

# Mathematical modeling and modification of an activated sludge benchmark process evaluated by multiple performance criteria

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**Abstract**—Optimal modification of an activated sludge process (ASP) evaluated by multiple performance criteria was studied. A benchmark process in BSM1 was taken as a target process. Four indexes of percentage of effluent violation (PEV), energy consumption (OCI), total volume of tanks (TV) and total suspended solid in tank5 (TSSa5), were criteria and eleven process parameters were decision variables, making up the multiple criteria optimization model, which was solved by non-dominated sorting genetic algorithm II (NSGA-II) in MATLAB. Pareto solutions were obtained; one solution (opt1) was selected based on the authors' decision for a further analysis. Results show that the process with opt1 strategy exhibits much better performance of PEV and OCI than with the default, being improved by 74.17% and 9.97% specifically under dry influent and without control. These results indicated that the multiple criterion optimization method is very useful for modification of an ASP.

**Keywords:** Modification of Benchmark Process, Multiple Performance Criteria, Effluent Quality, Energy Consumption, Volume of Tanks

## INTRODUCTION

Wastewater treatment plants (WWTP) based on the activated sludge process (ASP) are complex systems, subject to large disturbances in the influent flow rate and concentration of the pollutant; nevertheless, various physical and biological reactions are taking place in different tanks of WWTP [1-3]. These plants have to be designed and operated continuously to meet the effluent standards set by the local government.

Many control actions and control strategies have been proposed in the scientific literature for wastewater treatment plants, which may have great performance in the authors' studies alone. However, evaluation and comparison of these various strategies is very difficult, owing to the variability of influent, different plant layouts, the complexity phenomena inherent to the activated sludge process and, more importantly, the lack of standard evaluation criteria. To overcome these drawbacks, the framework of COST (European Cooperation in the field of Scientific and Technical Research) Actions 682 and 684 have proposed an unbiased benchmark simulation environment BSM1 [4], defining a plant layout (with nitrification and denitrification), a simulation model, influent loads (three weather conditions), two PI controllers controlling the dissolved oxygen (DO) and  $\text{NO}_3^-$ -N, more importantly, the test procedures and the evaluation criteria. The benchmark has successfully demonstrated its usefulness on a number of commercially available simulation software tools; the benchmark manual summarizes the common simulation platforms like BioWin, EFRO, FORTRAN, GPS-X, MATLAB/

SIMULINK, SIMBA, STOAT, WEST [5].

For the original benchmark process, many control methodologies have been tested and obtained good performance. Holenda et al. [6] applied model predictive control strategy (MPC) on the dissolved oxygen control problem, while Shen et al. [7] compared this method with the linear quadratic dynamic matrix control strategy and found that the former strategy could get better effluent quality. Besides these, Yong et al. [8] and Zhang [9] implemented the PI controllers with feedforward and cascade on benchmark to reduce the ammonia concentration in the effluent. And more, Coelho Belchior et al. [10] used the stable adaptive fuzzy control. Rojas et al. [11] used the multivariate virtual reference feedback tuning method for designing the control strategies for WWTPs.

Benchmark can compare different control strategies fairly, but it has its own shortcomings. The most important one is that its plant layout is a single-sludge continuous-flow process with the nitrification and denitrification. So, many developers are concentrating their interest on the extension scope of benchmark to include more processes. Jeppsson et al. [12] and Nopens et al. [13] extended the process-scale BSM1 to the plant-scale BSM2 through adding a primary clarifier, thickener, anaerobic digester (ADM1 description), etc. Aiming at the feature of perfect mixing in tanks of benchmark, Pons and Potier [14] considered the hydraulics in the benchmark and investigated how the performance assessment of several control strategies was affected when compared to the original benchmark; however, the result showed that the impact of hydraulics on the results was not significant and could be neglected. Gernaey and Jørgensen [15] added two anaerobic tanks in front of anoxic tanks and improved the influent quality to include P components, and then demonstrated the capabilities of a combined N and P removal simulation benchmark for evaluation and comparing control strategies. To promote

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more realistic simulations, Copp and Spanjers [16] considered a toxic influent and presented an evaluation of a toxicity mitigation control strategies based on the respiration rate measurement to the benchmark, while Rosen et al. [17] extended the original benchmark to a long-term benchmark simulation model, which took the seasonal changes into account, named BSM1\_LT.

Although many control strategies and expansions have been applied in the benchmark process, there are hardly any literatures analyzing the process parameters in benchmark, such as the hydraulic retention time (HRT) [18] for aerobic or anoxic tanks. Moreover, evaluation of a control strategy in ASP is especially difficult when more than one criterion must be taken into account, e.g., effluent quality, economic, technical and stability. Obviously, a fairly simple and straightforward objective of ASP is to achieve sufficient effluent with low concentrations of biodegradable matter and nutrients, with minimal sludge production at minimum operation cost [19]. So optimization of ASP is a multiple criterion problem relating to more than one criterion which may be conflicted; it's very difficult to calculate as well as for the decision makers (DM) to choose a satisfying solution. Therefore, most of the literatures converted the multiple criteria to single criterion by means of the weighting coefficient, such as transform effluent quality to a function of cost [20, 21]. One significant disadvantage is that the weights of different criteria are determined so the results cannot reflect the tradeoffs between them. In the last few years, very few published works [22-28] have considered multiple criteria in ASP directly. Among them, Flores et al. [22] presented a systematic procedure that assisted the designer of activated sludge plants to make critical decisions during the decision-making process, and demonstrated the new procedure with a case study in BSM1. Benedetti et al. [28] conducted Monte Carlo simulations and multi-criteria evaluation in analysis of BSM2, and found that the volume of the primary clarifier and the anoxic fraction of the reactor volume had an important impact on process performance.

The solution for the multiple criterion optimization problem (MCOP) is the Pareto optimal solution, also known as the compromise solution or non-dominated solution, where none of the criteria value can be improved without impairing at least one of the others. So there are many incomparable solutions for MOOP, which is totally different from the single criterion optimization problem having a unique optimal solution [29]. Thus the DM can examine and evaluate the tradeoffs between solutions and make the final decision based on his/her preference. Methods solving the MCOP are based on the genetic algorithm mostly [30], which as used in the paper is the non-dominated sorting genetic algorithm II (NSGA-II) [31].

Because of the fairness and comparability, we used the multiple criterion optimization method to analyze the original benchmark process to obtain a modified benchmark, and to illustrate the usefulness of multiple criterion optimization method in optimizing ASP. Four indexes of percentage of effluent violation (PEV), overall cost index (OCI), total volume (TV) and total suspended solid (TSS<sub>a5</sub>) were the criteria, the volumes (for each of five tanks), mixed liquor return rate, sludge return rate, excess sludge wasting rate, oxygen transfer coefficients (in each of three tanks) were the decision variables to establish the multiple criterion optimization model for benchmark. The first two criteria were the principal, while total volume was mainly used for optimizing the distribution of volume between anoxic with aerobic tanks and the last one could limit the sludge concentration in tanks. Solutions obtained were assessed under three influents and without/with control.

