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Relative energy stability analysis on the onset of Taylor-Görtler vortices in impulsively accelerating Couette flow

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Abstract−The onset of Taylor-Görtler vortices in impulsively accelerating Couette flows was analyzed by using the energy method. This model considers the growth rate of the kinetic energy of the base state and also that of disturbances. In the present system the primary transient Couette flow is laminar, but for the Reynolds number Re>Re_c secondary motion sets in at a certain time. For Re>Re_c the dimensionless critical time to mark the onset of vortex instabilities, τ_c, is presented as a function of Re. It is found that the predicted τ_c-value is much smaller than experimental detection time of first observable secondary motion. Therefore, small disturbances initiated at τ_c evidently require some growth period until they are detected experimentally. Since the present system is a rather simple one, the results will be helpful in comparing available stability models.

Keywords: Taylor-Görtler Vortex, Energy Method, Relative Stability

INTRODUCTION

In primary laminar flows along concavely curved walls, the destabilizing action of centrifugal forces can induce secondary motion in the form of vortices. The related hydrodynamic instabilities usually lead to Taylor-Görtler vortices in the flow near the wall. It is well-known that Taylor vortices appear for Ta≥1695 with a very narrow gap ($\eta\rightarrow 1$). For a fully-developed Couette flow the related stability analysis is described in detail by Chandrasekhar [1]. However, in real situations, the onset of a secondary flow driven by centrifugal forces under a developing period rather than a fully-developed state is common. Thus, many researchers have studied the onset of Taylor-Görtler vortices of transient laminar Couette flow in a rotating cylinder. Chen and colleagues [2-4] and Kasagi and Hirata [5] investigated the detection time of vortices caused by impulsive spin-up theoretically and experimentally. In this kind of flow system the characteristic conditions to mark the onset of secondary motion become an important question.

In the present system the primary transient Couette flow is laminar, but for the Reynolds number Re>Re_c secondary motion sets in at a certain time. Therefore, work to predict the dimensionless critical time, τ_c, which represents the onset of secondary motion, has been conducted. However, a conventional analysis of the equations governing the response of the flow field to small disturbances is complicated by very important aspect of the flow: the base flow profile is time variant. This aspect introduces the complication that any conclusion inferred about the stability of the system to small disturbances is only meaningful with respect to the temporal behavior of the base state. There are many fluid dynamical systems which exhibit this difficulty, and Shen's [6] original measure for investigating the relative stability of time variant base states proves a suitable framework for the present system.

Without considering the momentary instability concept, Chen et al. [2,3] and Kasagi and Hirata [5] analyzed the onset of Taylor-Görtler vortices linear theory: amplification theory and quasi-steady approach. The former method is the initial-value problem approach in which the time evolution of the initial disturbances assumed is monitored. They concluded that when the kinetic energy of disturbance has grown a thousand-fold (or ten thousand-fold) according to their amplification theory, the secondary flow pattern is clearly visible in comparison with their experimental observations. In the latter method, the instantaneous stability of the primary-velocity profile is analyzed and the state of motion at a particular instant examined. As discussed MacKerrell [7], this static approximation has two critical contradictions: First, that the growth rate of the disturbance is larger than that of base quantity under the quasi-steady state ansatz, and second that the growth rate of the disturbance is zero under the neutral stability condition. To overcome this contradiction, they propose the energy method as an alternative.

In this study, we analyzed the onset of Taylor-Görtler vortices in impulsively accelerating Couette flow by the energy method which has been used for the various problems [8-10]. The energy methods give the necessary conditions of the onset of secondary flows, since it introduces some finite, initial disturbances and traces the temporal growth of their kinetic energy. Here, without the quasi-steady state assumption, new stability equations were derived for the whole time region by introducing the relative energy and relative growth rate. Also, the effects of stability criteria on the critical conditions were examined. Since, for the whole domain of $\eta(=R_i/R_o)$ and time, the stability criteria are obtained without any assumption, the present study is the extension and complement of the previous studies based on the energy method.

