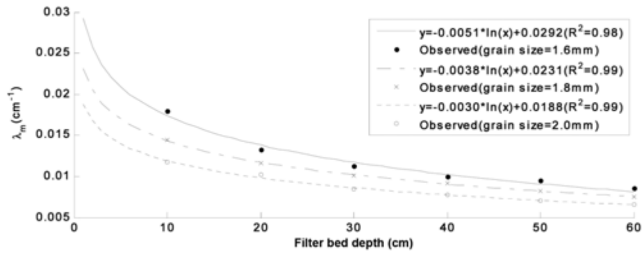

Fig. 12. The fitting curve of equivalent filter coefficient from the experiment with the first type of suspension.

Based on the fitting function and Eq. (4), not only can the filter coefficient λ_l be obtained easily, but also the first order derivative of λ_l. These results were considered to be fitting values.

To get the calculated value, the filter coefficient λ_l can be calculated by putting P(d), Q(d) and L_c in Eq. (10). Moreover, the first-order derivative of λ_l can be calculated by using Eq. (14) or Eq. (16). Using Eq. (20), the mean absolute percentage errors were counted and shown in Table 4. Furthermore, the comparisons between the

Table 4. Mean absolute percentage error of λ_l and $\sigma\lambda_l/\sigma l$

Suspension serial number	Error of λ_l			Error of $\sigma\lambda_l/\sigma l$		
	Filter bed grain sizes					
	1.6 mm	1.8 mm	2.0 mm	1.6 mm	1.8 mm	2.0 mm
I	0.307	0.233	0.222	0.604	0.599	0.579
II	0.129	0.195	0.447	0.273	0.228	0.283
III	0.246	0.162	0.354	0.376	0.280	0.446
IV	0.155	0.252	0.263	0.222	0.401	0.455

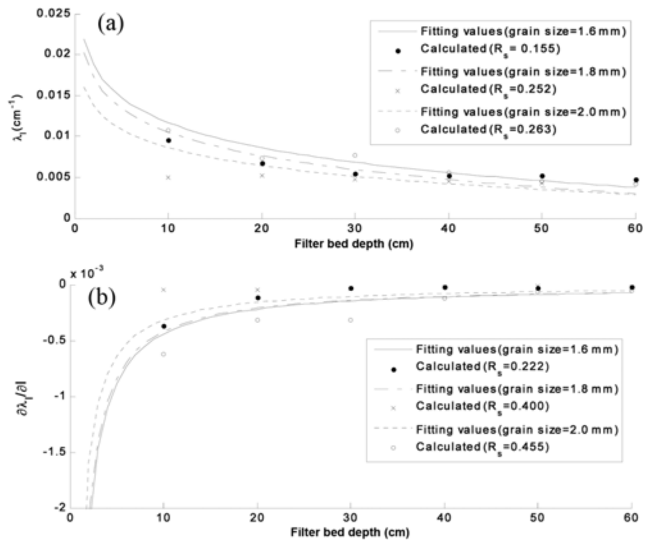

Fig. 13. Inter-comparison offitting values and calculating values with the 4th type of suspension.

(a) The λ_l inter-comparison of fitting values and calculated values with the 4th type of suspension

(b) The $\sigma\lambda_l/\sigma l$ inter-comparison offitting values and calculated values with the 4th type of suspension fitting values and the calculated values with type IV of suspension are illustrated in Fig. 13.

It can be seen from Table 4 and Fig. 13 that these results do not fit very well. This could be attributed to the experimental errors and the fitting function (Fig. 12). However, they have a similar tendency.

CONCLUSIONS

A stochastic method has been proposed to study the variation of filter coefficient in the initial stage of filtration when particles in the suspensions have different size. From Fig. 3, when the suspension was nonhomogeneous solution, the equivalent filter coefficient decreased with increasing depth of the filter bed. This phenomenon might be caused by the following reasons: The size distribution of particles in suspension varied with the depth of the filter bed during the filtration process, which could be known from Fig. 1. And, particles with different sizes have different captured probability those having higher probability always are easier to capture. Through analysis and comparison of multiple sets of experimental data, from Fig. 5, it was shown that p(d), as a function of suspension particle size, was also affected by the size of filter grain.