## MATERIALS AND METHODS

### 1. BSM1 Benchmark Process

The target used in this study was the BSM1 benchmark process, the layout of which is very simple (Fig. 1). Benchmark process consists of five reacting tanks (perfect mixing) and one clarifier. The volumes are 1,000 m<sup>3</sup>, 1,000 m<sup>3</sup>, 1,333 m<sup>3</sup>, 1,333 m<sup>3</sup>, 1,333 m<sup>3</sup> and 6,000 m<sup>3</sup>. Tank1 and tank2 are anoxic, where Nitrate Nitrogen (NO<sub>3</sub><sup>-</sup>-N) is converted to Nitrogen gas (N<sub>2</sub>), which is the denitrification; while the last three tanks are aerobic, changing Ammonia Nitrogen (NH<sub>4</sub><sup>+</sup>-N) to NO<sub>3</sub><sup>-</sup>-N, processing nitrification. Mixed liquor is returned from tank5 to tank1 to guarantee a good denitrification function, while part of the sludge is returned from the bottom of clarifier to tank1 to maintain sludge concentration. In addition, two PI controllers are implemented in benchmark, through adjusting oxygen transfer coefficient ( $K_{La5}$ ) and mixed liquor return rate to control the DO and NO<sub>3</sub><sup>-</sup>-N, which setpoints are 2 mg·L<sup>-1</sup> and 1 mg·L<sup>-1</sup> separately. The DO sensor is assumed to be ideal with no delay or noise; however, the NO<sub>3</sub><sup>-</sup>-N sensor is assumed to have a time delay of 10 minutes, with white, normally distributed (standard deviation of 0.1 mg·L<sup>-1</sup>), zero-mean noise [5].

BSM1 benchmark offers three weather influents files (dry, rain and storm), each file contains 14 days influent data at 15-minute intervals. The reacting tanks are modeled by ASM1 [32] and the clarifier is simulated with the solid flux model using the double-exponential setting velocity function of Takács [33]. ASM1 model contains 13 state variables: soluble inert organic matter S<sub>s</sub>, readily biodegradable substrate S<sub>ss</sub>, particulate inert organic matter X<sub>p</sub>, slowly biodegradable substrate X<sub>ss</sub>, active heterotrophic biomass X<sub>BH</sub>, active

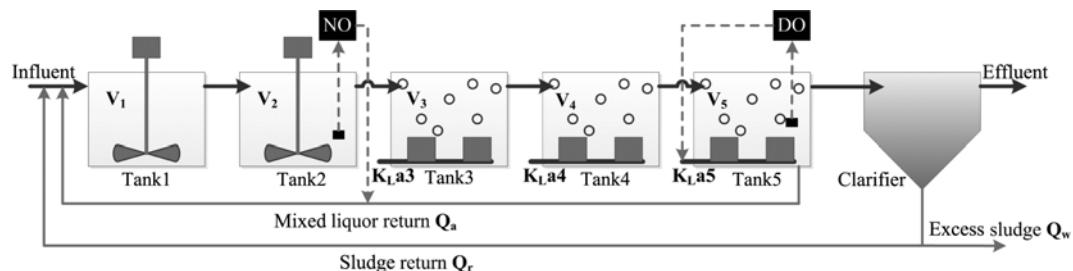


Fig. 1. Plant layout of benchmark process.

autotrophic biomass  $X_{BA}$ , particulate products arising from biomass decay  $X_p$ , oxygen  $S_o$ , nitrate and nitrite nitrogen  $S_{NO}$ , ammonia nitrogen  $S_{NH}$ , soluble biodegradable organic nitrogen  $S_{ND}$ , particulate biodegradable organic nitrogen  $X_{ND}$ , alkalinity  $S_{ALK}$ .

## 2. Performance Assessment

The criteria defined within the benchmark assess the overall performance of the applied control strategy, two performance levels include process performance and control loop performance. The first performance considers effluent quality, effluent violations and operational cost; while the second performance quantifies the effect of the control strategy on controller performance through different statistical criteria on the controlled and manipulated variables. The needed criteria in this study were introduced.

### 2-1. Process Performance

#### 2-1-1. Effluent Index: Percentage of Effluent Violation (PEV, %)

There are 14 days effluent data with the sampling time of 15 minutes according to 14 days influent data for each influent. The usable effluent data are the last seven days, so there are 672 ( $4*24*7$ ) data for every component. Benchmark sets the upper limits for some important effluent indexes, which are:  $COD \leq 100 \text{ mg} \cdot \text{L}^{-1}$ ,  $BOD_5 \leq 10 \text{ mg} \cdot \text{L}^{-1}$ ,  $NH_4^+ \text{-N} \leq 4 \text{ mg} \cdot \text{L}^{-1}$ ,  $TN \leq 18 \text{ mg} \cdot \text{L}^{-1}$  and  $TSS_e \leq 30 \text{ mg} \cdot \text{L}^{-1}$ . The first criterion was to calculate the sum percentage of violations for COD,  $BOD_5$ ,  $NH_4^+$ -N, TN and  $TSS_e$  among the total 3360 ( $5*672$ ) data.

#### 2-1-2. Energy Index: Overall Cost Index (OCI), the unit is $\text{kWh} \cdot \text{d}^{-1}$

The aeration energy, the pumping energy and the sludge treatment energy, made up the second criterion. The computing formulas were given as Eqs. (1)-(7) [5]:

**Aeration energy:** (AE, in units of  $\text{kWh} \cdot \text{d}^{-1}$ )

$$AE = \frac{24}{T} \int_{t=7 \text{ days}}^{t=14 \text{ days}} \sum_{i=1}^{i=5} [0.4032 \cdot K_L a_i(t)^2 + 7.8408 \cdot K_L a_i(t)] \cdot dt \quad (1)$$

where:  $K_L a_i(t)$ -the oxygen transfer coefficient in  $i^{\text{th}}$  aerobic tank at time  $t$  (in units of  $\text{hr}^{-1}$ )

**Pumping energy:** (PE, in units of  $\text{kWh} \cdot \text{d}^{-1}$ )

$$PE = \frac{0.04}{T} \int_{t=7 \text{ days}}^{t=14 \text{ days}} [Q_a(t) + Q_r(t) + Q_w(t)] \cdot dt \quad (2)$$

where:  $Q_a(t)$ -mixed liquor return rate at time  $t$  ( $\text{m}^3 \cdot \text{d}^{-1}$ )

$Q_r(t)$ -sludge return rate at time  $t$  ( $\text{m}^3 \cdot \text{d}^{-1}$ )

$Q_w(t)$ -excess sludge rate at time  $t$  ( $\text{m}^3 \cdot \text{d}^{-1}$ )

