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Design and optimization of industrial power systems for natural gas processing

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Abstract−This paper presents a novel design methodology for power systems. A superstructure-based modelling technique has been applied to identify the cost-effective match between available power generation equipment and energy consumers. Multi-period design is conducted to ensure accurate equipment performance estimation. The proposed MILP (mixed integer linear programming) optimization model is able to reflect the machinery performance variations affected by the environmental conditions, and to estimate the deteriorated machinery performance due to part-load operation. To maintain the linear nature of the overall mathematical model, machinery performance is linearized with reasonable accuracy. Moreover, the multi-period methodology is able to conduct synthesis of power systems for processes with non-constant energy demands. Case studies are illustrated to demonstrate the importance of considering the effect of ambient conditions and part-load operation on machinery performance. With the ability to satisfy varying energy demands, and more accurate description of the machinery performance, the optimal design yielded from the improved model would exhibit better flexibility and reliability.

Key words: Power Systems, Multi-period Design, Equipment Performance Deteriorations, Optimization

INTRODUCTION

A power system is necessary to provide the electrical and mechanical power required in process industries, for example, refrigeration cycles in industrial processes, which consume a considerable amount of power to drive compressors. Especially, in cryogenic processes, heating demand is comparatively low than power demand. For the interest of plant economy, it might be favorable to design a power system without a complex steam system, which involves a large number of supporting components, and consequently, a large amount of capital investment. With the simplification of excluding the steam system option, the power system only consists of two major parts, one for electricity supply, and the other for mechanical power supply. Generally, it might be cost-efficient to have an on-site power plant to meet the electricity demand for the entire plant, if there are many electricity consumers with considerable demand. Particularly, if electric motors are chosen for mechanical power supply, an on-site power plant is usually a good choice. Besides the electricity generation, mechanical power supply to refrigeration systems is another important task for the power systems. Typical mechanical drivers include direct drive gas turbines, electric motors and steam turbines. The current study focuses on the design of power systems and does not consider steam systems.

During the last three decades, many papers have been published to address the synthesis and design of utility systems. Mixed-integer linear programming was introduced in the optimization of utility systems by Papoulias and Grossmann [1]. Hui and Natori [2] presented an industrial application using mixed integer programming techniques for the optimization of site utility systems. A multi-period model was proposed to allow for the seasonal variations of steam and electricity demands. Some practical constraints were imposed to maximize the benefits of modifying an existing utility system. Iyer and Grossmann [3] proposed a universal MILP model for the multi-period design of utility systems, considering variations of energy demands. A decomposition algorithm was also presented to solve this problem, which requires less computation time than the usual MILP solvers. This work was later extended to include the concerns of global emissions of atmospheric pollutants [4]. A five-step systematic approach [5] had been presented for the synthesis and design of flexible site utility systems with varying energy demands. This methodology takes advantage of total site composite, thermodynamic analysis, as well as mathematical optimization. Thermodynamic analysis is employed to reduce the size of the superstructure and integrate only units required to achieve maximum performance.

Most work mentioned above focuses on the utility system design with the presence of a steam system, which might not be favorable for the design of power systems for low temperature processes. Del Nogal et al. [6,7] proposed a new systematic design approach, which is able to describe power systems more effectively for power dominated processes. It employs binary variables to account for the selection of gas turbine models with discrete sizes, and machine allocation is more flexible by allowing different compressor stages to be driven on a common drive shaft. Availability issues are also discussed with parallel compression. However, during the design, the equipment performance is fixed for specified operating conditions. And all the gas turbines and power plants are assumed to operate at full load. The design based on these assumptions could not reflect the performance variations caused by different equipment operating conditions, such as ambient conditions (Fig. 1) and operation load (Fig.

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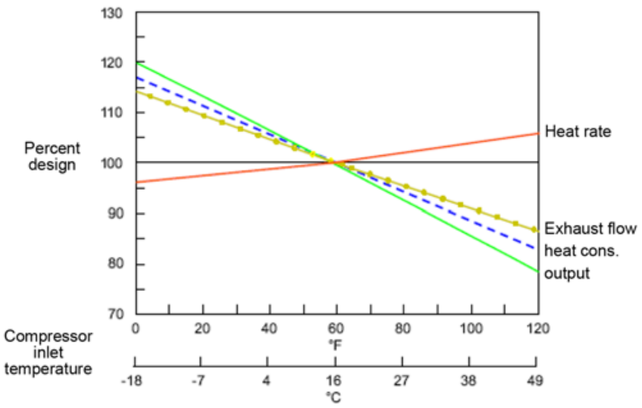

Fig. 1. Gas turbine performance variations due to ambient temperature change [8].

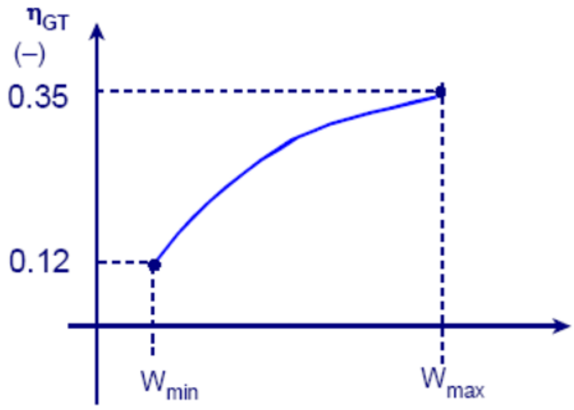

Fig. 2. Typical gas turbine performance variations due to part-load operation.

2). Moreover, the proposed model is not applicable for power system design with varying energy demands.

We have developed a new mathematical model for power system design with multi-period considerations. By splitting the whole time span into several time periods, the machinery performance could be evaluated for the operating conditions in each time period, instead of the overall average conditions, which makes the performance estimation more accurate. On the other hand, when gas turbine based equipment operates with partial load, the thermal efficiency of the equipment decreases significantly. Although the reduction of the equipment thermal efficiency varies from different models or equipment designs, the same partial load effect on equipment performance is assumed for different gas turbines models in this work. However, the methodology proposed in this paper is generic to accommodate different characteristics of partial load of individual equipment. Since the power plant models considered in this study are gas turbine based, the performance variations due to partial load operation for the power plant are estimated in the same way as gas turbines. With the multi-period methodology, the extended model is able to deal with power system designs with varying energy demands.

PROBLEM FORMULATION

A superstructure is established to cover all the possible design options for power systems as shown in Fig. 3. The model developed in this work can, in principle, accommodate any number of pieces of equipment, although in order to keep the design problem practically manageable, the maximum number, such as gas turbines, electric motors and power plants, can be specified before establishing the superstructure.

