

Genetic optimization of neural network and fuzzy logic for oil bubble point pressure modeling

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Abstract—Bubble point pressure is a critical pressure-volume-temperature (PVT) property of reservoir fluid, which plays an important role in almost all tasks involved in reservoir and production engineering. We developed two sophisticated models to estimate bubble point pressure from gas specific gravity, oil gravity, solution gas oil ratio, and reservoir temperature. Neural network and adaptive neuro-fuzzy inference system are powerful tools for extracting the underlying dependency of a set of input/output data. However, the mentioned tools are in danger of sticking in local minima. The present study went further by optimizing fuzzy logic and neural network models using the genetic algorithm in charge of eliminating the risk of being exposed to local minima. This strategy is capable of significantly improving the accuracy of both neural network and fuzzy logic models. The proposed methodology was successfully applied to a dataset of 153 PVT data points. Results showed that the genetic algorithm can serve the neural network and neuro-fuzzy models from local minima trapping, which might occur through back-propagation algorithm.

Keywords: Genetic Algorithm, Optimized Neural Network, Optimized Fuzzy Logic, Local Minima, Bubble Point Pressure of Crude Oils

INTRODUCTION

Bubble point pressure (Pb) is one of the most crucial pressure-volume-temperature (PVT) properties of reservoir fluid, which is widely used in petroleum engineering. Applications of this parameter include a variety of problems ranging from reservoir characterization and reserve estimation through surface production and facility design. The accuracy of above calculations is dominated by exactness of Pb measurement. Therefore, use of high accuracy data is inevitable. Precise values of PVT properties are obtained from laboratory experiments, such as constant mass expansion and differential vaporization tests on reservoir fluid samples at reservoir temperature [1]. However, these methods are very time-consuming and expensive. Many researchers attempt to find rapid and accurate ways to predict this parameter, and many of their empirical correlations confirm this hypothesis [2-10].

Intelligent systems are quick, accurate, and cheap methods for extracting the underlying dependency of a set of input/output data. Hitherto, researchers have used intelligent systems in their petroleum related problems [11-15]. Several researchers tried neural network models for estimation of bubble point pressure [16-22]. Others have proposed correlations for estimation of bubble point pressure from other PVT data [2-10]. Although these correlations result in satisfying results, further studies showed that neural networks and fuzzy logic models could produce more accurate results compared with correlations [12,16-18]. Nonetheless, these intelligent models still have some flaws. Neural network models with back-propagation

algorithm are in danger of sticking in local minima, i.e., the optimal formulation between input/output data might not be obtained through the back-propagation learning process [23]. This might occur for neuro-fuzzy systems due to the use of a back-propagation neural network for optimization of fuzzy logic. Genetic algorithms can extract the global minimum of a fitness function through a stochastic search process. The genetic algorithm is slower than the back-propagation algorithm. However, in petroleum related problems computing time is not as important as accuracy. It is preferred to spend some minutes more in order to achieve more accurate results instead. Therefore, in this study the genetic algorithm is used to extract the coefficients of embedded formulation of neural network and fuzzy logic models. Results showed that use of genetic algorithm eliminates the risk of sticking in local minima and improves the accuracy of final predictions. The propounded strategy was successfully applied to worldwide field data from open-literature sources.

BACKGROUND THEORIES

1. Fuzzy Logic

The basic idea of fuzzy logic (FL), or fuzzy set theory, was first presented by Zadeh [24]. Contrasting crisp logic (CL), in which a value may or may not belong to one class, fuzzy sets allow partial memberships. FL is appropriate for solving problems associated with uncertainty. Statistical methods try to abate and disregard this error, whereas fuzzy logic derives useful information from this error and uses it as a powerful parameter for increasing the accuracy of the predictions [11].