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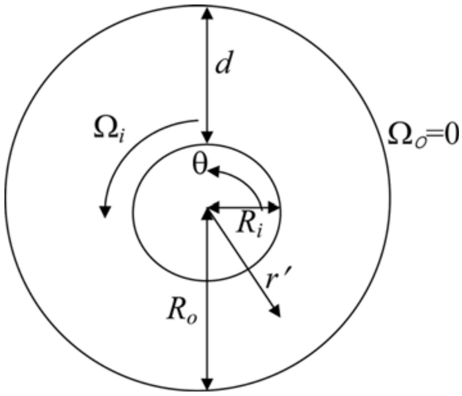

Fig. 1. Top view of the basic system considered here.

THEORETICAL ANALYSIS

1. Governing Equations

The system considered here is a Newtonian fluid confined between the two concentric cylinders of radii R_i and R_o. Let the axis of the inner cylinder be along the vertical z′-axis under the cylindrical coordinates (r′, θ, z′) and the corresponding velocities be U, V and W. Starting from time t=0, the inner cylinder is impulsively rotated at a constant surface speed $V_i(=R_i\Omega_i)$ and the outer cylinder is kept stationary ($\Omega_o=0$). A schematic diagram of the present system is shown in Fig. 1. For a high V_i, the secondary flow in form of Taylor-like vortices sets in at a certain time and the governing equations of the flow field are expressed as

$$\nabla\cdot\mathbf{U}=0, \tag{1}$$

$$\left\{\frac{\partial}{\partial t}+\mathbf{U}\cdot\nabla\right\}\mathbf{U}=-\frac{1}{\rho}\nabla P+\nu\nabla^2\mathbf{U}, \tag{2}$$

where **U**, P, ν and ρ represent the velocity vector, the dynamic pressure, the kinematic viscosity and the density, respectively.

The basic velocity field of the developing Couette flow is represented for the case of constant physical properties:

$$\frac{\partial V_0}{\partial t}=\nu D' D'_* V_0, \tag{3}$$

with the following initial and boundary conditions,

$$V_0=0 \text{ for } t\le 0, \tag{4a}$$
$$V_0=V_i \text{ at } r'=R_i \text{ for } t>0, \tag{4b}$$
$$V_0=0 \text{ at } r'=R_o \text{ for } t>0, \tag{4c}$$

where $D'=\partial/\partial r'$ and $D'_*=D'+1/r'$. The analytical, exact solution can be found in the work of Tranter [11].

$$v_0(r,\tau)=\frac{\eta}{\xi}\left(\frac{\xi^2-1}{\eta^2-1}\right)+\sum_{i=1}^{\infty}Q_i\left[J_1\left(\lambda_i\frac{\xi}{\eta}\right)Y_1(\lambda_i)-J_1(\lambda_i)Y_1\left(\lambda_i\frac{\xi}{\eta}\right)\right]\exp\left\{-\lambda_i^2\frac{(1-\eta)^2}{\eta^2}\tau\right\}, \tag{5}$$

where $v_0=V_0/V_i$, r=r′/d, $d=R_o-R_i$, and $\tau=\nu t/d^2$ and $\eta=R_i/R_o$. Here J_1 and Y_1 are the 1st kind Bessel functions, and $\xi=(1-\eta)r$ and $Q_i=\pi\{[J_1(\lambda_i)/J_1(\lambda_i/\eta)]^2-1\}^{-1}$. The λ_i's are the roots of the equation

$$J_1\left(\frac{\lambda}{\eta}\right)Y_1(\lambda)-J_1(\lambda)Y_1\left(\frac{\lambda}{\eta}\right)=0. \tag{6}$$

For the limiting case of very narrow gap ($\eta\to 1$), the curvature effects can be neglected and therefore, the primary-velocity field can be approximated as

$$v_0=\sum_{n=0}^{\infty}\left\{\text{erfc}\left(\frac{n}{\sqrt{\tau}}+\frac{y}{\sqrt{4\tau}}\right)-\text{erfc}\left(\frac{n+1}{\sqrt{\tau}}-\frac{y}{\sqrt{4\tau}}\right)\right\} \text{ for } \eta\to 1, \tag{7}$$

where $y=(r'-R_i)/d$.