**Sludge Production to be disposed:** (SP, in units of  $\text{kgSS} \cdot \text{d}^{-1}$ )

Amount of solid in the system at time  $t$ : TSS( $t$ )

$$TSS(t) = TSS_a(t) + TSS_s(t) \quad (3)$$

where:  $TSS_a(t)$  is the amount of solid in the tanks:

$$TSS_a(t) = 0.75 \cdot \sum_{i=1}^{i=n} (X_{S,i} + X_{I,i} + X_{BH,i} + X_{BA,i} + X_{P,i}) \cdot V_i \quad (4)$$

with  $n=5$

$TSS_s(t)$  is the amount of solid in the clarifier:

$$TSS_s(t) = 0.75 \cdot \sum_{j=1}^{j=m} (X_{S,j} + X_{I,j} + X_{BH,j} + X_{BA,j} + X_{P,j}) \cdot z_j \cdot A \quad (5)$$

with  $m=10$

$$SP = \frac{1}{T} \left( TSS(14 \text{ days}) - TSS(7 \text{ days}) + 0.75 \cdot \int_{t=7 \text{ days}}^{t=14 \text{ days}} (X_{S,w} + X_{I,w} + X_{BH,w} + X_{BA,w} + X_{P,w}) \cdot Q_w(t) \cdot dt \right) \quad (6)$$

Finally an overall cost index (OCI) is calculated:

$$OCI = AE + PE + 5 \cdot SP \quad (7)$$

### 2-2. Control Loop Performance

The DO and  $NO_3^-$ -N controller are implemented as Eq. (8).

$$u(t) = K_p \cdot \left[ e_j + \frac{1}{T_I} \int_0^t e_j \cdot dt \right] \quad (8)$$

where:  $e_j = Z_{j,\text{setpoint}} - Z_{j,\text{observed}}$ ,  $K_p$  represents proportional gain and  $T_I$  represents integral time constant. For DO controller,  $u(t)$  is  $K_d a_5$ ,  $K_p$  is 500,  $T_I$  is 0.001; and for  $NO_3^-$ -N controller,  $u(t)$  is  $Q_a$ ,  $K_p$  is 15000,  $T_I$  is 0.05.

The second level of assessment quantified the effect of the control strategy on controller performance and could be divided into two sub-levels:

#### (1) Controlled variables performance

##### a. Integral of the absolute error (IAE):

$$IAE_j = \int_{t=7 \text{ days}}^{t=14 \text{ days}} |e_j| dt \quad (9)$$

##### b. Integral of the squared error (ISE):

$$ISE_j = \int_{t=7 \text{ days}}^{t=14 \text{ days}} e_j^2 dt \quad (10)$$

##### c. Maximum deviation from setpoint:

$$\max(e_{CV}) = \max|e_j| \quad (11)$$

##### d. Variance in the controlled variable error:

$$\text{Var}(e_j) = \overline{e_j^2} - (\overline{e_j})^2 \quad (12)$$

$$\text{where: } \overline{e_j} = \frac{\int_{t=7 \text{ days}}^{t=14 \text{ days}} e_j dt}{T}, \quad \overline{e_j^2} = \frac{\int_{t=7 \text{ days}}^{t=14 \text{ days}} e_j^2 dt}{T}$$

#### (2) Manipulated variables performance

##### a. Maximum deviation in the manipulated variable:

$$\max(e_{MV}) = u_{j,max} - u_{j,min} \quad (13)$$

##### b. Maximum deviation in the change in manipulated variable:

$$\max(\Delta u_j) = \max(|u_j(t+dt) - u_j(t)|) \quad (14)$$

##### c. Variance in the change in manipulated variable:

$$\text{Var}(\Delta u_j) = \overline{\Delta u_j^2} - (\overline{\Delta u_j})^2 \quad (15)$$

$$\text{where: } \overline{\Delta u_j} = \frac{\int_{t=7 \text{ days}}^{t=14 \text{ days}} \Delta u_j dt}{T}, \quad \overline{\Delta u_j^2} = \frac{\int_{t=7 \text{ days}}^{t=14 \text{ days}} \Delta u_j^2 dt}{T}$$

## 3. Formula of Multiple Criterion Optimization Model

### 3-1. Criteria

We selected four criteria: PEV, OCI, total volume (TV) and total

**Table 1. Lower, upper limits, the default and optimized values for the decision variables**

Decision variables	$V_1$ (m <sup>3</sup> )	$V_2$ (m <sup>3</sup> )	$V_3$ (m <sup>3</sup> )	$V_4$ (m <sup>3</sup> )	$V_5$ (m <sup>3</sup> )	$Q_a$ (m <sup>3</sup> ·d <sup>-1</sup> )	$Q_r$ (m <sup>3</sup> ·d <sup>-1</sup> )	$Q_w$ (m <sup>3</sup> ·d <sup>-1</sup> )	$K_{L3}$ (d <sup>-1</sup> )	$K_{L4}$ (d <sup>-1</sup> )	$K_{L5}$ (d <sup>-1</sup> )
Default value	1000	1000	1333	1333	1333	55338	18446	385	240	240	84
Lower limit	300	300	300	300	300	0	0	0	0	0	0
Upper limit	1500	1500	2000	2000	2000	70000	25000	700	300	300	300
opt1	1093	879	1314	1478	1370	47637	17982	230	227	238	87

suspended solid of tank5 (TSS<sub>a5</sub>). The first two criteria were the most important, while TV was mainly used for optimizing the volume distribution between anoxic and aerobic tanks and the last one could limit the sludge concentration in tanks, avoiding excessive loading for the clarifier.

### 3-2. Decision Variables

We had eleven decision variables ( $V_1, V_2, V_3, V_4, V_5, Q_a, Q_r, Q_w, K_{L3}, K_{L4}$  and  $K_{L5}$ , shown as bold in Fig. 1) for the multiple criteria optimization model, and the limits of these decision variables mainly made up the constraints (see Table 1).  $V_i$  (m<sup>3</sup>) represents the volume of the  $i^{\text{th}}$  tank and  $K_{L3}, K_{L4}, K_{L5}$  represent the oxygen transfer coefficients of tank3, tank4 and tank5, respectively. The lower limits of volumes were set to 300 m<sup>3</sup> instead of 0, which guaranteed that there were five tanks. Others were set 0, which might seem unreasonable as the corresponding behavior of the plant was unstable (especially for  $Q_w$ ), but the genetic algorithm was able to handle this without any problem. The upper limits of eleven decision variables were estimated according to default values. In fact, the goal was not to constrain too much the search space so that the algorithm would not miss any promising solution [23].

### 3-3. Multiple Criteria Optimization Model

$$\text{Minimize } f = (\text{PEV}, \text{OCI}, \text{TV}, \text{TSS}_a)^T$$

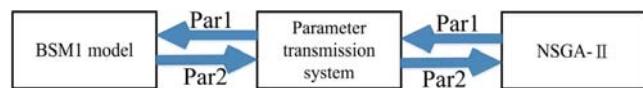
Subject to  $X \in S$

$$\text{PEV} < 14.88\%$$

where the  $X = (V_1, V_2, V_3, V_4, V_5, Q_a, Q_r, Q_w, K_{L3}, K_{L4}, K_{L5})^T$ , which were the eleven decision variables, and  $S$  was the feasible zone made up by constraints (lower and upper limits) of decision variables (see Table 1). The constraint PEV<14.88% guaranteed that the calculated PEV were all less than 14.88%, which was the PEV value of using the default strategy (all decision variables used the default values) without control. This constraint could avoid unavailable solutions for PEV and made the model more realistic.