A fuzzy inference system (FIS) is the method of formulating from a given input to an output using fuzzy logic [25]. There are different types of FIS, but in this study the Takagi-Sugeno fuzzy inference

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Table 1. Statistics, including minimum (Min), maximum (Max), mean, and standard deviation (STD) for each Dataset used in this study

Datasets	Min	Max	Mean	STD
Temperature °F	90	266	177.8911	51.5028
Oil gravity °API	19.3	45.42	27.8855	5.2691
Rs/GG (Scf/STB)	158.5726	1280.1	591.8624	309.2044
Bubble point pressure (Psi)	915	4256	2428	934.4976

system (TS-FIS) was employed to predict the bubble point pressure from PVT data. In Takagi and Sugeno [26], output membership functions might be either constant or linear, which are mined by fuzzy clustering processes.

2. Back-propagation Neural Network

An artificial neural network (ANN) is a powerful intelligent tool for handling non-linear problems. One of the most conventional methods of training for ANN to learn how to carry out a particular task is back-propagation (BP). It's a recommended learning method, i.e., it needs a set of training data that has the corresponding output for any given input. In this method, the network calculates the deviation of predicted output and corresponding wanted output from the training data set. Then the error is propagated backward through the network and the weights are adjusted during a number of iterations, named epochs. The training stops when the calculated output values best approximate the desired values [27].

3. Neuro-fuzzy System

So far, the key roles of neural network (NN) and fuzzy logic (FL) have been described. FL is associated with explicit knowledge, while NN deals with implicit knowledge. Neuro-fuzzy modeling is a technique for describing the behavior of a system by means of fuzzy inference rules within a neural network construction. Adaptive neuro-fuzzy inference system (ANFIS) uses a given input/output data set, and constructs an FIS whose membership functions' parameters are tuned by the back-propagation algorithm [25]. Thus, the fuzzy inference system could train and learn from the modeling data.

4. Evolutionary Computing

Evolutionary computing uses some known mechanism of evolution as main elements in algorithmic design for computing. There are various suggested algorithms, but they are all based upon simulating the evolution of individual structure via processes of parent selection, mutation, crossover and reproduction. The most popular one is the genetic algorithm (GA) [28]. GA is a well-organized global optimization method for solving discontinuous and non-linear problems.

Genetic algorithm (GA) starts with discovering the parameters of a given estimator as chromosomes (binary or floating-point). This is tracked by populating a variety of possible solutions. Each chromosome is assessed by a fitness function. The better parent solutions are reproduced and the next generation of solutions (children) are created by applying the genetic operators (crossover and mutation). The children solutions are evaluated and the whole cycle repeats until the best solution is achieved.

STATISTICS OF DATASETS

To construct the planned model, 153 data points, taken from several papers [29-32] were employed. These PVT data include the proportion of solution-gas-ratio over gas specific gravity (Rs/GG),

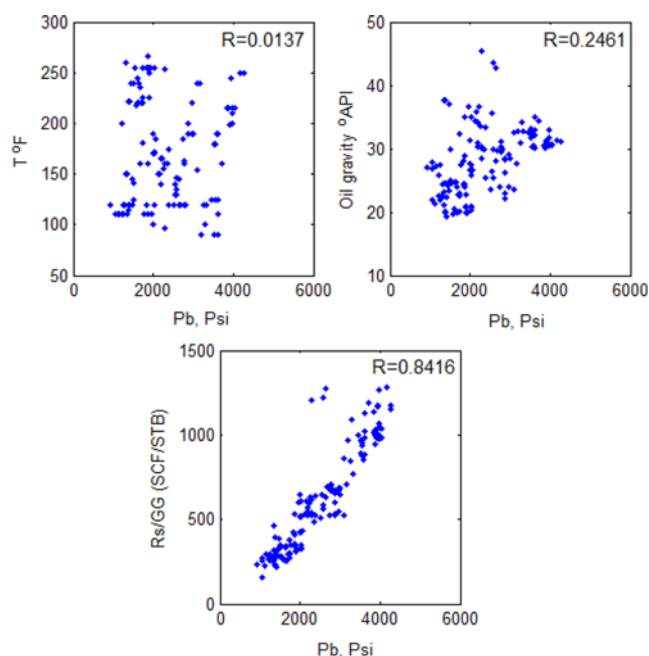


Fig. 1. Cross-plots showing the relationship between bubble point pressure and PVT data. As this figure shows there is a strong direct relationship between bubble point pressure and Rs/GG and oil gravity, while the direct relationship between temperature and bubble point pressure is weak.