For the case of the unbounded domain of $R_o\to\infty$, i.e. $\eta\to 0$, using the Laplace transform method, the base velocity field can be represented as [11]

$$v_0=1+\frac{2}{\pi}\int_0^{\infty}\exp(-\varpi^2\bar{\tau})\frac{J_0(\varpi\bar{r})Y_0(\varpi)-Y_0(\varpi\bar{r})J_0(\varpi)}{J_0^2(\varpi)+Y_0^2(\varpi)}\frac{d\varpi}{\varpi}, \tag{8}$$

where $\bar{r}=r'/R_i$ and $\bar{\tau}=\nu t/R_i^2$. For small values of $\bar{\tau}$, the above velocity profile can be approximated as

$$v_0=\left(\frac{1}{\bar{r}}\right)^{1/2}\text{erfc}\left(\frac{\bar{r}-1}{\sqrt{4\bar{\tau}}}\right)+\left\{\frac{(\bar{r}-1)\sqrt{\bar{\tau}}}{4\bar{r}^{3/2}}\right\}\text{ierfc}\left(\frac{\bar{r}-1}{\sqrt{4\bar{\tau}}}\right)+\left\{\frac{(9-2\bar{r}-7\bar{r}^2)\bar{\tau}}{32\bar{r}^{5/2}}\right\}\text{i}^2\text{erfc}\left(\frac{\bar{r}-1}{\sqrt{4\bar{\tau}}}\right)+\cdots. \tag{9}$$

And, for the limiting case of $\tau\ll 1$, the above velocity profiles can be simplified as:

$$v_0=\left(\frac{\eta}{\eta+(1-\eta)y}\right)^{1/2}\text{erfc}\left(\frac{y}{\sqrt{4\tau}}\right) \text{ for } \tau\ll 1. \tag{10}$$

For $\eta=0.2$, the above solution is compared with the exact solutions in Fig. 2. For $\tau\le 10^{-2}$, Eq. (10) is almost exact, as shown in the figure. With increasing η but decreasing τ, the solution of Eq. (5) can be further approximated as:

$$v_0=\text{erfc}\left(\frac{y}{4\tau}\right) \text{ for } \tau\to 0. \tag{11}$$

Otto [13] and MacKerrell et al. [7] used this profile in their linear

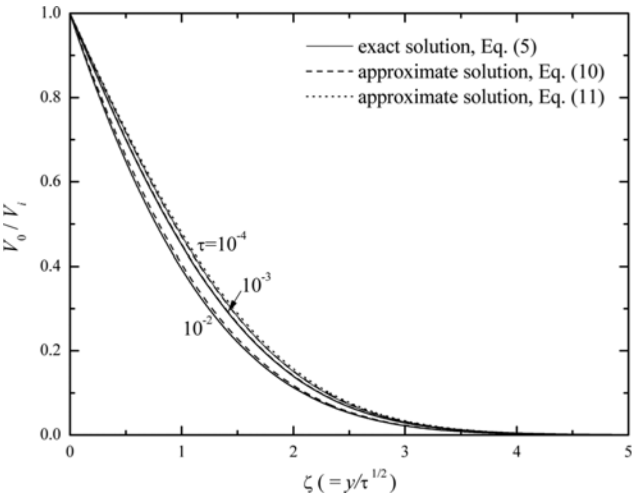

Fig. 2. Primary-velocity profiles.

stability analyses.

2. Energy Method

The disturbance equations are obtained in the usual way by assuming physical quantities consisting of the basic quantities and disturbance ones. Following the work of Serrin [7], the energy identity is written as

$$\frac{\mathrm{dE}}{\mathrm{d}\tau}=\mathrm{ReI}-\mathrm{D}, \tag{12}$$

where $\mathrm{E}=\langle \mathbf{u}_1\cdot\mathbf{u}_1\rangle/2$, $\mathrm{I}=\langle \mathrm{u}_1\mathrm{v}_1\phi\rangle$, $\phi=(1/\mathrm{r}-\partial/\partial\mathrm{r})\mathrm{v}_0$ and $\mathrm{D}=\langle\nabla\mathbf{u}_1:\nabla\mathbf{u}_1\rangle$. Here $\mathbf{u}_1(=\mathbf{U}_1/\mathrm{V}_i)$ is the dimensionless velocity vector, $\mathrm{Re}(=\mathrm{V}_i\mathrm{d}/\nu)$ is the Reynolds number and $\langle\cdot\rangle$ represents the average over the system. The conventional energy method determines the critical times to mark the onset of secondary motion at which E is the minimum, *i.e.*

$$\frac{\mathrm{dE}}{\mathrm{d}\tau}=0 \text{ at } \tau=\tau_S. \tag{13}$$

This condition is known as the strong stability criterion.