In the superstructure, gas turbines and electric motors are used as main mechanical drivers, and power plants are main electricity

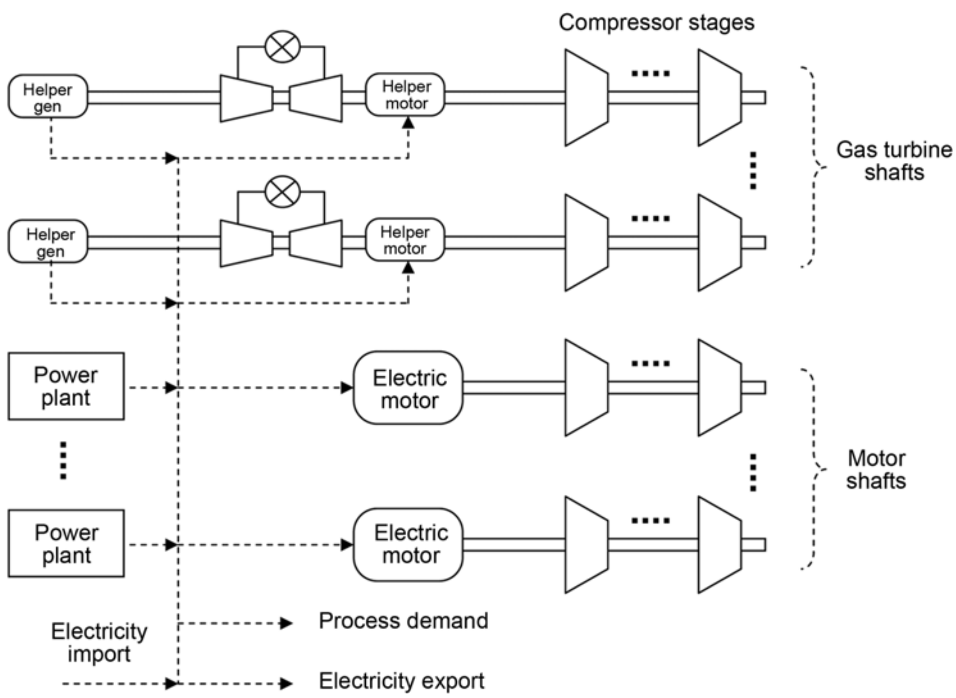

Fig. 3. Superstructure for power system design.

suppliers. Other equipment or options can be added for mechanical drivers and electricity suppliers in the superstructure. For the shafts driven by the gas turbines, the starters, which are necessary to support the start-up of gas turbines, can operate continuously as either helper motors or helper generators after starting the gas turbines. Helper motors are able to supplement part of the mechanical power supply. And helper generators are able to supplement part of the electrical power supply. On each drive shaft, only one driver serves for mechanical power supply, but more than one compressor stage is allowed to be allocated on a single drive shaft. Besides the electrical power demand from the electric motors and helper motors, there is a lumped electrical power demand from the background process. Electricity could be either imported from the external power grid, or exported if it is surplus. Each compressor stage is allowed to run on any of the shafts driven by gas turbines or electric motors, with each compressor split into different casings if necessary.

Generally, once the connections between the compressor stages and the drivers are specified, then the power system configuration is fixed. However, equipment output is allowed to vary in different time periods.

1. The Objective Function

The goal of optimization is to find the best power system configuration and operating conditions in each time period, which could achieve the minimum total cost over the whole time span. The total cost (TCOST) is calculated in Eq. (1), which includes annualized capital cost (CCAP), fuel cost (CFUEL) and electricity cost (CELEC). By minimizing the total cost, the balance between the capital investment and the operating cost, determined by fuel consumption and electricity demand is able to be evaluated, and performance of the power system is optimized.

$$\mathrm{TCOST}=\mathrm{CCAP}+\mathrm{CFUEL}+\mathrm{CELEC} \tag{1}$$

2. Costs

The annualized capital cost, calculated in Eq. (2), includes the capital investment for gas turbines (CGT), electric motors (CEM), power plants (CPP), helper motors and generators (CHMG), and refrigerant compressors (CRC). With the use of annualized factor $\overline{\mathrm{AnnuFact}}$, the total capital cost is converted to annualized total cost. The annualized factor could be calculated by Eq. (3), with the specification of interest rate ($\overline{\mathrm{IR}}$) and plant life ($\overline{\mathrm{LIFE}}$).

$$\mathrm{CCAP}=(\mathrm{CGT}+\mathrm{CEM}+\mathrm{CPP}+\mathrm{CHMG}+\mathrm{CRC})\cdot\overline{\mathrm{AnnuFact}} \tag{2}$$

$$\overline{\mathrm{AnnuFact}}=\frac{\overline{\mathrm{IR}}\cdot(1+\overline{\mathrm{IR}})^{\overline{LIFE}}}{(1+\overline{\mathrm{IR}})^{\overline{LIFE}}-1} \tag{3}$$

$$\mathrm{CGT}=\sum_{GT}\sum_{GTOP}(\mathrm{YGTOP}_{GT,GTOP}\cdot\overline{\mathrm{CGTOP}}_{GTOP}) \tag{4}$$

$$\mathrm{CPP}=\sum_{PP}\sum_{PPOP}(\mathrm{YPPOP}_{PP,PPOP}\cdot\overline{\mathrm{CPPOP}}_{PPOP}) \tag{5}$$

The set of binary variables, $\mathrm{YGTOP}_{GT,GTOP}$, indicates whether the gas turbine model considered in the optimization problem (GTOP) is selected on the gas turbine shaft (GT). Thus, Eq. (4) accounts for the cost of all the actual gas turbines present in the final design, and is formulated in a discrete form. The cost of the power plants (CPP) is calculated in a similar way by Eq. (5).

The capital costs for other equipments are calculated by using the six-tenths factor rule to reflect the economy of scale in Eqs. (6) to (12), but more suitable equipment-specific exponents other than 0.6 are used instead (either from correlated data or from Remer and Chai [9]).

$$\mathrm{CEM}=\sum_{EM}\mathrm{EMCost}_{EM} \tag{6}$$

$$\frac{\mathrm{EMCost}_{EM}}{\overline{\mathrm{EMCost0}}}=\left(\frac{\mathrm{EMSize}_{EM}}{\overline{\mathrm{EMSize0}}}\right)^{0.70} \tag{7}$$

$$\mathrm{EMSize}_{EM}\geq\mathrm{WEM}_{EM,T} \tag{8}$$

$$\mathrm{CHMG}=\sum_{GT}\mathrm{HMGCost}_{GT} \tag{9}$$

$$\frac{\mathrm{HMGCost}_{GT}}{\overline{\mathrm{HMGCost0}}}=\left(\frac{\mathrm{HMGSize}_{GT}}{\overline{\mathrm{HMGSize0}}}\right)^{0.70} \tag{10}$$

$$\mathrm{HMGSize}_{GT}\geq\mathrm{WSM}_{GT}+\mathrm{WHMEx}_{GT,T}+\mathrm{WHGEx}_{GT,T} \tag{11}$$

$$\mathrm{CRC}=\sum_{C}\left(\frac{\sum_{CS\in C}\overline{\mathrm{CSSize}}_{CS}}{\overline{\mathrm{WRC0}}_{C}}\right)^{0.82}\cdot\overline{\mathrm{CRC0}}_{C}\cdot(1+(\mathrm{NCAS}_{C}-\overline{\mathrm{NCAS0}}_{C})\cdot\overline{\mathrm{PCAS}}) \tag{12}$$

$$\overline{\mathrm{CSSize}}_{CS}=\max_{T}\{\overline{\mathrm{WCS}}_{CS,T}\} \tag{13}$$

The cost of electric motors (CEM) is scaled according to their size, maximum electrical power demand over all the time periods. The motor size (EMSize) could be determined by Eq. (8). With the minimization of the total annualized cost, the optimization solver will be able to find the minimum value for motor size, which is equal to the maximum power requirement over all the time periods.