temperature (T), stock-tank oil gravity (API), and bubble point pressure (Pb). Statistics of these parameters are mentioned in Table 1. Although some papers have utilized Rs and GG as separate parameters for their modeling, many others proposed Rs/GG is a more appropriate choice for feeding models meant to predict bubble point pressure [2-10]. All these PVT data are in field unit system due to its prevalence in the petroleum industry. However, conversions from field units to SI units are stated in A.1. There is a logical correlation between Pb and other mentioned parameters. The correlation coefficient between Pb and other PVT parameters could qualitatively illustrate the supremacy of dependency between these properties (Fig. 1). According to Fig. 1, Rs/GG and API of crude oil show a strong direct relationship with bubble point pressure, while the direct relationship between temperature and bubble point pressure is weak.

CASE STUDY

1. Neural Network Model and Genetic Optimized Neural Network

A three-layered neural network with back-propagation algorithm was used for construction of an intelligent model which is meant

to estimate bubble point pressure from PVT data. A neural network with back-propagation algorithm can effectively estimate bubble point pressure if TANSIG and PURELIN transfer functions are used for hidden layer and output layer, respectively [11,12]. Consequently, in this study TANSIG and PURELIN transfer functions were used for hidden layer and output layer, correspondingly. The network was trained using Bayesian regularization training function (trainBR). To achieve the optimal number of neurons in the hidden layer of neural network models, several neural network models with different number of neurons in their hidden layer were constructed and the performance of the models was evaluated at each stage. Investigations showed that the optimal model is achieved if the number of neurons in the hidden layer is specified as five neurons. After training of the network, associated weights and biases reveal the dependency of input data and bubble point pressure. By applying the test data in the trained neural network, it is possible to estimate bubble point pressure for unseen data. A justification based on Asoo-deh and Bagheripour [19] proves the high probability of the neural network to be trapped in local minima. In general, modification of weights and biases follows the equation,

$$w(t+1) = w(t) - \alpha_t g_t \quad (1)$$

where, $w(t+1)$ and $w(t)$ refer to weights and biases of $(t+1)^{th}$ and t^{th} iterations, respectively. α_t and g_t are learning rate and gradient in t^{th} iteration, correspondingly. Judgment based on Eq. (1) confirms that in local minima (where $g_t=0$) no modification occurs for weights and biases. In other words, the neural network sticks in local minima. Although proper/adequate adjustment of the network parameters can prevent this, it's a big challenge how to determine appropriate adjustment for networks.

At the next stage of this study, a genetic algorithm was employed for extracting the weights and biases of the neural network. In fact, the genetic algorithm was used instead of back-propagation algorithm and Bayesian regulation training function for finding the relationship between input/output data. For this purpose, the same structure as above three-layered back-propagation neural network was introduced to the genetic algorithm. This network is composed of five hidden neurons and one output neuron, which produces twenty weights along with six biases that should be determined by genetic algorithm. A mean square error (MSE) function for training data was used as the fitness function of the genetic algorithm, while an MSE function for test data was used as nonlinear constraint function to prevent over-fitting of the neural network.

Having run the genetic algorithm with pattern search hybrid function (Fig. 2), optimal connection weights and biases were obtained.