### 4. Simulation Platform, Solving Method and Calculating Flow

The program of multiple criteria optimization model was done in M-files/SIMULINK of MATLAB and was solved by the non-dominated sorting genetic algorithm II (NSGA-II), which was provided by the optimization toolbox in MATLAB. The ordinary differential equations for describing activated sludge reactions and the clarifier were completed by C-program in S-function of SIMULINK. The M-file was used for building a link between SIMULINK and the optimization toolbox. Fig. 2 is the calculating flow chart: NSGA-II generates the value of eleven decision variables, which is transferred by the parameter transmission system (M-file) to BSM1 model (SIMULINK); then BSM1 model computes the results of criteria transferred to NSGA-II through parameter transmission system, through analysis, selecting the parents and then generate the child



**Fig. 2. Diagram of the data flow in multiple criteria optimization model (Par1 represent the eleven decision variables and Par2 are the results of criteria).**

**Table 2. Settings for NSGA-II parameters**

NSGA-II (parameters)	Parameter values (type)
Population size	165
Number of generation	100
Selection size	2 (tournament)
Crossover probability	0.8
Mutation probability	0.01
Migration probability	0.2
Migration interval	10

and this process is repeated until obtaining the Pareto solutions. Table 2 is the settings for NSGA-II parameters.

## RESULTS AND DISCUSSION

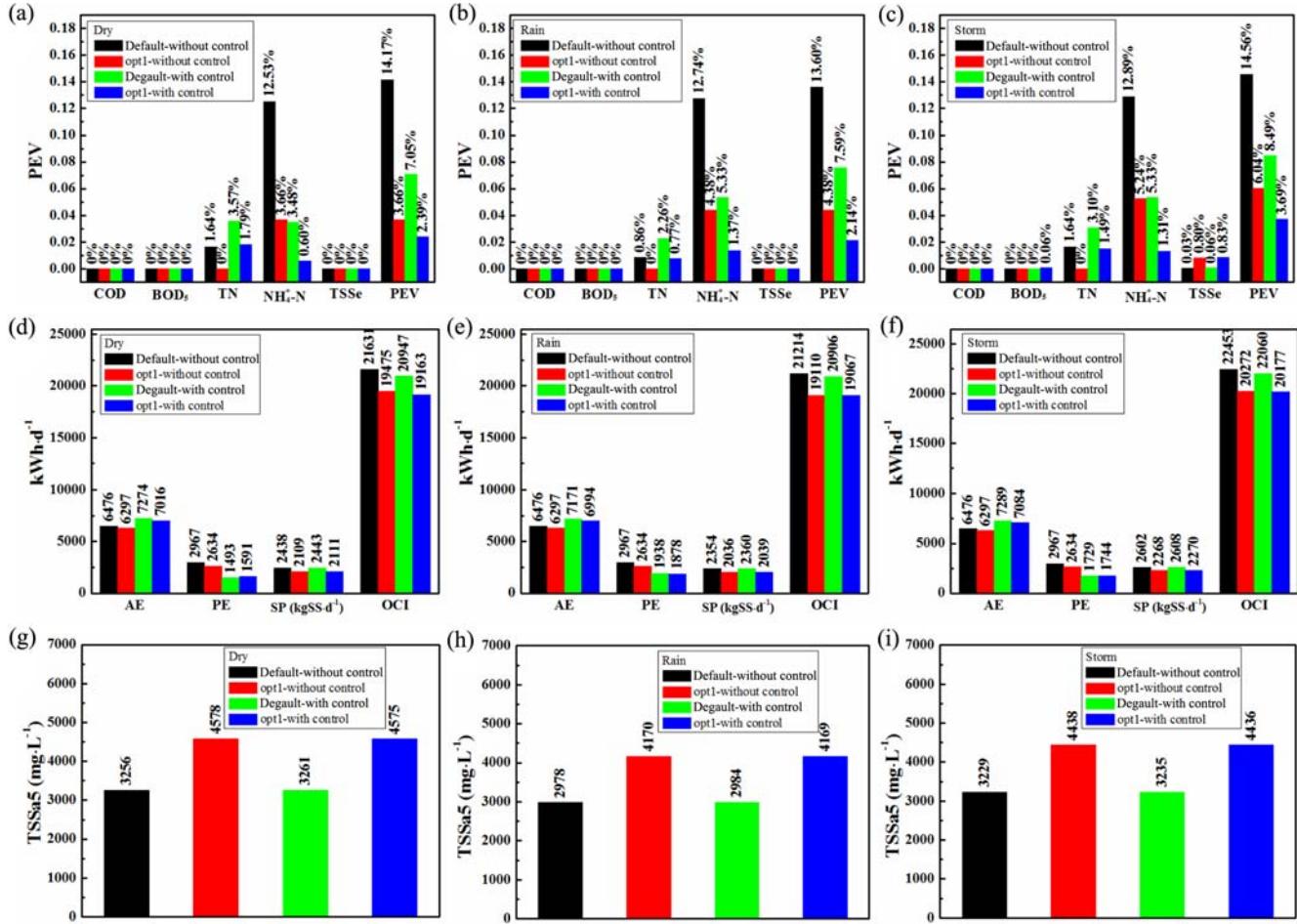
### 1. Solution Selected Procedure

Many (58) Pareto solutions (shown in Supplementary data file) were obtained under dry influent and without control in this study, so we selected one strategy (opt1) for a further analysis according to the following procedure: first all solutions were ascending sorted according to PEV, and then the final solution was selected which has as smaller PEV and OCI as possible, simultaneously, has the sufficient low TV and TSS<sub>a5</sub>. We considered that different readers might have different choices because the selecting procedure was very general and subjective.

### 2. Process Performance Assessment

Decision variable values are listed in Table 1 while criteria values of the default and selected strategy (opt1) are shown in Fig. 3 except TV, which are constant and can be calculated easily from Table 1, being 5,999 m<sup>3</sup> and 6,136 m<sup>3</sup> for the default and opt1, respectively.