Even though the temporal evolution of the disturbance energy can be described by Eq. (9), care must be taken in defining the stability criterion for the system having time-dependent base states. Shen [6] first observed time dependent parallel shear flow; if the kinetic energy of a perturbation decreases in time but that of the base state decreases at a faster rate, then the kinetic energy of perturbation will appear amplified in time. Conversely, if the kinetic energy of a perturbation increases in time but that of the base state increases faster still, then the kinetic energy of perturbation will appear to decay in time. To determine the stability characteristics of perturbations of time variant base states, Shen [6] introduced the concept of relative stability where the neutral stability of the system is determined when

$$\frac{\mathrm{dE}_R}{\mathrm{d}\tau}=0 \text{ at } \tau=\tau_R. \tag{14}$$

Here

$$\mathrm{E}_R=\frac{\mathrm{E}}{\mathrm{E}_0} \tag{15}$$

is called the relative energy [14] and Eq. (14) has been known as relative stability [15]. Here E and E_0 are the kinetic energies of the disturbance and base flow, respectively. For the present system, E_0 is defined as

$$\mathrm{E}_0=\frac{1}{2}\left(\int_0^1\mathrm{v}_0^2\mathrm{dz}\right). \tag{16}$$

With these definitions, the criterion for momentary stability of unsteady base state is given by

$$\frac{1}{\mathrm{E}_R}\frac{\mathrm{dE}_R}{\mathrm{d}\tau}=\sigma-\sigma_0. \tag{17}$$

Here σ and σ_0 are the growth rates of the disturbance and base energy defined as

$$\sigma=\frac{1}{\mathrm{E}}\frac{\mathrm{dE}}{\mathrm{d}\tau} \text{ and } \sigma_0=\frac{1}{\mathrm{E}_0}\frac{\mathrm{dE}_0}{\mathrm{d}\tau}. \tag{18}$$

Based on the velocity profile of Eq. (5), the growth rate of basic kinetic energy is given as,

$$\sigma_0=\frac{-2\left(\frac{1-\eta}{\eta}\right)^2\sum_{i=0}^{\infty}\exp(-\lambda_i^2\tau')\left\{(\lambda_i\mathrm{Q}_i)^2\exp(-\lambda_i^2\tau')\mathrm{b}_i+\frac{\eta\lambda_i^2\mathrm{Q}_i}{\eta^2-1}\mathrm{c}_i\right\}}{\mathrm{f}(\eta)+\sum_{i=1}^{\infty}\exp(-\lambda_i^2\tau')\left\{\mathrm{Q}_i^2\exp(-\lambda_i^2\tau')\mathrm{b}_i+2\frac{\mathrm{Q}_i\eta}{\eta^2-1}\mathrm{c}_i\right\}}, \tag{19}$$

where $\mathrm{f}(\eta)=\eta^2/(\eta^2-1)^2(-3/4-\eta^4/4+\eta^2-\ln\eta)$, $\tau'=\tau\{(1-\eta)/\eta\}^2$ and b_i and c_i are

$$\mathrm{b}_i(\lambda_i,\eta)=\int_\eta^1\xi\left[\mathrm{J}_1\left(\lambda_i\frac{\xi}{\eta}\right)\mathrm{Y}_1(\lambda_i)-\mathrm{J}_1(\lambda_i)\mathrm{Y}_1\left(\lambda_i\frac{\xi}{\eta}\right)\right]^2\mathrm{d}\xi,$$

and

$$\mathrm{c}_i(\lambda_i,\eta)=\int_\eta^1(\xi^2-1)\left[\mathrm{J}_1\left(\lambda_i\frac{\xi}{\eta}\right)\mathrm{Y}_1(\lambda_i)-\mathrm{J}_1(\lambda_i)\mathrm{Y}_1\left(\lambda_i\frac{\xi}{\eta}\right)\right]\mathrm{d}\xi.$$

For the limiting case of $\eta\to1$, σ_0 is obtained from Eq. (7) as

$$\sigma_0=\frac{8\sum_{n=1}^{\infty}\exp(-\mathrm{n}^2\pi^2\tau)\{1-\exp(-\mathrm{n}^2\pi^2\tau)\}}{1/3-\sum_{n=1}^{\infty}\exp(-\mathrm{n}^2\pi^2\tau)/(\mathrm{n}\pi)^2\{2-\exp(-\mathrm{n}^2\pi^2\tau)\}}. \tag{20}$$