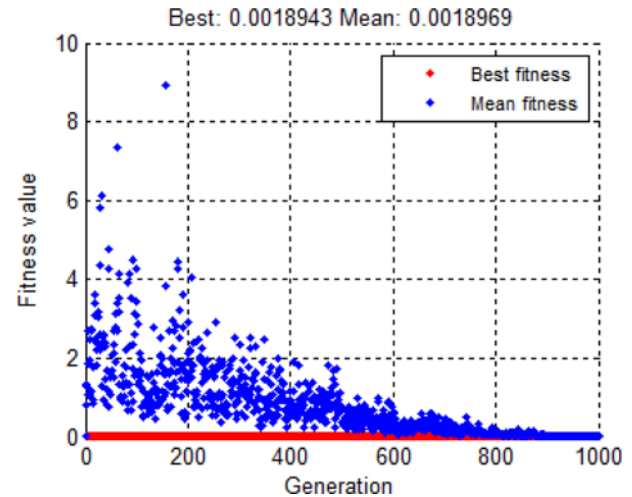


Fig. 2. Cross-plot showing the mean and best value of fitness function for optimization of neural network during 1000 generations. Final best fitness value is equal to 0.00189 which refers to mean square error of prediction by GONN model.

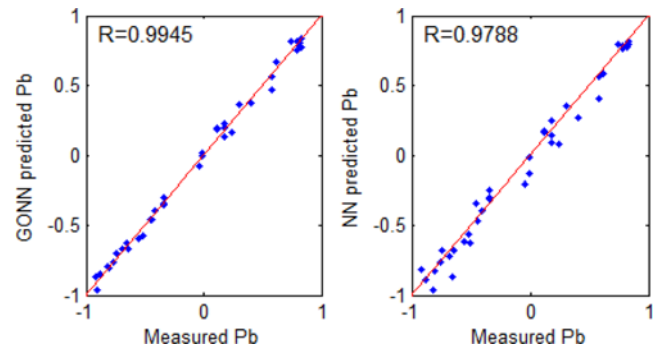


Fig. 3. Cross-plots showing the correlation coefficient between measured and predicted values. This figure illustrates GONN provides more reliable results compared with non-optimized traditional neural network (NN).

Regulations, done before run of genetic algorithm, are provided in Table 2. By dint of applying the weights to data points, it is possible to estimate final Pb. A comparison between neural network with back-propagation algorithm and genetic optimized neural network (GONN) illustrates that GONN performed more satisfyingly compared with back-propagation neural network. Fig. 3 compares these two methods. As is seen, the correlation coefficient of GONN method for prediction of bubble point is equal to 0.9945, which is much

Table 2. Regulations done before run of genetic algorithm for optimizing neural network

Parameter/Setting	Type/Value	Parameter/Setting	Type/Value
Population type	Double vector	Mutation function	Gaussian
Population size	50 Chromosomes	Crossover function	Scattered
Initial range	[0 1]	Hybrid function	Pattern search
Scaling function	Rank	Generations	1000
Selection function	Roulette	Stall generations	500
Elite preservation	2	Fitness tolerance	1.0 E -6
Crossover fraction	0.6	Time limit	Infinity

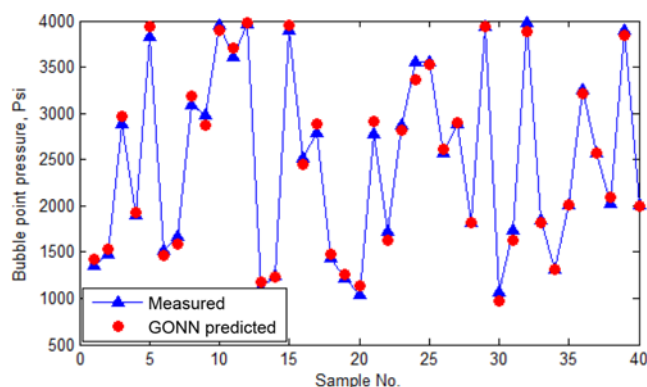


Fig. 4. A comparison between GONN predicted bubble point pressure and measured values. This figure proves there is a good agreement between predicted values and reality.

more than the correlation coefficient of the back-propagation neural network (0.9788). Mean square error (MSE) is another evidence for comparing performance of GONN and back-propagation NN models. Lower value of MSE for GONN (i.e., 72.73) relative to MSE of back-propagation NN (i.e., 155.89) proves superiority of GONN. A comparison between the GONN predicted bubble point pressure and measured value is shown in Fig. 4. This figure demonstrates that the predicted values are in good agreement with reality, i.e., the GONN method is an efficient way for producing high accuracy results.