The cost of helper motors or helper generators (CHMG) is scaled according to their size as well. The size of helper devices consists of two terms as formulated in Eq. (11). The first part (WSM_{GT}) is fixed once a gas turbine is chosen and represents the shaft power requirement for start-up. The second term ($\mathrm{WHMEx}_{GT,T}$ or $\mathrm{WHGEx}_{GT,T}$) becomes active only when the continuous helper contribution (motor or generator) is larger than the start-up requirements and precisely accounts for the amount of additional power generated or consumed. Additional constraints activating these variables are presented later. The cost of motors and generators is calculated using the same equation, Eq. (10), since they are essentially the same kind of equipment but operate in different modes.

In the case of refrigerant compressor cost (CRC) estimation, compressors of the same type with similar operating conditions are used as the cost reference (e.g., a four-stage centrifugal propane compressor with similar compression ratio). Two factors affect the compressor cost in Eq. (12). The first is the overall compressor size relative to that of the reference. The second considers a fixed fractional penalty per additional casing (or a credit, if a compressor ends up with fewer casings than the reference). This second factor is estimation and extends the applicability of the six-tenths factor rule in order to explicitly allow for the effect of casing arrangement on compressor costs.

The total fuel consumption in each time period (QTotal_T) could be calculated by Eq. (14), which includes the contributions for both gas turbines ($\mathrm{QGT}_{GT,T}$) and power plants ($\mathrm{QGT}_{PP,T}$). Two different types of fuel are considered in this model, fresh fuel (FF) and end flash fuel (EFF). The total fuel demand could be satisfied by a blend of different fuels, as formulated in Eq. (15). However, there might be a maximum available amount of end flash fuel ($QEEF_T$) in a plant, which is defined in Eq. (16). The annual fuel cost (CFUEL) is calculated in Eq. (17) by summing up the fuel cost in each time period.

$$QTotal_T = \sum_{GT} QGT_{GT,T} + \sum_{PP} QPP_{PP,T} \quad (14)$$

$$QTotal_T = QFF_T + QEFF_T \quad (15)$$

$$QEFF_T \le \overline{MaxEFF}_T \quad (16)$$

$$CFUEL = \sum_T ((QFF_T \cdot \overline{UCFF} + QEFF_T \cdot \overline{UCEFF}) \cdot \overline{PT}_T) \cdot \overline{OprTime} \quad (17)$$

The electricity cost (CELEC) calculation is formulated in Eq. (18), which allows electricity to be either imported from an external source ($EImp_T$) or exported to gain a credit ($EExp_T$) in each time period.

$$CELEC = \sum_T ((EImp_T \cdot \overline{UCEImp} - EExp_T \cdot \overline{UCEExp}) \cdot \overline{PT}_T) \cdot \overline{OprTime} \quad (18)$$

3. Equipment Performance Estimation

All the equipment in a power system is modelled in this section to evaluate the respective performance, including operating conditions and fuel or power demand.

4. Gas Turbines

Gas turbines are considered as mechanical driver options, and they are able to supply mechanical power by consuming fuels. As gas turbine models only have discrete size, formulations have to take this into account with discrete elements. Binary variable set $YGTOP_{GT,GTOP}$ is introduced to account for the presence of gas turbine model GTOP at gas turbine shaft GT. If $YGTOP_{GT,GTOP}=1$, then gas turbine model GTOP is employed in the shaft GT. For this study, only one gas turbine model is allowed to operate on each drive shaft, as given in Eq. (19). In each time period, the output of a gas turbine model, $WGTOP_{GT,GTOP,T}$, can be varied over the specified operating range between the minimum output $\overline{WGTOP}^{Min}_{GTOP,T}$ and the maximum output $\overline{WGTOP}^{Max}_{GTOP,T}$. And the outputs for different time periods are allowed to be different.

With the variations of ambient conditions, the output of gas turbine models changes when operating in different time periods. It could be estimated before the optimization, according to the performance deterioration data shown in Fig. 1. A maximum output could be imposed such that gas turbines are required to operate at partial load. The upper bound of output could be defined as a proportion of full-load output with a ratio $\overline{\lambda}_{GTUB}$ from 0.5 to 1. And the minimum output $\overline{WGTOP}^{Min}_{GTOP,T}$ is defined as half of the full-load output, $\overline{WGTOP}^{Min}_{GTOP,T} = 0.5 \cdot \overline{WGTOP}^{Max}_{GTOP,T}$, in this study.

Shaft power supply by gas turbines on each gas turbine shaft ($WGT_{GT,T}$) is calculated in Eq. (20). Although all the gas turbines models are potential candidates, only one of them will be selected for each gas turbine shaft. The output of inactive gas turbine models will be set to 0 automatically with constraints Eqs. (19), (21) and (22).

$$\sum_{GTOP} YGTOP_{GT,GTOP} \le 1 \quad (19)$$

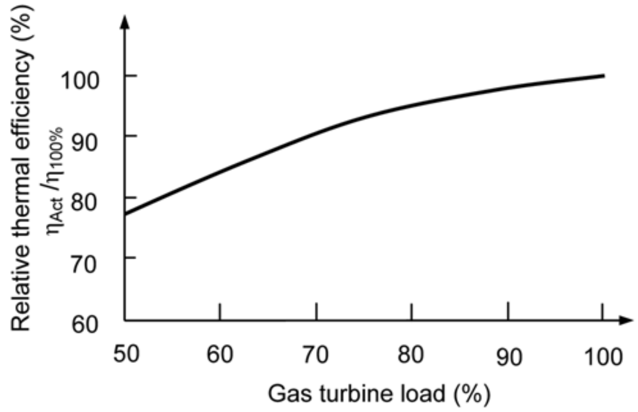

Fig. 4. Relative thermal efficiency of gas turbines as a function of load [10].

$$WGT_{GT,T} = \sum_{GTOP} WGTOP_{GT,GTOP,T} \quad (20)$$

$$WGTOP_{GT,GTOP,T} - YGTOP_{GT,GTOP} \cdot \overline{\lambda}_{GTUB} \cdot \overline{WGTOP}^{Max}_{GTOP,T} \le 0 \quad (21)$$

$$WGTOP_{GT,GTOP,T} - YGTOP_{GT,GTOP} \cdot \overline{WGTOP}^{Min}_{GTOP,T} \ge 0 \quad (22)$$

As mentioned before, the performance of gas turbines deteriorates significantly when operating at partial load. This feature should be taken into account in the model during the optimization. The thermal efficiency of gas turbines is a non-linear function of load, as shown in Fig. 4, which is taken from reference [10]. To linearize the relationship between relative fuel consumption and load, three typical operation loads, 50%, 75% and 100%, are selected as reference points to form two piecewise lines, which approximate the non-linear performance variations. The relative fuel consumption at each operation load is calculated according to the thermal efficiency shown in Fig. 5. Once the selected three points are specified, the relationship between relative fuel consumption and operation load is formulated in Eqs. (23) to (25). The reference points chosen here are those shown in Fig. 5, (0.5, 0.633), (0.75, 0.809) and (1, 1).