Under dry influent and without control, opt1 strategy has better performance on PEV than default strategy, improved 74.17%. Effluent COD, BOD<sub>5</sub>, TN and TSS<sub>e</sub> all meet the standards, while PEV of default strategy on effluent TN is 1.64%. Effluent NH<sub>4</sub><sup>+</sup>-N PEV reduces about 70.79% comparing opt1 strategy 3.66% with default strategy 12.53%. Fig. 4(a), (b) is the dynamic variation of effluent NH<sub>4</sub><sup>+</sup>-N and TN without control. Although the effluent tendency of the two strategies is almost the same, effluent concentrations of NH<sub>4</sub><sup>+</sup>-



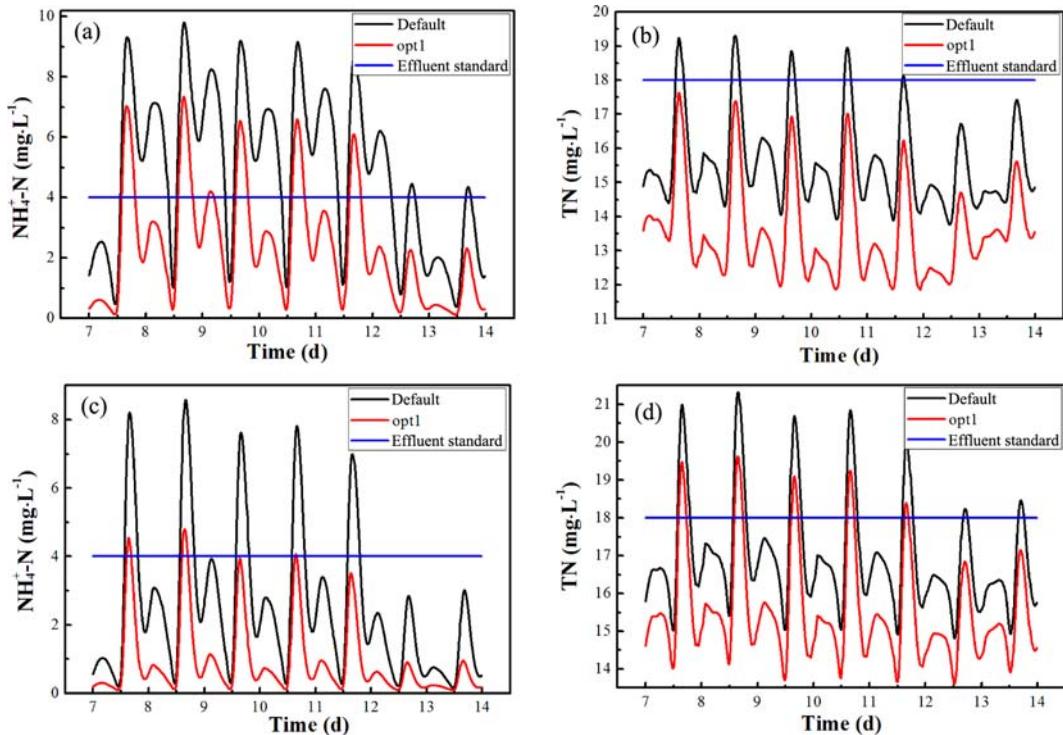
**Fig. 3. Comparison of process performance criteria PEV (a), (b), (c), OCI (d), (e), (f) and TSSa5 (g), (h), (i) under three influents and without/with control.**

N and TN in opt1 are all smaller than the default, especially for effluent TN. Average concentrations of  $\text{NH}_4^+$ -N and  $\text{NO}_3^-$ -N in each tank during the last seven days are shown in Fig. 5. From tank2 to tank5, concentration of  $\text{NH}_4^+$ -N reduces  $5.97 \text{ mg}\cdot\text{L}^{-1}$  for default strategy; however, concentration of  $\text{NH}_4^+$ -N reduces  $7.19 \text{ mg}\cdot\text{L}^{-1}$  for opt1 strategy, performance of nitrification is enhanced about 20.44%. Average concentration of  $\text{NO}_3^-$ -N is increased  $6.05 \text{ mg}\cdot\text{L}^{-1}$  and  $7.02 \text{ mg}\cdot\text{L}^{-1}$  from tank2 to tank5 separately. The results show that opt1 strategy obtained from the multiple criteria optimization has better N removal efficiency under dry influent.

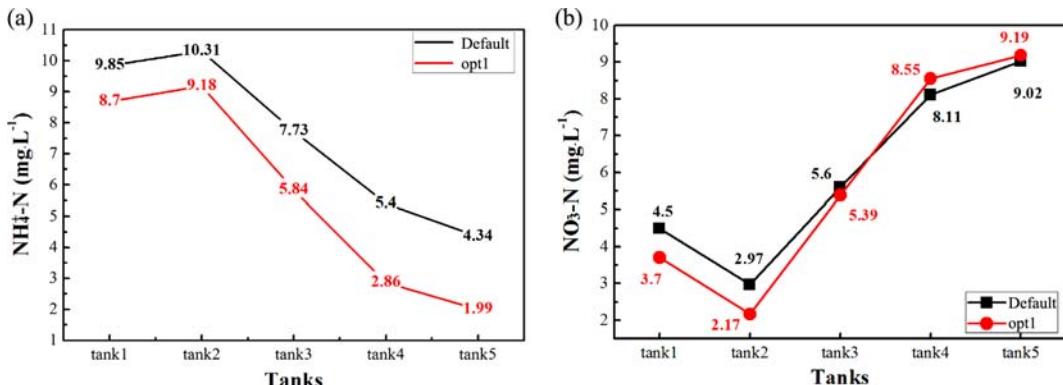
Process parameters of the default and opt1 strategy were compared, as shown in Table 1. For the volume of the aerobic tanks, the default is  $3,999 \text{ m}^3$ , while that of the opt1 increases 4.10%, equaling  $4,162 \text{ m}^3$ . Increasing volume and extension of the HRT of the aerobic tanks will enhance the performance of nitrification in BSM1 benchmark. For the volume of anoxic tanks, opt1 is  $1,972 \text{ m}^3$  compared with the default  $2,000 \text{ m}^3$ . So opt1 shortens the HRT of anoxic tanks about 1.36%, especially the HRT of tank2. However, the concentration of  $\text{NO}_3^-$ -N is reduced by the same  $1.53 \text{ mg}\cdot\text{L}^{-1}$  from tank1 to tank2 (see Fig. 5(b)) for both default and opt1 strategy, which shows that the denitrification hadn't been weakened as the result of shortening HRT in anoxic tanks. Another decision variable having big changes is the  $Q_w$ , which value reduces 40.21% compared with

the default strategy. Diminishment of  $Q_w$  will lower SP, extend the sludge age, increase the sludge concentration in tanks, and then decrease the sludge loading, which has the promoting effect on nitrification and denitrification. As a result, the optimized criterion TSSa5 reaches  $4,578 \text{ mg}\cdot\text{L}^{-1}$ , while the default is about  $3,256 \text{ mg}\cdot\text{L}^{-1}$  (Fig. 3(g)). This may be having a bad effect on the clarifier. What's more,  $Q_a$  equals  $47,637.58 \text{ m}^3\cdot\text{d}^{-1}$ ,  $Q_r$  is  $17,982.25 \text{ m}^3\cdot\text{d}^{-1}$  and the total oxygen transfer coefficient is  $553.80 \text{ d}^{-1}$ , reducing 13.92%, 2.51%, 1.81% respectively, so partial PE and AE can be saved. On the premise of guaranteeing the effluent quality, in fact getting much better, the criterion OCI reduces 9.97%. So the opt1 strategy exhibits better performance on effluent quality as well as energy consumption.