2. Adaptive Neuro-fuzzy Inference System & Adaptive Genetic-fuzzy Inference System

To construct an adaptive neuro-fuzzy inference system, a Takagi-Sugeno fuzzy inference system (FIS) was developed and then a hybrid optimization method, which combines least squares estimations with back-propagation, was used to adjust the membership functions' parameters. Previous studies [33] showed that Takagi-Sugeno [26] is more efficient than other types of FIS such as Mamdani [34,35] and Laesen [36] FISs. Subtractive clustering is an effective approach to estimate the number of fuzzy clusters and centers in Takagi-Sugeno fuzzy inference system [37]. Clustering radius, varying between the range of [0 1], is a critical design parameter which has an important role in construction of a fuzzy inference system [15]. Specifying a smaller cluster radius will usually yield more and smaller clusters in the data (resulting in more rules). A large cluster radius yields a few large clusters in data [38]. Details of subtractive clustering can be found in Chiu [38], Chen and Wang [39], and Jarrah and Halawani [37].

To extract the optimal number of clusters, a set of clustering radii, ranging from 0 to 1, was introduced to ANFIS. At the same time, the performance of the model for test data at each stage was evaluated. The model with the lowest error was chosen as optimal FIS (Table 3). Accordingly, specification of 1 for clustering radius yielded the optimal ANFIS, which is meant to predict bubble point pressure. Each input was bunched into two Gaussian clusters (Fig. 5). Output membership functions were linear equations expressed as:

$$OMF_i = \beta_{1i}T + \beta_{2i}API + \beta_{3i}\frac{Rs}{GG} + \beta_{4i} \quad (2)$$

Coefficients corresponding to the inputs (β_1 , β_2 , and β_3) and con-

Table 3. Variation of correlation coefficient (R), mean square error (MSE), and number of rules for ANFIS model versus clustering radius. By specification of clustering radius of 1, the optimal model is achieved. As this table shows specification of same number of rules (5 rules or 2 rules) provides different values of MSE and R. This shows that ANFIS model has been stuck in local minima in these regions

Clustering radius	No. of rules	R	MSE
0.1	62	0.9562	0.0081
0.2	20	0.80281	0.0587
0.3	11	0.9457	0.0104
0.4	7	0.99	0.0016
0.5	5	0.98952	0.0020
0.6	5	0.9897	0.0018
0.7	5	0.9893	0.0020
0.8	3	0.9915	0.0015
0.9	2	0.9923	0.0014
1	2	0.9936	0.0011