In Eqs. (23) to (25), SOS2 variable set, $LGTWQ_{GT,GTOP,T}(i)$ (i=1, 2, 3), is employed to implement the linearization. The variables de-

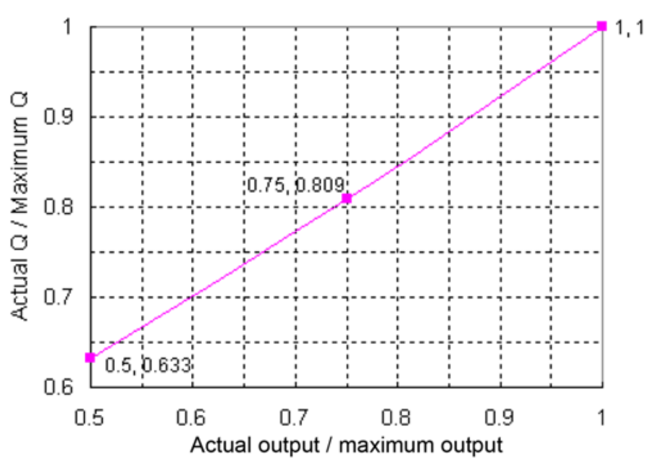

Fig. 5. Part-load performance for gas turbines and power plants.

fined in this special set have features that at most two variables can have non-zero values and they must be adjacent. This performance linearization is activated only when the gas turbine model is used, $YGTOP_{GT,GTOP}=1$. The fuel consumption for each gas turbine shaft ($QGT_{GT,T}$) is calculated in Eq. (26).

$$\frac{WGTOP_{GT,GTOP,T}}{\overline{WGTOP^{Max}_{GTOP,T}}}=LGTWQ_{GT,GTOP,T}(1)\cdot 1.00 + LGTWQ_{GT,GTOP,T}(2)\cdot 0.75 + LGTWQ_{GT,GTOP,T}(3)\cdot 0.50 \quad (23)$$

$$\frac{QGTOP_{GT,GTOP,T}}{\overline{QGTOP^{Max}_{GTOP,T}}}=LGTWQ_{GT,GTOP,T}(1)\cdot 1.00 + LGTWQ_{GT,GTOP,T}(2)\cdot 0.809 + LGTWQ_{GT,GTOP,T}(3)\cdot 0.633 \quad (24)$$

$$\sum_{i=1}^{3} LGTWQ_{GT,GTOP,T}(i)=YGTOP_{GT,GTOP} \quad (25)$$

$$QGT_{GT,T}=\sum_{GTOP} QGTOP_{GT,GTOP,T} \quad (26)$$

5. Helper Motors and Helper Generators

Starters serve as motors to provide the required mechanical power during the start-up of gas turbines. The shaft power requirement for start-up is determined by the size of gas turbines. In this work, it is assumed to be 15% of the gas turbine maximum output at ISO conditions. After the start-up, starters are allowed to operate continuously either as helper motors or helper generators to balance the shaft power on the drive shaft. When used in a continuous mode, the helper motor or generator can have a power output less than or greater than the starting requirements. In the first case, the size of a starter is determined by the shaft power requirement for start-up. On the other hand, if a starter is installed to operate continuously as a helper with a capacity greater than the starting requirements, and operates at partial capacity during start-up, the starter size should be determined by the peak output over all the time periods. The maximum capacity of helper motors or helper generators must have an upper limit associated with the gas turbine models. In this work, it is defined as 25% of the gas turbine maximum output at ISO conditions.

The shaft power demand for gas turbine start-up (WSM_{GT}) is calculated by Eq. (27). The excessive power of helpers beyond the start-up requirement ($WHMEx_{GT,T}$ or $WHGEx_{GT,T}$) is limited by Eqs. (28) to (29), one for helper motors, and the other for helper generators. On each gas turbine shaft, the starter can only operate in one mode, either helper motor or helper generator, which is formulated in Eqs. (32) to (33). With the minimization of total annualized cost, the optimizer will minimize the size of starters, which is equivalent to minimizing $WHMEx_{GT,T}$ or $WHGEx_{GT,T}$. With Eqs. (30) to (31), $WHMEx_{GT,T}$ or $WHGEx_{GT,T}$ is active only when capacity of a starter, $WHM_{GT,T}$ or $WHG_{GT,T}$, is larger than its start-up requirement WSM_{GT}. Otherwise, they are set to zero.

$$WSM_{GT}=\sum_{GTOP}(YGTOP_{GT,GTOP}\cdot\overline{WSMGTOP_{GTOP}}) \quad (27)$$

$$WHMEx_{GT,T}\le\sum_{GTOP}(YGTOP_{GT,GTOP}\cdot\overline{WHMExGTOP_{GTOP}}) \quad (28)$$

$$WHGEx_{GT,T}\le\sum_{GTOP}(YGTOP_{GT,GTOP}\cdot\overline{WHGExGTOP_{GTOP}}) \quad (29)$$

$$WHM_{GT,T}\le WSM_{GT}+WHMEx_{GT,T} \quad (30)$$

$$WHG_{GT,T}\le WSM_{GT}+WHGEx_{GT,T} \quad (31)$$

$$WHM_{GT,T}\le YHM_{GT}\cdot\overline{WHMGUB} \quad (32)$$

$$WHG_{GT,T}\le(1-YHM_{GT})\cdot\overline{WHMGUB} \quad (33)$$

$$PowHM_{GT,T}=\frac{WHM_{GT,T}}{\overline{HMEff}} \quad (34)$$

$$PowHG_{GT,T}=WHG_{GT,T}\cdot\overline{HGEff} \quad (35)$$

The electrical power consumption for helper motors ($PowHM_{GT,T}$) is calculated in Eq. (34) with a given motor efficiency. The electrical power production from helper generators ($PowHG_{GT,T}$) is calculated in Eq. (35) with a given generator efficiency.

6. Electric Motors

An electric motor is another important option for mechanical drivers. They consume electricity, which can be generated on-site by power plants, or imported from external power grid. Different from gas turbines, electric motors can be designed for given shaft power requirement, which means continuous size of electric motors is available. However, since the electric motors might operate at partial load in some of the time periods, the peak output over all the time periods is regarded as the electric motor size. In practice, the largest electric motors with acceptable reliability can supply maximum 60 MW shaft power [11,12]. And this limitation is formulated in Eq. (36). Binary variable set YEM_{EM} accounts for the presence of electric motor of electric motor shaft EM.

Electrical power consumption for electric motors ($PowEM_{EM,T}$) is simply calculated by Eq. (37), with the specification of motor efficiency $\overline{EMEff}$.

$$WEM_{EM,T}-YEM_{EM}\cdot\overline{MaxWEM}\le 0 \quad (36)$$

$$PowEM_{EM,T}=\frac{WEM_{EM,T}}{\overline{EMEff}} \quad (37)$$

7. Power Plants

The main responsibility for power plants is electrical power generation. Similar to gas turbines, power plants only have discrete size models, and the performance varies when operating under different conditions. Since most of the power plants models are gas turbine based, the formulations of power plants, Eqs. (38) to (41), are similar to those of gas turbines.