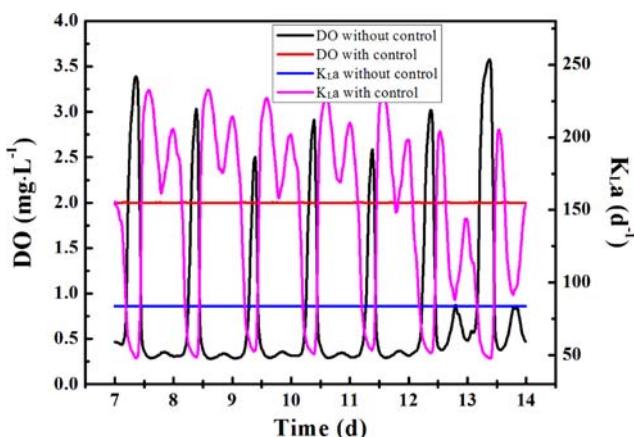
Criteria values of strategies after adding two PI controllers are also shown in Fig. 3. Only  $\text{NH}_4^+$ -N and TN exceed the effluent standards under dry influent. Fig. 4(c), (d) shows that the dynamic variation of effluent  $\text{NH}_4^+$ -N and TN with control in the last seven days. It is obvious that the effluent concentrations of  $\text{NH}_4^+$ -N and TN are lower in opt1 strategy than default strategy. Comparing the concentration variation of  $\text{NH}_4^+$ -N and TN without/with PI controllers, effluent concentration of  $\text{NH}_4^+$ -N for both opt1 and default strategy is lower when using the PI controllers. Taking opt1 for example, PEV of  $\text{NH}_4^+$ -N reduces to 0.60% from 3.66% after using the DO PI controller, which controls the DO of tank5 to a setpoint of 2.0



**Fig. 4. Dynamic variation of effluent  $\text{NH}_4^+ \text{-N}$  and TN under dry influent, without control (a), (b) and with control (c), (d).**



**Fig. 5. Average concentrations of  $\text{NH}_4^+ \text{-N}$  (a) and  $\text{NO}_3^- \text{-N}$  (b) in different tanks under dry influent.**



**Fig. 6. Variations of DO and  $K_{La}$  without/with PI for the default strategy under Dry influent.**

$\text{mg}\cdot\text{L}^{-1}$ . Fig. 6 shows the DO and  $K_{La}$  variations without/with DO controller for the default strategy. Without DO controller,  $K_{La}$  is maintained at the default value  $84 \text{ d}^{-1}$  and range of DO concentration is wide, from  $0.25 \text{ mg}\cdot\text{L}^{-1}$  to  $3.5 \text{ mg}\cdot\text{L}^{-1}$ , average value is  $0.82 \text{ mg}\cdot\text{L}^{-1}$ ; after using the DO controller, concentrations of DO maintain at  $2.0 \text{ mg}\cdot\text{L}^{-1}$  by adjusting  $K_{La}$  from  $47.79 \text{ d}^{-1}$  to  $232.83 \text{ d}^{-1}$ ,  $K_{La}$  will increase when DO is below  $2.0 \text{ mg}\cdot\text{L}^{-1}$  and vice versa. Nitrification will have better performance at higher DO concentration; that's why effluent concentration of  $\text{NH}_4^+ \text{-N}$  gets lower after using the DO controller. The effect of controller can also be used for explaining why the effluent TN concentration gets higher after implementing the  $\text{NO}_3^- \text{-N}$  controller in tank2, whose setpoint is  $1.0 \text{ mg}\cdot\text{L}^{-1}$ . Without control, PEV of effluent TN for opt1 is 0, while with control it becomes  $1.79\%$ .  $Q_a$  can affect the efficiency of denitrification, which is the positive correlation between them.  $Q_a$  is constant  $47,637 \text{ m}^3 \cdot \text{d}^{-1}$  without control; however, the average value

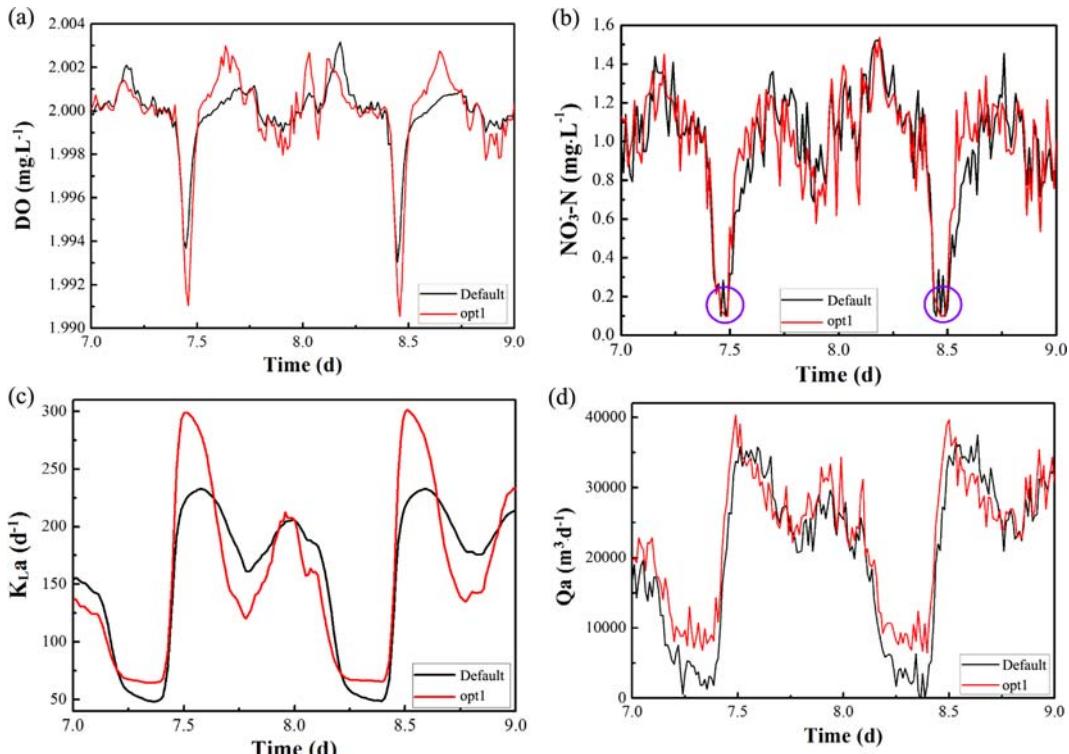
becomes  $21,565.57 \text{ m}^3 \cdot \text{d}^{-1}$  when using the  $\text{NO}_3^-$ -N controller. The variation of  $Q_a$  is shown in Fig. 7(d).