It is well-known that the present system with $\mathrm{Re}>\mathrm{Re}_c$ is asymptotically unstable [7]. Therefore, our primary concern is the instantaneous instability, which is defined as

$$\sigma>\sigma_0, \tag{21}$$

under the relative instability concept [6]. The neutral stability condition under the relative instability can be determined from

$$\sigma_0\mathrm{E}=\mathrm{ReI}-\mathrm{D}. \tag{22}$$

And, therefore the relative stability limit can be obtained as

$$\frac{1}{\mathrm{Re}}=\max\left[\frac{\mathrm{I}}{\mathrm{D}+\sigma_0\mathrm{E}}\right]. \tag{23}$$

under the condition of D=1.

Under the normal mode analysis the typical axisymmetric disturbances, which have been observed experimentally [2-5], are well represented by

$$(\mathrm{u}_1,\mathrm{v}_1,\mathrm{p}_1)=(\mathrm{u}',\mathrm{v}',\mathrm{p}')\cos az, \tag{24a}$$
$$\mathrm{w}_1=\mathrm{w}'\sin az, \tag{24b}$$

where a is the dimensionless wavenumber representing the periodicity in the z′-direction, z=z′/d and the primed quantities representing disturbance amplitudes are a function of r and τ. Here we assume the infinitely long cylinder and neglect the end wall effects. Then this maximum problem can be solved by the variational technique [7]. By eliminating the Lagrangian multiplier, the Euler-Lagrange equations for the relative stability model are obtained:

$$-\frac{1}{2}\mathrm{a}^2\mathrm{Re}\phi\mathrm{v}'+\left\{\frac{\partial}{\partial\mathrm{r}}\left(\frac{\partial}{\partial\mathrm{r}}+\frac{1}{\mathrm{r}}\right)-\mathrm{a}^2\right\}^2\mathrm{u}'=\frac{\sigma_0}{2}\frac{\partial^2\mathrm{u}'}{\partial\mathrm{r}^2}, \tag{25}$$

$$\frac{1}{2}\mathrm{Re}\phi\mathrm{u}'+\left\{\frac{\partial}{\partial\mathrm{r}}\left(\frac{\partial}{\partial\mathrm{r}}+\frac{1}{\mathrm{r}}\right)-\mathrm{a}^2\right\}\mathrm{v}'=\frac{\sigma_0}{2}\mathrm{v}'. \tag{26}$$

The proper boundary conditions are

$$u'=\frac{\partial u'}{\partial r}=v'=0 \text{ at } r=\eta/(1-\eta) \text{ and } 1/(1-\eta). \tag{27}$$

For the case of $\tau\rightarrow\infty$, the above stability equations with $\sigma_0=0$ degenerate into the strong stability formulation. And, for the limiting case of $\tau\rightarrow 0$, based on the velocity profile of Eq. (11), it is found that $\sigma_0=(1/E_0)(dE_0/d\tau)=1/2\tau$ and therefore, $\sigma_0/2$ becomes $1/4\tau$.

3. Solution Method

The stability Eqs. (25)-(27) are solved by employing the shooting method. To integrate these stability equations the proper values of $\partial^2 u'/\partial r^2$, $\partial^3 u'/\partial r^3$ and $\partial v'/\partial r$ at $r=\eta/(1-\eta)$ are assumed for a given τ and a. Since the stability equations and their boundary conditions are all homogeneous, the value of $\partial^3 u'/\partial r^3$ at $r=\eta/(1-\eta)$ can be assigned arbitrarily and the value of the parameter Re is assumed. This procedure can be understood easily by taking into account the characteristics of eigenvalue problems. After all the values at r=1 are provided, this eigenvalue problem can be proceeded numerically. Integration is performed from r=1 to the axis of rotation with the fourth order Runge-Kutta-Gill method. If the guessed values of Re, $\partial^3 u'/\partial r^3$ and $\partial v'/\partial r$ at $r=\eta/(1-\eta)$ are correct, u', $\partial u'/\partial r$ and r' will vanish at $r=1/(1-\eta)$. The minimum Re-value is found in the plot of Re vs. a.