The amount of electrical power generated by power plants at each power plant place ($PowPP_{PP,T}$) is calculated in Eq. (38). Only one of the power plant models can be selected at each power plant place, and the power output of inactive power plant models will be set to 0 with the constraint Eqs. (39) to (41).

$$PowPP_{PP,T}=\sum_{PPOP} PowPPOP_{PP,PPOP,T} \quad (38)$$

$$PowPPOP_{PP,PPOP,T}-YPPOP_{PP,PPOP}\cdot\overline{\lambda_{PPUB}}\cdot\overline{PowPPOP^{Max}_{PPOP,T}}\le 0 \quad (39)$$

$$PowPPOP_{PP,PPOP,T}-YPPOP_{PP,PPOP}\cdot\overline{PowPPOP^{Min}_{PPOP,T}}\ge 0 \quad (40)$$

$$\sum_{PP}\text{YPPOP}_{PP,PPOP}\leq 1 \tag{41}$$

The performance of power plants is estimated with the same method as the one used for gas turbines. The same performance deterioration data are employed and the same reference points are picked for performance linearization, as formulated in Eqs. (42) to (44). Fuel consumption of power plants ($\text{QPP}_{PP,T}$) is calculated in Eq. (45).

$$\frac{\text{PowPPOP}_{PP,PPOP,T}}{\overline{\text{PowPPOP}^{Max}_{PPOP,T}}}=\text{LPPPowQ}_{PP,PPOP,T}(1)\cdot 1.00 + \text{LPPPowQ}_{PP,PPOP,T}(2)\cdot 0.75 + \text{LPPPowQ}_{PP,PPOP,T}(3)\cdot 0.50 \tag{42}$$

$$\frac{\text{QPPOP}_{PP,PPOP,T}}{\overline{\text{QPPOP}^{Max}_{PPOP,T}}}=\text{LPPPowQ}_{PP,PPOP,T}(1)\cdot 1.00 + \text{LPPPowQ}_{PP,PPOP,T}(2)\cdot 0.809 + \text{LPPPowQ}_{PP,PPOP,T}(3)\cdot 0.633 \tag{43}$$

$$\sum_{i=1}^{3}\text{LPPPowQ}_{PP,PPOP,T}(i)=\text{YPPOP}_{PP,PPOP} \tag{44}$$

$$\text{QPP}_{PP,T}=\sum_{PP}\text{QPPOP}_{PP,PPOP,T} \tag{45}$$

8. Shaft Power Balance

Eqs. (46) and (47) calculate the shaft power demand for each gas turbine shaft ($\text{WGTSD}_{GT,CS,T}$) and electric motor shaft ($\text{WEMSD}_{GT,CS,T}$) respectively. Only one mechanical driver can be allocated to run a compressor stage as formulated in Eq. (48).

$$\text{WGTSD}_{GT,CS,T}=\text{YGTCS}_{GT,CS}\cdot\overline{\text{WCS}}_{CS,T} \tag{46}$$

$$\text{WEMSD}_{EM,CS,T}=\text{YEMCS}_{EM,CS}\cdot\overline{\text{WCS}}_{CS,T} \tag{47}$$

$$\sum_{GT}\text{YGTCS}_{GT,CS}+\sum_{EM}\text{YEMCS}_{EM,CS}=1 \tag{48}$$

On the drive shafts, including gas turbine shafts and electrical motor shafts, the shaft power supplied by the mechanical drivers must be equal to the shaft power consumed by the compressors. To account for the actual efficiency of mechanical transmission, a fraction of mechanical power loss ($\overline{\text{MLoss}}$) is defined in Eqs. (49) and (50).

For each gas turbine shaft, the mechanical drivers include a gas turbine and a helper motor, if the starter operates in a motor mode. The shaft power consumers include a group of compressor stages and a helper generator, if the starter operates in a generator mode. So the shaft power balance for gas turbine shaft is formulated as Eq. (49).

$$(\text{WGT}_{GT,T}+\text{WHM}_{GT,T})\cdot(1-\overline{\text{MLoss}}) = \sum_{CS}\text{WGTSD}_{GT,CS,T}+\text{WHG}_{GT,T} \tag{49}$$

For each electric motor shaft, the only possible mechanical driver is an electric motor. And the possible shaft power consumers are a group of compressor stages. So the shaft power balance for electric motor shaft is formulated as Eq. (50).

$$\text{WEM}_{EM,T}(1-\overline{\text{MLoss}})=\sum_{CS}\text{WEMSD}_{EM,CS,T} \tag{50}$$

9. Electricity Balance

Besides the shaft power, in each time period, the supply and demand for electricity should be balanced as well, as formulated in Eq. (51). A fraction of electrical power loss ($\overline{\text{ELoss}}$) is defined to account for the actual efficiency of electrical power distribution.

Electricity can be generated by on-site power plants and helper generators on gas turbine shafts. If necessary, it can also be imported from the external power grid. The electricity is consumed by electric motors, helper motors on gas turbine shafts and some other equipment in the plant. If surplus electrical power is available in the power system, it is possible to export electricity to gain some credits. However, the power system can only have one active electricity trade option at a time, either import or export, which is constrained by Eqs. (52) to (53).

$$\overline{\text{BED}}_T+\sum_{\text{EM}}\text{PowEM}_{EM,T}+\sum_{GT}\text{PowHM}_{GT,T}+\text{EExp}_T = \left(\sum_{PP}\text{PowPP}_{PP,T}+\sum_{GTD}\text{WHG}_{GTD,T}\cdot\overline{\text{HGEff}}\right)\cdot(1-\text{ELoss})+\text{EImp}_T \tag{51}$$

$$\text{EExp}_T\leq\text{YEExp}_T\cdot\overline{\text{MaxEExp}} \tag{52}$$

$$\text{EImp}_T\leq(1-\text{YEExp}_T)\cdot\overline{\text{MaxEImp}} \tag{53}$$

10. Compressor Casing Arrangement

The purpose of Eqs. (54) to (60) is to calculate the number of casings in which each compressor can be built. Stages of the same compressor that are to be run on a common shaft, driven by either gas turbines, or electric motors, can be merged into one compressor casing provided they are of the same nature (e.g., centrifugal), which is an economic incentive. Eq. (54) finds the number of stages of compressor C ($\text{NCSGT}_{C,GT}$) that are driven by each gas turbine, and Eq. (55) identifies whether gas turbine GT is driving at least one stage of such a compressor or not. Then Eq. (56) is able to calculate the number of different gas turbine shafts on which the stages of the compressor C are running (NGTS_C). Similarly, Eqs. (57) to (59) account for the number of electric motor shafts serving for each compressor. In Eq. (60), the number of casings for each compressor is calculated. It is assumed that those stages belonging to compressor C on the same shaft are merged into the same casing.