The comparison between opt1 and the default strategy was further analyzed under the rain and storm influents. Criteria values are shown in Fig. 3. It is obvious that opt1 strategy has much better per-

formance on PEV and OCI than the default most of the time under three influents. Effluents TN are all satisfied with the standards when without control. Even though exceeding standard occurs when using control, effluent concentrations are lower than the default; the reasons have been discussed above. Note that with the storm influent

**Table 3. Performance of DO and  $\text{NO}_3^-$ -N controllers under three influents**

	IAE	ISE	$\max(e_{cv})$	$\text{Var}(e_j)$	$\max(e_{mv})$	$\max(\Delta u_j)$	$\text{Var}(\Delta u_j)$
<b>DO controller</b>							
<b>Dry</b>							
Default	0.0063	0.000014	0.0069	0.000002	185	35	33
opt1	0.0084	0.000026	0.0095	0.000004	237	50	64
<b>Rain</b>							
Default	0.0054	0.000011	0.0063	0.000002	188	34	25
opt1	0.0076	0.000021	0.0090	0.000003	255	49	52
<b>Storm</b>							
Default	0.0062	0.000013	0.0069	0.000002	208	35	30
opt1	0.0086	0.000026	0.0094	0.000004	260	50	62
<b><math>\text{NO}_3^-</math>-N controller</b>							
<b>Dry</b>							
Default	1.5497	0.628568	0.9000	0.089785	38902	8350	2240945
opt1	1.4156	0.533760	0.9000	0.076248	34816	10752	2608465
<b>Rain</b>							
Default	1.8742	0.876971	1.0605	0.125188	77659	9135	2263984
opt1	1.6098	0.665302	0.9000	0.095043	58226	10752	2742162
<b>Storm</b>							
Default	1.8663	0.893192	1.3380	0.127597	72913	10648	2386076
opt1	1.5678	0.654083	0.9000	0.093402	61229	10278	2511302



**Fig. 7. Variation of controlled variables (a), (b) and manipulated variables (c), (d) in two PI controllers with dry influent.**

and without control, PEV of  $TSS_e$  for opt1 is 0.80%, which is larger than the default 0. The reasons are that the  $Q_w$  in opt1 strategy is very small, leading to a higher TSSa5 in tanks. Under storm influent, when the flow rate increases sharply, the HRT in benchmark process will be reduced, as well as for the clarifier, loading in where will be increased, and then there's not enough HRT for sedimentation of the sludge. Fortunately, the phenomenon of exceeding standard for TSSe is transitory.

### 3. Control Loop Performance Assessment

Performance assessment of DO controller and  $\text{NO}_3^-$ -N controller are shown in Table 3. Fig. 7 is the variation of controlled variables (a), (b) and manipulated variables (c), (d) in two PI controllers with dry influent. Concentrations of DO in opt1 strategy fluctuate wildly and the range is larger than the default, which can be quantified by the maximum deviation from setpoint  $\max(e_{cv})$  (Eq. (11)) and variance in the controlled variable error  $\text{Var}(e)$  (Eq. (12)) in Table 3. Variation of  $\text{NO}_3^-$ -N in opt1 strategy varied gently compared with the default; this phenomenon is especially evident when the concentration of  $\text{NO}_3^-$ -N reaches the valleys (circle marking). It can be found the reasons in Fig. 7(c): maximum value of  $K_{t,a5}$  is almost  $300 \text{ d}^{-1}$  in opt1 strategy, while the default value is  $230 \text{ d}^{-1}$  and the minimum value is  $63 \text{ d}^{-1}, 45 \text{ d}^{-1}$  separately. Maximum value of  $Q_o$  in opt1 is bigger; meanwhile the maximum deviation in the manipulated variable for opt1 is  $34,816 \text{ m}^3 \cdot \text{d}^{-1}$ , which is lower than the default  $38,902 \text{ m}^3 \cdot \text{d}^{-1}$ .

Performance of controllers under the other two influents has the same discipline, which can be seen from Table 3. In summary, comparing opt1 with the default strategy,  $\text{NO}_3^-$ -N controller is improved while DO controller gets worse. But the differences are very small because the type of the two controllers has not been changed generally.

The benefits of this research include:

- i) Different criteria can be considered independently and evaluated by multiple criterion optimization method instead of weighting them into a single criterion;
- ii) All process parameters in BSM1 are set as the decision variables, indicating that this approach can be used for designing the APS;
- iii) There are other criteria that are important for evaluating an ASP except effluent quality and energy consumption, for example TSSa5, which will limit the clarifier loading;
- iv) There are numerous solutions in the multiple criterion optimization problem, and the final strategy can be selected by DM based on one's decision;
- v) Performance of the optimized process is much better than with the default parameters.

### CONCLUSIONS

We optimized the BSM1 benchmark process using the multiple criterion optimization method. Four indexes of percentage of effluent violation (PEV), overall cost index (OCI), total volume (TV) and total suspended solid (TSSa5) were the criteria, the volumes (for each of five tanks), mixed liquor return rate, sludge return rate, excess sludge rate, oxygen transfer coefficients (in each of three tanks) were the decision variables to establish the multiple criteria optimization model for benchmark. Finally, a modified benchmark process was obtained. Reducing volume of anoxic 1.36%, increasing

volume of aerobic 4.10% and decreasing the excess sludge wasting rate 40.21%, the PEV and OCI are improved 74.16%, 9.97% separately under Dry influent and without control; meanwhile, nitrification is enhanced 20.44%, and the effluent TN all meet the effluent standard. Under all influents or with control loops, the performance of PEV and OCI by using the optimized strategy is also better than by using the default strategy. So the modified benchmark process exhibits better performance on effluent quality and energy consumption.

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## Supporting Information

# Mathematical modeling and modification of an activated sludge benchmark process evaluated by multiple performance criteria

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Table 1 shows all of the Pareto solutions for the multiple criteria optimization model, and solution 8 (in red) is selected as the opt1

in the manuscript. Table 2 shows values of eleven decision variables corresponding to solutions shown in Table 1.

**Table 1. Pareto solutions for the multiple criteria optimization model**

	PEV	TV (m <sup>3</sup> )	OCI (kWh·d <sup>-1</sup> )	TSSa5 (mg·L <sup>-1</sup> )
1	2.62%	6242	18747	5097
2	2.80%	6289	18834	5012
3	2.95%	6405	18866	4828
4	3.07%	6602	18693	4878
5	3.18%	6531	18906	4722
6	3.33%	6581	19758	4129
7	3.60%	6241	18487	5017
8	3.66%	6136	19475	4578
9	3.75%	6281	18744	4906
10	4.14%	6400	18261	5056
11	4.52%	6322	19044	4600
12	4.76%	6239	19276	4620
13	4.82%	6290	18331	5072
14	5.12%	6235	20344	3887
15	5.24%	6051	19492	4651
16	5.36%	6033	18722	5109
17	5.45%	6212	18184	5083
18	5.89%	5995	19805	4414
19	6.01%	6013	18557	5145
20	6.07%	6146	18286	5171
21	6.43%	6146	19216	4797
22	6.55%	6399	17848	4907
23	6.64%	6157	18203	5127
24	6.73%	6115	19205	4643
25	6.82%	6336	20236	3828
26	7.14%	6169	19980	4048
27	7.35%	6134	20268	3880
28	7.68%	6399	17732	4915
29	7.77%	6045	18951	4762