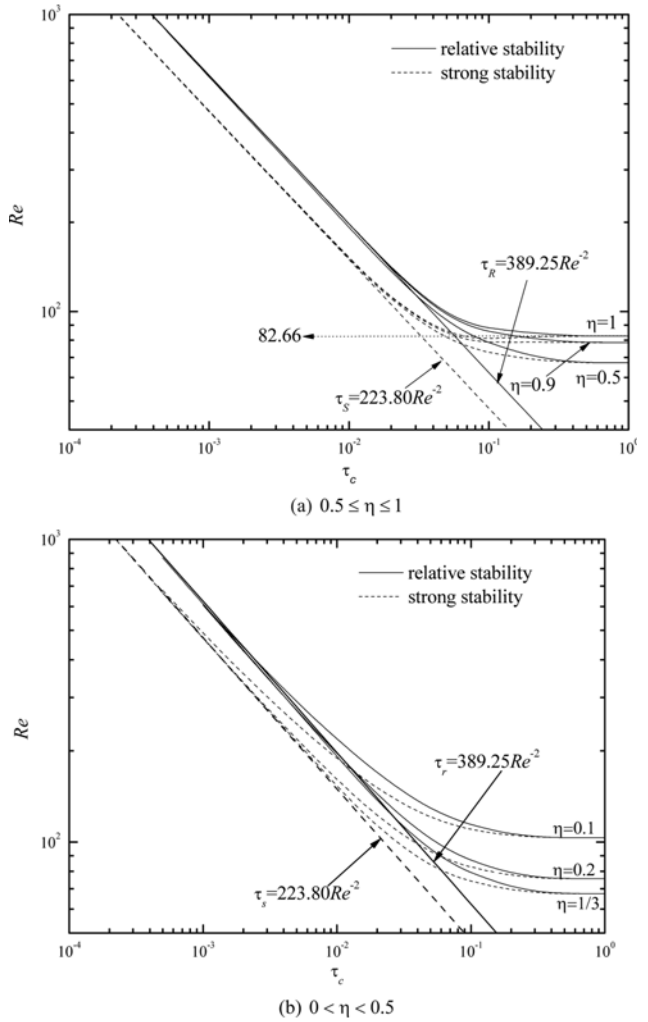

Fig. 3. Characteristic stability curve in the Re−τ diagram. (a) 0.5≤ η≤1 and (b) 0<η<0.5.

RESULTS AND DISCUSSION

The stability conditions obtained from the present momentary and the strong stability model are illustrated in Fig. 3. The approximation (10) produces the same values as those from Eq. (5) as time decreases. As discussed below Eq. (24), the present model yields the strong stability limit as $\tau\rightarrow\infty$. For the limiting case of $\tau\rightarrow\infty$, the stability Eqs. (25)-(27) are reduced as

$$-\frac{1}{2}a^2 \mathrm{Re}\, v'+\left\{\frac{d^2}{dy^2}-a^2\right\}^2 u'=0, \tag{28}$$

$$\frac{1}{2}\mathrm{Re}\, u'+\left\{\frac{d^2}{dy^2}-a^2\right\} v'=0. \tag{29}$$

under the very narrow gap condition, *i.e.* $\eta\rightarrow 1$, where $\phi\rightarrow 1$ and $\sigma_0\rightarrow 0$ from Eqs. (5) and (27), respectively. The proper boundary conditions are

$$u'=\frac{du'}{dy}=v'=0 \text{ at } y=0 \text{ and } 1. \tag{30}$$

According to the calculation of Chandrasekhar [1], the critical condition is $(1/2\mathrm{Re}_c)^2=1708$ and $a_c=3.117$, i.e. $\mathrm{Re}_c=82.66$ for $\tau\rightarrow\infty$ and $\eta\rightarrow 1$. And, for the another limiting case of small τ, the critical time τ_c to mark the onset of a fastest growing instability decreases with increasing Re. Based on the base velocity field given in Eq. (8), they approach

$$\tau_R=389.27\mathrm{Re}^{-2} \text{ as } \tau\rightarrow 0, \tag{31a}$$

for the momentary stability, and

$$\tau_S=223.80\mathrm{Re}^{-2} \text{ as } \tau\rightarrow 0, \tag{31b}$$

for the strong stability, as are illustrated in Fig. 3. These stability limits suggest that the proper length scale for the Reynolds number is the Rayleigh layer thickness, $\sqrt{\nu t}$, which was employed as a length scale for the Taylor number in the linear stability analysis [13].