$$\text{NCSGT}_{C,GT}=\sum_{CS\in C}\text{YGTCS}_{GT,CS} \tag{54}$$

$$\text{NCSGT}_{C,GT}\leq\text{YGTC}_{GT,C}\cdot\overline{\text{NCS}}_C \tag{55}$$

$$\text{NGTS}_C=\sum_{GT}\text{YGTC}_{GT,C} \tag{56}$$

$$\text{NCSEM}_{C,EM}=\sum_{CS\in C}\text{YEMCS}_{EM,CS} \tag{57}$$

$$\text{NCSEM}_{C,EM}\leq\text{YEMC}_{EM,C}\cdot\overline{\text{NCS}}_C \tag{58}$$

$$\text{NEMS}_C=\sum_{EM}\text{YEMC}_{EM,C} \tag{59}$$

$$\text{NCAS}_C=\text{NGTS}_C+\text{NEMS}_C \tag{60}$$

If a compressor consists of stages of a different nature (e.g., axial and centrifugal), it is more convenient for formulation purposes to treat it as different compressors with each one featuring only stages of the same type. In this way, the solver can avoid merging stages of different types. Also, the compressor costing can be more accurate, as the costs of compressors of different nature are calculated separately.

In the model formulated in Eqs. (1) to (57), most of the constraints are linear equations, although some of them involve binary variables. Only those equations accounting for cost estimations have some non-linear terms. To achieve the linear nature of the whole problem formulations, those non-linear cost functions are piecewise linearized with the application of SOS2 variables.

CASE STUDY

In this section, the model proposed above is applied to the power system design for two different geographical locations, one for Qatar, and one for the U.K. The average ambient conditions in Qatar are quite different from the ISO conditions. However, in the U.K., the average ambient conditions are almost the same as ISO conditions. As ambient conditions have a significant impact on equipment performance, in each case the equipment performance is evaluated according to the local ambient conditions.

The MILP version of the power system model was formulated in GAMS. All the input data and parameters were provided in a spreadsheet interface and transferred automatically to GAMS using a Visual Basic macro in order to centralize the input information and to make running different cases much easier.

1. Input Data

The mechanical and electrical power demands from an LNG plant based on the study [7] are shown in Table 1. There are two refrigeration compressors, with each comprised of several compression stages. At each compression stage, a certain amount of shaft power is required to implement the compression. The mechanical transmission loss on the drive shaft is assumed to be 1.5% of the total amount. The plant also has a lumped electrical power demand as shown in the table, and 2% of the electricity is assumed to be lost in the distribution.

Other design parameters are summarized in Table 2. The candidate gas turbine drivers and power plants are from reference [13], and their performance was de-rated to account for ageing and fouling. The plant is intended to operate for 25 years. 8000 operation hours per year are assumed initially. The plant will not be allowed to trade electricity, and consequently its import is set to zero. And there is no credit for exporting electricity.

Table 1. Mechanical and electrical power demands from an LNG plant[7]

Compressor	Stage	Mechanical power demand (kW)
C3	C3-1	2500
	C3-2	6960
	C3-3	12350
	C3-4	32250
MR	MR-1	57640
	MR-2	18740
	MR-3	27340
Fraction of mechanical transmission loss		1.5%
Electrical power demand (kW)		42,648
Fraction of electricity distribution loss		2%

Table 2. Other design parameters

	Parameters	Value
Equipment	Maximum number of gas turbine drivers	3
	Maximum number of electric motor drivers	3
	Maximum number of power plants	2
	Maximum size of electric motor	60 MW
	Maximum helper motor size	25% of GT
	Maximum helper generator size	25% of GT
	Shaft power required for gas turbine start-up	15% of GT
	Electric motor efficiency	95%
	Helper generator efficiency	95%
Electricity	Maximum electricity export	25 MW
	Maximum electricity import	0 MW
	Price of sold electricity	0$/kWh
	Cost of imported electricity	0.06$/kWh
Fuel	Cost of end flash fuel	0$/kWh
	Cost of fresh fuel	0.0158$/kWh
	Max. end flash fuel available	0 MW
Other	Plant lifetime	25 yr
	Interest rate	4%

Table 3. Equipment performance correction data in UK

Time period (i)	1	2	3	4
Ambient temp. (°C)	5	15	25	15
Output(i)/Output (ISO)	1.07	1.00	0.94	1.00
HR(i)/HR (ISO)	0.98	1.00	1.02	1.00

2. UK Case

In this case, the equipment performance in each time period is estimated for the average ambient conditions in the U.K. The equipment performance correction data, shown in Table 3, are calculated according to the performance deterioration data shown in Fig. 1. The whole operation year is split into four time periods with equal length. In the time period of the highest ambient temperature, the equipment features the lowest output and highest heat rate. With the correction data, the full-load performance in the U.K. of all the equipment is evaluated more accurately with the consideration of ambient conditions.

To gain higher system reliability, all the equipment is required to operate at maximum 85% load in each time period. All the design options are fed to the model in GAMS. After optimization, the optimal solution is obtained. The detailed configuration and operating conditions in each time period are shown in Fig. 6.

In the optimal design, one gas turbine driver LM1600PA and three electric motors are selected as mechanical drivers. Power plant GUD1S84-3A is selected to generate electricity for both electric motors and consumers in the LNG plant. As shown in Fig. 6, gas turbine and power plant operate at partial load, which means it is

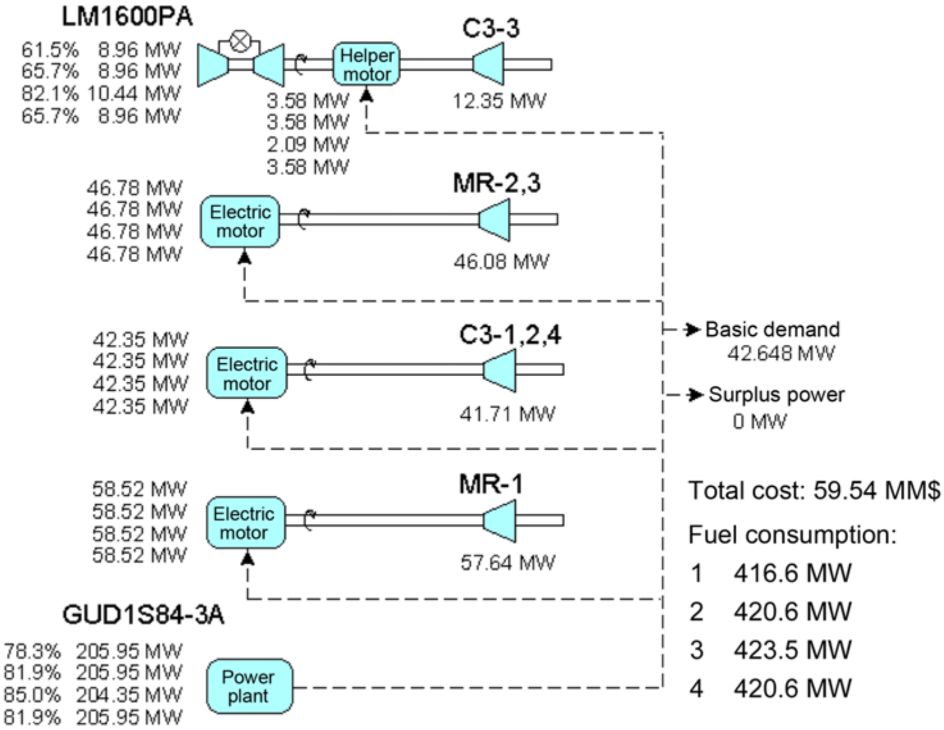

Fig. 6. Best design for UK case.

necessary to consider performance deterioration in this case. Since there is no credit for power export, the solver is trying to find a solution with no surplus power.