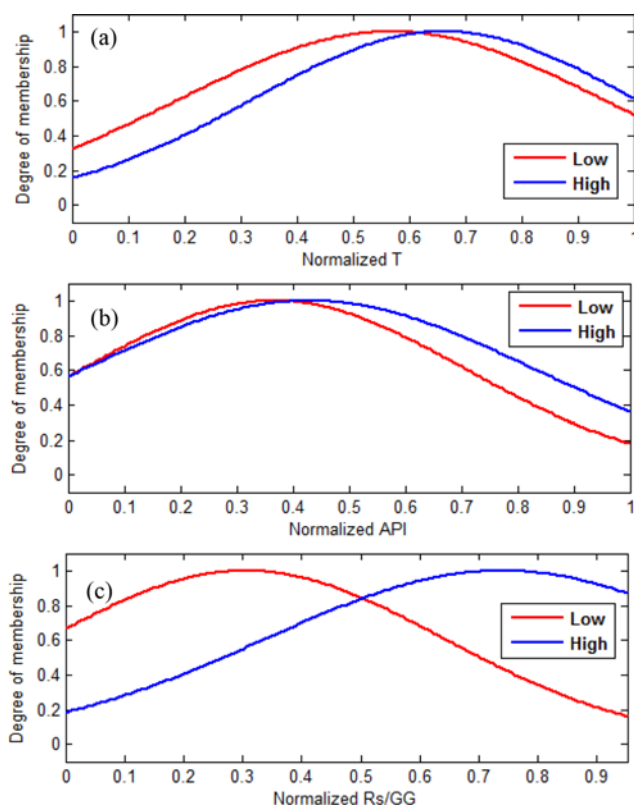


Fig. 5. ANFIS generated Gaussian membership functions for input data. ANFIS model provides two rules for handling quantitative formulation between PVT data and bubble point pressure.

stant coefficient (β_i) for output MFs are listed in Table 4. The generated 'if-then' rules are as below:

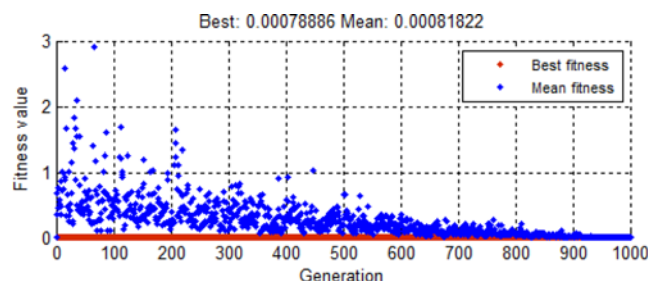
Rule 1: If (T is low) and (API is Low) and (Rs/GG is low) Then (Pb is low)

Rule 2: If (T is high) and (API is high) and (Rs/GG is high) Then (Pb is high).

As seen in Table 3, the adaptive neuro-fuzzy inference system

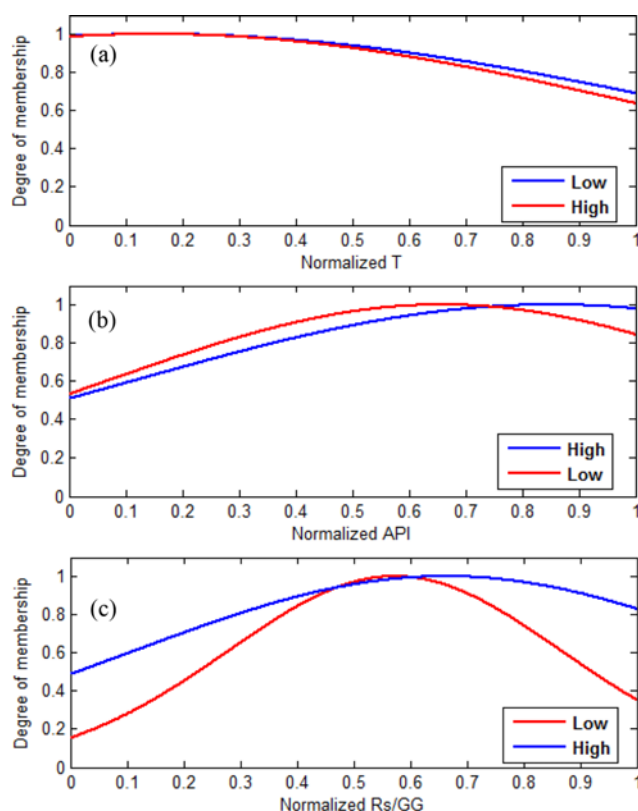
Table 4. Parameters associated with output linear membership functions (MFs) of ANFIS model

$\text{Out} = \beta_1 (T) + \beta_2 (\text{API}) + \beta_3 (\text{Rs/GG}) + \beta_4$				
Adaptive Neuro-Fuzzy Inference System (ANFIS)				
MF	β_1	β_2	β_3	β_4
Low cluster	0.461	1.765	-1.248	-0.5789
High cluster	0.07557	-0.3981	-0.8638	1.693