**Table 1. Continued**

	PEV	TV (m <sup>3</sup> )	OCI (kWh·d <sup>-1</sup> )	TSSa5 (mg·L <sup>-1</sup> )
30	8.10%	5895	20826	3877
31	8.21%	6042	19907	4220
32	8.60%	6007	20434	3865
33	8.81%	6102	20089	3936
34	9.08%	5924	20921	3731
35	9.23%	5924	20905	3731
36	9.38%	6235	20689	3554
37	9.67%	6176	20880	3488
38	9.79%	6079	20878	3623
39	10.09%	6036	19850	4162
40	10.39%	5939	20383	3916
41	10.65%	6170	21238	3380
42	10.92%	6467	20604	3312
43	11.16%	6215	21007	3324
44	11.28%	5984	20902	3602
45	11.76%	6186	21163	3340
46	12.11%	6260	21424	3240
47	12.50%	6365	21073	3166
48	12.65%	6352	20980	3161
49	12.71%	6150	20982	3308
50	13.30%	6200	20983	3236
51	13.33%	6272	21317	3101
52	13.60%	6344	21090	3073
53	13.69%	6222	21273	3149
54	13.81%	6072	21243	3286
55	14.38%	5801	21350	3414
56	14.55%	5700	21466	3471
57	14.82%	5740	21455	3407
58	14.88%	5737	21440	3408

**Table 2. Values of decision variables for the Pareto solutions**

Decision variables	V <sub>1</sub> (m <sup>3</sup> )	V <sub>2</sub> (m <sup>3</sup> )	V <sub>3</sub> (m <sup>3</sup> )	V <sub>4</sub> (m <sup>3</sup> )	V <sub>5</sub> (m <sup>3</sup> )	Q <sub>a</sub> (m <sup>3</sup> ·d <sup>-1</sup> )	Q <sub>r</sub> (m <sup>3</sup> ·d <sup>-1</sup> )	Q <sub>w</sub> (m <sup>3</sup> ·d <sup>-1</sup> )	K <sub>L</sub> a3 (d <sup>-1</sup> )	K <sub>L</sub> a4 (d <sup>-1</sup> )	K <sub>L</sub> a5 (d <sup>-1</sup> )
1	1193	675	1524	1452	1398	54468	17809	189	232	233	73
2	1166	712	1513	1442	1455	54462	17364	193	230	234	70
3	1236	765	1515	1462	1427	54410	15662	193	228	234	69
4	1232	882	1527	1455	1507	54378	16983	198	220	234	66
5	1224	850	1526	1459	1472	54378	16942	211	221	233	65
6	1222	925	1509	1453	1473	54463	16650	254	231	236	71
7	1155	737	1519	1428	1402	54589	16617	185	224	234	68
<b>8</b>	<b>1093</b>	<b>879</b>	<b>1314</b>	<b>1478</b>	<b>1370</b>	<b>47637</b>	<b>17982</b>	<b>230</b>	<b>227</b>	<b>238</b>	<b>87</b>
9	1197	686	1521	1408	1470	54452	16360	194	221	234	65
10	1182	822	1498	1449	1450	54502	17078	183	221	233	63
11	1152	767	1495	1439	1468	54387	15800	214	219	232	64
12	1161	739	1497	1370	1471	54464	17669	227	223	235	65
13	1108	800	1482	1428	1473	54511	17148	183	223	235	65
14	1154	745	1481	1391	1465	54554	17085	287	233	236	74
15	1124	720	1403	1405	1400	54678	17363	225	229	235	69
16	1073	811	1416	1374	1359	54794	17648	188	228	237	69
17	1172	645	1525	1444	1427	54367	17094	181	226	232	64
18	1116	692	1444	1365	1379	54677	17819	248	228	236	70
19	1053	783	1441	1365	1370	54737	17747	184	234	233	76
20	1072	795	1467	1423	1388	54817	18089	183	223	237	66
21	1125	838	1443	1354	1386	54680	17773	216	226	234	68
22	1251	636	1535	1469	1508	54228	15341	177	215	230	57
23	1076	791	1469	1427	1394	54661	17628	183	221	233	62
24	1087	716	1405	1428	1480	54604	16569	219	221	234	63
25	1208	838	1504	1367	1419	54580	18021	299	225	236	72
26	1149	707	1528	1369	1417	54253	17256	273	233	233	58
27	1168	724	1431	1399	1413	54616	16681	286	220	238	78
28	1251	636	1534	1470	1508	54228	15343	175	215	230	57
29	1071	638	1406	1432	1499	54384	15757	205	219	231	58
30	1019	779	1392	1343	1361	54737	17957	301	239	238	82
31	1092	829	1355	1399	1368	54575	17222	258	224	233	77
32	1102	711	1440	1363	1391	54672	17576	297	228	236	73
33	1093	770	1429	1417	1394	54609	17225	285	221	235	68
34	1093	686	1403	1352	1391	54695	17664	314	237	239	79
35	1093	686	1403	1352	1391	54695	17664	314	236	239	79
36	1164	806	1453	1416	1396	54476	17365	328	230	236	72
37	1087	794	1465	1381	1449	54510	16925	333	234	235	75
38	1129	778	1455	1356	1360	54716	17510	323	237	238	75
39	1055	799	1368	1423	1391	54564	17491	265	224	233	63
40	1106	710	1393	1356	1374	54697	17776	294	227	235	72
41	1149	784	1467	1381	1390	54663	17483	354	242	240	74
42	1224	812	1505	1447	1479	54409	16260	345	221	234	68
43	1137	775	1495	1407	1401	54627	17142	358	232	234	73
44	1043	790	1418	1363	1370	54611	17231	325	234	239	73
45	1168	781	1467	1359	1412	54659	17969	364	237	236	76
46	1109	870	1437	1398	1446	54584	18144	380	242	244	72
47	1191	801	1486	1417	1469	54493	16094	367	234	234	70
48	1160	809	1502	1416	1465	54427	16204	369	227	234	70
49	1169	735	1422	1403	1420	54692	16504	354	223	237	77
50	1138	755	1461	1384	1462	54522	17633	376	223	235	70
51	1135	806	1468	1394	1468	54492	16165	380	235	235	77

**Table 2. Continued**

Decision variables	V <sub>1</sub> (m <sup>3</sup> )	V <sub>2</sub> (m <sup>3</sup> )	V <sub>3</sub> (m <sup>3</sup> )	V <sub>4</sub> (m <sup>3</sup> )	V <sub>5</sub> (m <sup>3</sup> )	Q <sub>a</sub> (m <sup>3</sup> ·d <sup>-1</sup> )	Q <sub>r</sub> (m <sup>3</sup> ·d <sup>-1</sup> )	Q <sub>w</sub> (m <sup>3</sup> ·d <sup>-1</sup> )	K <sub>L</sub> a3 (d <sup>-1</sup> )	K <sub>L</sub> a4 (d <sup>-1</sup> )	K <sub>L</sub> a5 (d <sup>-1</sup> )
52	1222	772	1471	1400	1479	54481	15956	381	220	235	77
53	1168	791	1441	1404	1418	54555	16558	378	234	235	75
54	1134	709	1373	1380	1477	54437	16690	361	237	239	72
55	1063	686	1342	1351	1358	54798	17963	360	237	238	77
56	1006	681	1335	1336	1342	54808	18197	357	239	240	82
57	1012	689	1338	1351	1351	54802	18136	364	238	239	78
58	1011	688	1337	1351	1351	54801	18136	364	237	239	78