In Fig. 4, the present predictions are compared with the previous ones based on the linear stability theory. For the case of a rather wide gap $\eta\leq 0.2$, the quasi-static analysis based on the linear stability theory gives the subcritical region where the global minimum of the critical Reynolds number, Re_G is lower than the steady state one, Re_c. In the range of $\mathrm{Re}_G\leq\mathrm{Re}\leq\mathrm{Re}_c$, the instability is transient and decays out as τ increases. For the limiting case of $\eta\rightarrow 1$, the strong stability limit base on the energy method also shows the gloabal minumum. However, the present momentary stability limit does not show the global minimum for the whole region of η. As expected, the present predictions bound those obtained from the linear stability analysis. Since, in the energy method, finite disturbances are introduced into the basic flow and their growth monitored, it is natural that the energy method bound the linear stability theory.

Now, the above results are compared with the experimental data of Chen et al. [2,3] and Hirata et al. [5]. Since no experimental points lie to the left of the corresponding curve, the present stability limits bound the experimental data. However, as expected, none of the

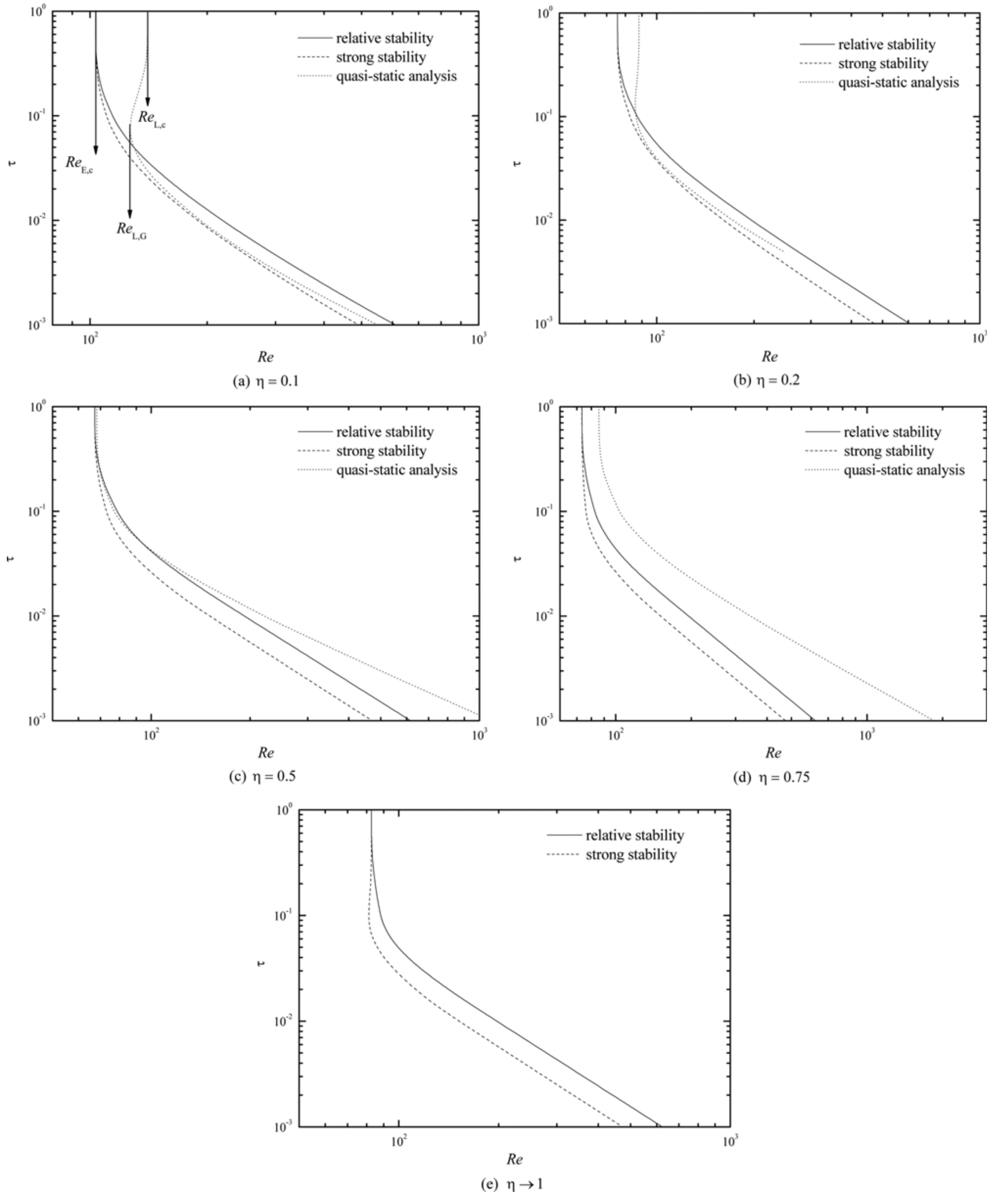

Fig. 4. Comparison of critical condition with the previous theoretical predictions for the various η. The subscripts L and E mean the linear stability theory and the energy method, respectively.

stability criteria predict the experimental results quantitatively. In the energy method, an overly broad class of disturbances, which need not satisfy the dynamic equations such as Navier-Stokes, equation is employed and therefore, this disturbance does not represent the actual disturbance present during the experiments. This is one of the major sources of discrepancy. Another one is that the time for disturbances to grow to finite amplitude before being observed is necessary. Therefore, further study on the proper disturbance which