**Fig. 6. Cross-plot showing the mean and best value of fitness function for optimization of fuzzy logic during 1000 generations. Final best fitness value is equal to 0.000789 which refers to mean square error of prediction by AGFIS model.**

(ANFIS) yielded different results for the same number of rules. It is expected that for the same number of rules the same values of membership functions' parameters, and consequently the same R and MSE, are achieved. These results show the ANFIS model has wedged in local minima. To avoid this feature of ANFIS, the constructed Sugeno type fuzzy inference system is optimized by virtue of genetic algorithm in the next step. The same above TS-FIS is employed to be optimized by means of genetic algorithm. A TS-FIS with two rules and three inputs provides 12 parameters of input membership functions along with eight parameters of output membership functions for optimization. These parameters are typically mined in companion of back-propagation algorithms and subtractive clustering. However, the back-propagation algorithm is usually encountered with local minima trapping. To erase this flaw, a genetic algorithm is used and optimal values of mentioned parameters are derived. For this purpose, an MSE function for training data was used as fitness function, while MSE function for test data was employed as nonlinear constraint function to prevent over-fitting on training sets.

After running the genetic algorithm (Fig. 6), optimal parameters of input and output membership functions were obtained. The genetic algorithm was regulated according to the information of Table 5 be-

**Fig. 7. AGFIS generated Gaussian membership functions for input data. This figure shows that both mean and spread of membership functions are optimized by means of genetic algorithm.**

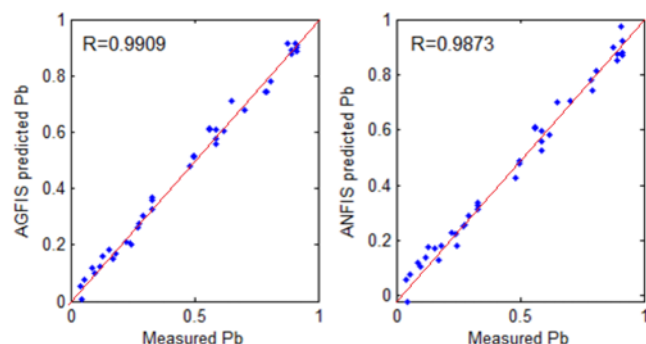
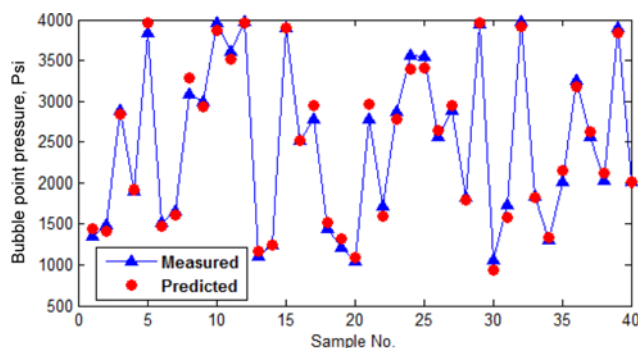
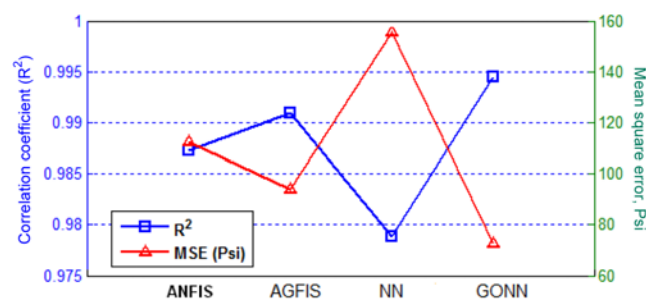
fore having run. By applying these parameters to data points, it is possible to estimate final Pb. Fig. 7 shows the input Gaussian membership functions. Parameters associated with output membership functions are listed in Table 6. A comparison between adaptive neuro-fuzzy inference system (ANFIS) and adaptive genetic-fuzzy inference system (AGFIS) shows that AGFIS performed more satisfactorily compared with ANFIS. Fig. 8 compares these two methods. As seen, the correlation coefficient of AGFIS method for prediction of bubble point is equal to 0.9903 which is much more than the correlation coefficient of ANFIS (0.9873). Mean square error (MSE) of AGFIS model is equal to 94.33, which is lower than MSE of ANFIS (112.76). This criterion provides another evidence for superiority of AGFIS model compared with ANFIS. A comparison between AGFIS predicted bubble point pressure and measured value