We first defined the characteristic length of filter bed L_c and the standard capture probability function P(d). These two definitions separate the influenceof the size of filter grain on filter coefficient from the influence of the size distribution ofsuspended particles.

We assumed that P(d) was decided by the textures and surface roughness of filter grain and of suspension particles. And hence, P(d) is not influenced by the size of filter grain. So, P(d) is universally applied in different filter beds which have similar textures and surface roughness. Moreover, L_c is the reflection of attributes of the filter bed, and not affected by the suspended particles.

According to the definitionand the sets of observed data, P(d) and L_c could be estimated at first. Based on the estimated result, the result of the effluent distribution, the ratio of C_{out} to C_{in} in any depths and the equivalent filter coefficient λ_m during the filtering process could be deduced. The calculated results were well fitted to the observed data from the comparison.

Basis on these research results, we also studied the filter coefficient λ_l and the first order derivative of λ_l during the experiment progress. Restricted by the condition of experiment, the study work did not look at infinitely small particles sizes less than 1 μm. But, for the infinitesimal particles, it should followthe same rule in the filtration, because different size of these particles also has different captured probability [10].

NOMENCLATURE

A : filtering surface area
C : particles concentration $[mg\cdot l^{-1}]$
C_{in} : influent concentration $[mg\cdot l^{-1}]$
C_{out} : effluent concentration $[mg\cdot l^{-1}]$
d : particle diameter [m]
D(Y) : the variance of Y
l : axial distance from the top of the filter bed [m]
L : length of filter bed [m]
L_c : characteristic length of filter bed
M : total mass of all particles [mg]
$m(x\rightarrow x+\Delta x)$: mass of particles with the diameter from x to x+ Δx [mg]
p(d) : capture probability function
P(d) : standard capture probability function
q(d) : the probability the particle is not captured
Q(d) : mass density function
Q_{in}, Q_{out} : denote the mass density of particles of influents and effluents respectively
$Q_{\Delta L}$: mass density of particles after suspension passing through the filter bed
R_s : mean absolute percentage error
t : time [s]
U : filtration velocity $[m\cdot s^{-1}]$
y : the observed value
y′ : the calculated value
y_{max}, y_{min} : the maximum and the minimum of observed values respectively
X, Y : random variable [m]

Greek Symbols

λ : filter coefficient
λ_0 : initial filter coefficient
λ_l : filter coefficient at $l$cm depth of filter bed
λ_m : equivalent filter coefficient
Θ : the corrected time for the initial filtration at different filter depths
θ : the porosity of the filter
σ : retention rate

REFERENCES

1. R. Vaidyanathan and C. Tien, *Chem. Eng. Sci.*, **46**, 967 (1991).
2. S. Vigneswaran, H. Prasanthi and H. B. Dharmappa, *Korean J. Chem. Eng.*, **13**, 565 (1996).
3. V. Jegatheesan, S. Vigneswaran and S.-H. Lee, *Korean J. Chem. Eng.*, **22**, 142 (2005).
4. J. P. Herzig, D. M. Leclerc and Goff P. Le, *Ind. Eng. Chem.*, **62**, 8 (1970).
5. C. S. P. Ojha and N. J. D. Graham, *Water Res.*, **26**, 145 (1992).
6. L. A. Spielman and S. L. Goren, *Environ. Sci. Technol.*, **4**, 135 (1970).
7. K. M. Yao, M. T. Habibian and C. R. O'Melia, *Environ. Sci. Technol.*, **5**, 1105 (1971).
8. T. Iwasaki, *J. AWWA.*, **29**, 1591 (1937).
9. S. Osmak, D. Gosak and A. Glasnovi, *Comput. Chem. Eng.*, **21**, 763 (1997).
10. V. Jegatheesan and S. Vigneswaran, *Water Res.*, **31**, 2910 (1997).