In Fig. 6, four rows of numbers next to a gas turbine and a power plant show percentage of part load for and power produced by the unit for each time period, while four rows of numbers next to electric motors are the amount of power consumed for each period. The number below the compressor is the specified duty for compression, which is met by either direct drive from the gas turbine or indirect drive from the electric motor. For each shaft, the power balance is met between power producers and consumers, with the consideration of mechanical power loss. The dotted line represents the transmission of electricity, which should reflect loss during transmission.

Regarding the fuel consumption, the third time period, which features the highest ambient temperature, requires the largest amount.

Table 4. Equipment performance correction data in Qatar

Time period (i)	1	2	3	4
Ambient temp. (ºC)	15	25	35	25
Output(i)/Output(ISO)	1	0.94	0.87	0.94
HR(i)/HR(ISO)	1	1.02	1.04	1.02

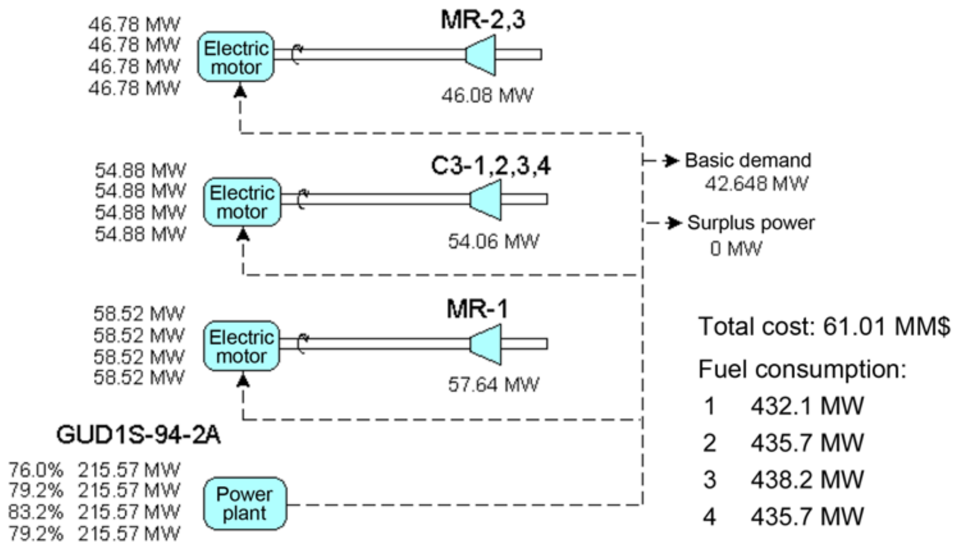

Fig. 7. Best design for Qatar case.

This is mainly because the equipment has the lowest thermal efficiency in this time period.

3. Qatar Case

In this case, the equipment performance in each time period is estimated for the average ambient conditions in Qatar. The equipment performance correction data are shown in Table 4. They are calculated according to the performance deterioration data shown in Fig. 1.

To gain higher system reliability, all the equipment is required to operate at maximum 85% load in each time period. The best design for Qatar is shown in Fig. 7. In this design, no gas turbine driver is present, and three electric motors are selected as mechanical drivers. Power plant GUD1S94-3A is selected to generate electricity for all the electric motors and consumers in the LNG plant. As shown in Fig. 7, the power plant operates at partial load, which means it is necessary to consider performance deterioration in this case. Since there is no credit for power export, the solver is trying to find a solution with no surplus power.

Impacts of the part-load performance on the overall energy efficiency can be explained with the power plant selected in the optimal solution. The power plant GUD1S94-3A is producing the same power for all the time periods, although working load for each time period is different between 76% and 83.2%, which results in different fuel consumption for each time period. 215.57 MW of power is produced at 76% part load for the first time period, while the same amount of power is produced at higher working load (i.e., 79.2%) due to high ambient temperature and, consequently, poor efficiency.

Compared with the U.K. design, a larger power plant is selected in the Qatar design due to worse equipment performance deterioration, which also results in an increased fuel consumption and annualized cost in this design. The refrigeration compressors for propane pre-cooling, C3-1 to 4, are driven together by a single electric motor in this design. So the ambient conditions have a significant impact on power system design not only for driver selection, but also for machine allocation. It is also necessary to consider the equipment performance deterioration due to ambient condition change in the design stage.

CONCLUSIONS

As can be seen in the case study, ambient conditions have a significant impact on power system design. It is important to reflect the equipment performance variations when ambient conditions change. Multi-period methodology effectively allows for the effect of ambient conditions in the design. Although the way of machine allocation is fixed throughout the whole time span, the output of drivers and power plants is able to vary and meet the demand in each time period. In the time period of a higher ambient temperature, more fuel consumption is required due to lower thermal efficiency of the power generation equipment. For geographical locations with a higher ambient temperature, more or larger equipment is likely to be selected in power systems to compensate for worse performance deterioration.

On the other hand, equipment performance deterioration due to partial load operation should also be considered during the design. The piecewise approximation applied in this work is able to account for the equipment partial load performance and to estimate fuel consumption more accurately.

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NOMENCLATURE

Indices

C : refrigeration compressor
CS : compressor stage
T : time period
GT : gas turbine shaft
GTOP : gas turbine option (actual model)
EM : electric motor shaft
PP : power plant place
PPOP : power plant option (actual model)

Parameters

$\overline{\lambda_{GTUB}}$: maximum load ratio for gas turbine drivers [-]
$\overline{\lambda_{PPUB}}$: maximum load ratio for power plants [-]
$\overline{\text{AnnuFact}}$: annualized factor [1/yr]
$\overline{\text{BED}}$: basic electricity demand. Electricity for helper or main electric motors is not included [kW]
$\overline{\text{CGTOP}}$: cost of gas turbine options [k$]
$\overline{\text{CPPOP}}$: cost of power plant options [k$]
$\overline{\text{CRC0}}$: reference cost of compressors [k$]
$\overline{\text{CSSize}}$: size of compressor stages [kW]
$\overline{\text{ELoss}}$: fraction of electricity loss in electrical power distribution
$\overline{\text{EMCost0}}$: reference cost of electric motors [k$]
$\overline{\text{EMEff}}$: electric motor efficiency
$\overline{\text{EMSize0}}$: reference size of electric motors [kW]
$\overline{\text{HGEff}}$: helper generator efficiency
$\overline{\text{HMEff}}$: helper motor efficiency
$\overline{\text{HMGCost0}}$: reference cost of helper motors and generators [k$]
$\overline{\text{HMGSize0}}$: reference size of helper motors and generators [kW]
$\overline{\text{IR}}$: interest rate
$\overline{\text{LIFE}}$: plant life [yr]
$\overline{\text{MaxEExp}}$: maximum electricity export [kW]
$\overline{\text{MaxEFF}}$: maximum amount of available end flash fuel [kW]
$\overline{\text{MaxEImp}}$: maximum electricity import [kW]
$\overline{\text{MaxWEM}}$: maximum size of electric motors [kW]
$\overline{\text{MLoss}}$: fraction of shaft power loss in mechanical transmission
$\overline{\text{NCAS0}}$: reference number of compressor casings
$\overline{\text{NCS}}$: number of compressor stages
$\overline{\text{OprTime}}$: total operation time in the whole time span [hr/yr]
$\overline{\text{PCAS}}$: factor of cost penalty per additional compressor casing
$\overline{\text{PowPPOP}^{Max}}$: maximum electrical power output for power plant