Table 5. Regulations done before run of genetic algorithm for optimizing fuzzy model

Parameter/Setting	Type/Value	Parameter/Setting	Type/Value
Population type	Double vector	Mutation function	Gaussian
Population size	20 Chromosomes	Crossover function	Scattered
Initial range	[0 1]	Hybrid function	Pattern search
Scaling function	Rank	Generations	1000
Selection function	Roulette	Stall generations	500
Elite preservation	4	Fitness tolerance	1.0 E -6
Crossover fraction	0.8	Time limit	Infinity

Table 6. Parameters associated with output linear membership functions (MFs) of AGFIS model

Out= β_1 (T)+ β_2 (API)+ β_3 (Rs/GG)+ β_4				
Adaptive genetic-fuzzy inference system (AGFIS)				
MF	β_1	β_2	β_3	β_4
Low cluster	0.0833020482	0.350087307	-0.342584096	-0.02355611357
High cluster	0.214851889	1.37745156	0.10660672	0.540845011

**Fig. 8. Cross-plots showing the correlation coefficient between measured and predicted values. This figure is an evidence to prove superiority of AGFIS model to ANFIS model.****Fig. 9. A comparison between AGFIS predicted bubble point pressure and measured values. This figure shows there is a satisfying match between predicted values and reality.****Fig. 10. Graph showing a comparison between different methods versus MSE and correlation coefficient. To make judgment based on this figure, genetic optimized methods performed more efficiently compared with non-optimized methods.**

is shown in Fig. 9. This figure shows that the predicted values are in good agreement with reality. Fig. 10 provides an opportunity to compare constructed models in this study using correlation coefficient

and mean square error. This figure proves that the genetic algorithm is capable of enhancing the accuracy of neural networks and fuzzy model compared with traditional methods such as back-propagation algorithm and subtractive clustering.

CONCLUSIONS

Bubble point pressure is a critical property of oil samples which plays an important role in reservoir evaluation and production calculations. In this study, two improved strategies, including genetic optimized neural network (GONN) and adaptive genetic-fuzzy inference system (AGFIS), were introduced for the estimation of bubble point pressure. Back-propagation learning algorithm, which is embedded in adaptive neuro-fuzzy inference systems and neural network itself, is highly at risk of sticking in local minima. Since the genetic algorithm provides stochastic search ability for finding the global minimum of a fitness function, by using the genetic algorithm for training of neural networks and optimization of fuzzy logic model the risk of being stuck in local minima was eliminated. To make a judgment based on correlation coefficient and MSE, it was shown that GONN performed more effectively compared with AGFIS. In addition, the optimized methods performed more satisfactorily compared with non-optimized methods. It indicated that the genetic algorithm is a potent tool for curve fitting and optimization purpose. It is possible to utilize the robustness and search capability of genetic algorithm to optimize any formulation between input/output data space. Eventually, implementation of the proposed strategies provides an accurate, quick and cost-effective way of estimating bubble point pressure from PVT data.

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APPENDIX 1. CONVERSIONS FORM BRITISH FIELD UNITS TO SI UNITS

Conversions	Quantity
$^{\circ}\text{C}=1.8(^{\circ}\text{F})+32$	Temperature
$\lambda=141.5/(^{\circ}\text{API}+131.5)$	Density
$\text{Pascal}=6.895 \times 10^3 \times \text{Psi}$	Pressure
$(\text{v/v})=0.1782 \times (\text{Scf/STB})$	Solution gas-oil ratio over specific gas gravity