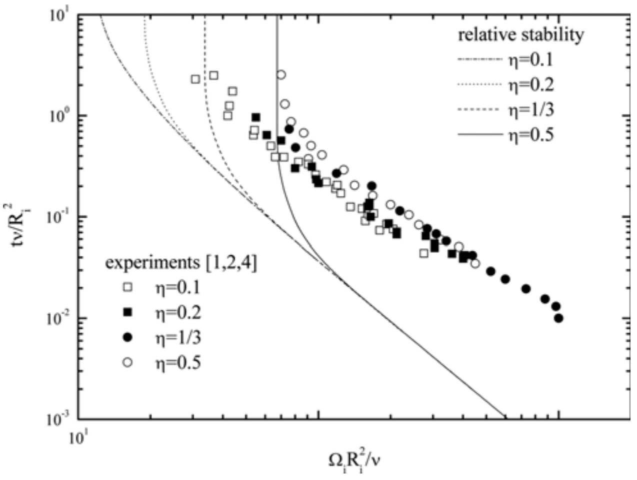

Fig. 5. Comparison of critical condition with the available experimental data.

explains the experimental process and its growth is needed.

CONCLUSIONS

The onset of Taylor-Görtler vortices in impulsively accelerating Couette flows has been analyzed without the quasi-steady state assumption. The present analysis yields more conservative stability limits than that from the linear stability theory. For Re>Re$_c$ the strong stability criterion yields the lowest bound of the critical time among available models. The present momentary stability model shifts the stability limit to the more stable direction than the strong stability model: $\tau_S<\tau_R$. Unlike the previous studies based on the quasi-steady state assumption or the conventional energy method, the sub-critical region, where the global minimum of the critical Reynolds number is lower than the steady state one, has not been observed in the present analysis. All the predicted critical times to mark the onset of vortices, which are shown in this study, are much smaller than available experimental data. It seems evident that disturbances require some growth period until they are detected experimentally.

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NOMENCLATUE

a : dimensionless wavenumber
d : gap size (=R$_o$−R$_i$) [m]
E : dimensionless kinetic energy
P : pressure [Pa]
p : dimensionless pressure disturbance
R : radius of cylinder [m]
Re : Reynolds number, $V_i d/\nu$
(U, V, W) : velocities in cylindrical coordinates [m/s]
(u′, v′, w′) : velocity disturbance amplitudes in cylindrical coordinates
(r′, θ, z′) : cylindrical coordinates

Greek Letters

η : radius ratio, (−R$_i$/R$_o$)
ν : kinematic viscosity [m^2/s]
ρ : density [kg/m^3]
σ : growth rate
τ : dimensionless time, (= $\nu t/d^2$)
Ω : angular velocity [rad/s]

Subscripts

c : critical condtions
i : inner cylinder
o : outer cylinder
0 : basic quantities
1 : disturbance quantities

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