options [kW]
PowPPOP^{Min} : minimum electrical power output for power plant options [kW]
PT : length of time period. Defined by a fraction of the total time span
QGTOP^{Max} : fuel consumption when gas turbine options operate with maximum shaft power output [kW]
QPPOP$^{Max}_{PPOP,T}$: fuel consumption when power plant options operate with maximum power output [kW]
UCEExp : unit cost of electricity export [k$/kW·h]
UCEFF : unit cost of end flash fuel [k$/kW·h]
UCEImp : unit cost of electricity import [k$/kW·h]
UCFF : unit cost of fresh fuel [k$/kW·h]
WCS : shaft power requirement for compression stage CS in time period T [kW]
WGTOP^{Max} : maximum shaft power output for gas turbine options [kW]
WGTOP^{Min} : minimum shaft power output for gas turbine options [kW]
WHGExGTOP : maximum excessive output for helper generators over the start-up requirement [kW]
WHMExGTOP : maximum excessive output for helper motors over the start-up requirement [kW]
WHMGUB : upper bound of shaft power output for helper motors and generators [kW]
WRC0 : reference size of compressors [kW]
WSMGTOP : shaft power requirement for start-up of gas turbine options [kW]

Continuous, Positive Variables
CCAP : annualized capital cost [k$/yr]
CGT : total cost of gas turbine drivers [k$]
CELEC : electricity cost per year [k$/yr]
CEM : total cost of electric motors [k$]
CFUEL : fuel cost per year [k$/yr]
CHMG : total cost of gas turbine starters/helper devices [k$]
CPP : total cost of power plants [k$]
CRC : total cost of refrigerant compressors [k$]
EExp : amount of electricity export [kW]
EImp : amount of electricity import [kW]
EMCost : electric motor cost on each motor shaft [k$]
EMSize : electric motor size on each motor shaft [kW]
HMGCost : helper motor or generator cost on each gas turbine shaft [k$]
HMGSize : helper motor or generator size on each gas turbine shaft [kW]
NCAS : number of compressor casings
NCSGT : number of compressor stages belonging to the same compressor on each gas turbine shaft
NCSEM : number of compressor stages belonging to the same compressor on each electric motor shaft
NGTS : number of gas turbine shafts driving compressor stages belonging to the same compressor
NEMS : number of electric motor shafts driving compressor stages belonging to the same compressor
PowEM : electrical power consumption of electric motors [kW]
PowHG : electrical power generation of helper generators [kW]
PowHM : electrical power consumption of helper motors [kW]
PowPP : electrical power generation of power plants [kW]
PowPPOP : electrical power generation of power plant options [kW]
QGT : fuel consumption of gas turbine drivers [kW]
QGTOP : fuel consumption of gas turbine options [kW]
QEFF : end flash fuel consumption [kW]
QFF : fresh fuel consumption [kW]
QPP : fuel consumption of power plants [kW]
QPPOP : fuel consumption of power plant options [kW]
QTotal : total fuel consumption [kW]
TCOST : total annualized cost [k$/yr]
WGT : shaft power output of gas turbine drivers [kW]
WGTOP : shaft power output of gas turbine options [kW]
WGTSD : total shaft power demand on gas turbine shafts [kW]
WEM : shaft power output of electric motors [kW]
WEMSD : total shaft power demand on electric motor shafts [kW]
WHG : actual output of helper generators [kW]
WHGEx : excessive output of helper generators over the start-up requirement [kW]
WHM : actual output of helper motors [kW]
WHMEx : excessive output of helper motors over the start-up requirement [kW]
WSM : shaft power requirement for gas turbine start-up [kW]

Binary Variables
YGTC$_{GT,C}$: indicates whether gas turbine shaft GT is supplying power to compressor C
YGTCS$_{GT,CS}$: indicates whether gas turbine shaft GT is supplying power to compressor stage CS
YGTOP$_{GT,GTOP}$: indicates whether the gas turbine option GTOP (actual model) is selected to drive the gas turbine shaft GT
YEExp : to allow or not allow exporting electricity
YEM$_{EM}$: indicates whether electric motor EM is active
YEMCS$_{EM,CS}$: indicates whether electric motor shaft EM is supplying power to compressor stage CS
YEMC$_{EM,C}$: indicates whether electric motor shaft EM is supplying power to compressor C
YHM$_{GT}$: existence of continuous helper motor on gas turbine shaft GT
YPPOP$_{PP,PPOP}$: indicates whether the power plant option PPOP (actual model) is selected to occupy the power plant place PP

SOS2 Variables
LGTWQ(i) : linearization of gas turbine performance variation under part load operation
LPPPowQ(i) : linearization of power plant performance variation under part load operation

REFERENCES

1. S. A. Papoulias and I. E. Grossmann, *Comput. Chem. Eng.*, **7**, 695 (1983).
2. C. W. Hui and Y. Natori, *Comput. Chem. Eng.*, **20**, S1577 (1996).
3. R. R. Iyer and I. E. Grossmann, *Comput. Chem. Eng.*, **22**, 979 (1998).
4. F. A. P. Oliveira and H. A. Matos, *Computer Aided Chem. Eng.*, **14**, 233 (2003).
5. Z. Shang and A. Kokossis, *Chem. Eng. Sci.*, **60**, 4431 (2005).

6. F. Del Nogal, J. Kim, S. Perry and R. Smith, *AIChE J.*, **56**, 2356 (2010).
7. F. Del Nogal, J. Kim, S. Perry and R. Smith, *AIChE J.*, **56**, 2377 (2010).
8. F. J. Brooks, GE Gas Turbine Performance Characteristics (GER-3567H), GE Power Systems, New York (2013).
9. D. S. Remer and L. H. Chai, *Chem. Eng. Progress*, **August**, 72 (1990).
10. R. Kurz, Proceedings of the Thirty-Fourth Turbomachinery Symposium 2005, 131 (2005).
11. R. A. Fallaize and R. S. Phillips, Gas Processors Association Europe, London, UK (2002).
12. S. Shu, F. Christiano and M. Harrison, *Oil Gas J.*, **100**, 60 (2002).
13. X. Zheng and J. Kim, *Ind. Eng. Chem. Res.*, **50**, 11201 (2011).