Korean J. Chem. Eng., 33(1), 25-32 (2016)
DOI: 10.1007/s11814-015-0133-8

pISSN: 0256-1115
eISSN: 1975-7220

Dynamic optimization of maintenance and improvement planning for water main system: Periodic replacement approach

Jong Woo Kim*, Go Bong Choi*, Jung Chul Suh, and Jong Min Lee*,†**

*School of Chemical and Biological Engineering, Institute of Chemical Processes, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul 151-744, Korea
**Samchully Corporation, 42 Gukjegeumyung-ro, 6-gil, Yeongdeungpo-gu, Seoul 150-885, Korea
(*Received 17 December 2014 • accepted 20 June 2015*)

Abstract−This paper proposes a Markov decision process (MDP) based approach to derive an optimal schedule of maintenance, rehabilitation and replacement of the water main system. The scheduling problem utilizes auxiliary information of a pipe such as the current state, cost, and deterioration model. The objective function and detailed algorithm of dynamic programming are modified to solve the periodic replacement problem. The optimal policy evaluated by the proposed algorithm is compared to several existing policies via Monte Carlo simulations. The proposed decision framework provides a systematic way to obtain an optimal policy.

Keywords: Markov Decision Processes, Monte Carlo Simulation, Periodic Replacement, Survival Analysis, Dynamic Programming

INTRODUCTION

Various policies for the appropriate combination of maintenance and improvement of infrastructure facilities are implemented. Maintenance activity is a set of actions that corrects minor defects and collects information on the state of infrastructure facilities by inspection. Improvement activity returns the facility's condition to its best state and is classified as replacement and rehabilitation in general. Replacement is to set up a new facility while rehabilitation takes place in situ.

Heuristics and myopic policies are the most widely implemented for the water main system management. The former is to improve the pipe at fixed time intervals, i.e., every 15-30 years, and the latter is to improve the pipe when a failure occurs in a reactive manner. However, in both policies, the trade-off of the expenses from improvement period and potential failure threat is not considered systematically. The objective of this study is to suggest a systematic decision-making framework for deriving an optimal maintenance and improvement policy that minimizes the total cost, where the overall state of pipe is reflected.

A mathematical model describing the deterioration of water pipe is necessary since the direct investigation of a water main system is time-consuming and costly. A statistical approach, survival analysis, has been suggested to explain the failure risk of the water pipe [1,2]. Weibull model is commonly used in survival analysis and has been shown to give the best estimation performance [3]. Survival analysis is applied for a number of real data sets [4].

†To whom correspondence should be addressed.
E-mail: jongmin@snu.ac.kr
‡This article is dedicated to Prof. Hwayong Kim on the occasion of his retirement from Seoul National University.

The deterioration model is then combined with the cost model to formulate the water main system scheduling problem. Loganathan et al. use threshold break rate, which concerns the failure history of a single pipe [5]. Kleiner et al. add the analysis of hydraulic pressure to extend a single pipe problem into a pipeline network [6,7]. Both studies use the exponential failure model. Additional criteria such as pipe resilience or production rate are also added to formulate multiobjective optimization problems [8-10]. Since both the network and multiobjective optimization problems are nonlinear and nonconvex, they are often solved by heuristic optimization algorithms. Among them are Monte Carlo simulation, genetic algorithm, and simulated annealing [11-13].

Another state-of-the-art scheduling method is Markov decision process (MDP). Markov process is one of the most widely used model to describe the pipe deterioration, because it can describe the transitions among multiple states systematically. It classifies a pipe into a set of finite states given some criteria such as the cumulative failure number [14] or the current performance of pipe [15]. Once the state classification criteria are determined, the state transition probability can be obtained in two ways. The first is a data-driven regression method to calculate the transition probabilities [15-21]. The result is the time-invariant transition probability, which is inappropriate for describing age-dependent transitions of the water main system. The second combines survival analysis with Markov model to obtain a series of time-varying transition probabilities over the decision horizon [14,22].

Basic concepts of applying MDP to the scheduling problem are suggested in [17,23,24] where linear programming (LP) based selection problems between maintenance and replacement are solved. Madanat and Ben-Akiva utilize the concept of Latent Markov decision process (LMDP) to alleviate the uncertainty of observation data [25].

Dynamic programming (DP) is the most general and systematic framework for solving multi-horizon dynamic optimization problems. DP decomposes a multi-horizon optimization problem into smaller subproblems using recurrence property. Once the solutions of subproblems are obtained, they can provide an overall solution. This bottom-up approach reduces the repeated calculation and complexity of a large-scale optimization problem. Nevertheless, DP has not been utilized for solving multiple action scheduling problems for water main system owing to inherent difficulties in formulation.

This paper proposes a method to apply DP to the MDP based scheduling problem by separating the replacement actions from the rest. We also extend the optimization of two actions (maintenance and replacement) into multiple actions (maintenance, rehabilitation, and replacement), which has not been studied in the literature. To develop more accurate scheduling, the Markov model combined with survival analysis is employed rather than the exponential model used in previous works.

The rest of the paper is organized as follows. In Section 2, the problem statement and MDP formulation are introduced. In Section 3, survival analysis is combined with the Markov model to evaluate state transition probabilities. In Section 4, DP as an optimization method is introduced and the problem modification into infinite horizon periodic replacement is presented. Section 5 presents simulation and sensitivity analysis results with example data, which are compared with several existing policies via Monte Carlo simulations. Simulation results and practical aspects of the model are discussed in the final section of the paper.

WATER MAIN SYSTEM SCHEDULING PROBLEM FORMULATION

1. Problem Statement Using Markov Decision Process (MDP)

Consider a single pipe in the water main system. Pipe state can be categorized into several classes, and the classification criteria have been suggested by several studies and municipal governments [15,16,19]. The condition rating system, the most prominent state classification method, is to score the water pipe based on several criteria, which represents the pipe state.

One of the three actions is implemented on the water pipe at each decision epoch based on the information of the system. The three actions are maintenance, rehabilitation, and replacement. With an assumption that the system follows Markov property, Markov decision process (MDP) can be employed to describe temporal transitions of the system. The detailed indices and data for the problem description are given as follows:

T={1, …, N} is a set of decision epochs. S={1, …, |S|} is a set of finite states; state 1 denotes a new pipe and state |S| denotes the failure. A={1, …, |A|} is a set of finite actions; 1: maintenance, 2: rehabilitation, and 3: replacement. θ_t is the pipe age at decision epoch t. $C_t(s_{t+1}|s_t, a_t, \theta_t)$ is the single stage cost of pipe age θ_t and decision epoch t, when the current state is s_t and the successor state is s_{t+1} with action a_t taken. $p_t(s_{t+1}|s_t, a_t, \theta_t)$ is the transition probability of pipe age θ_t and decision epoch t. When the state is s_t and the action a_t is taken, the subsequent state will be in s_{t+1} with the probability of $p_t(s_{t+1}|s_t, a_t, \theta_t)$.

At decision epoch t, the state of the water pipe s_t is observed or calculated and action a_t is implemented with the accompanying

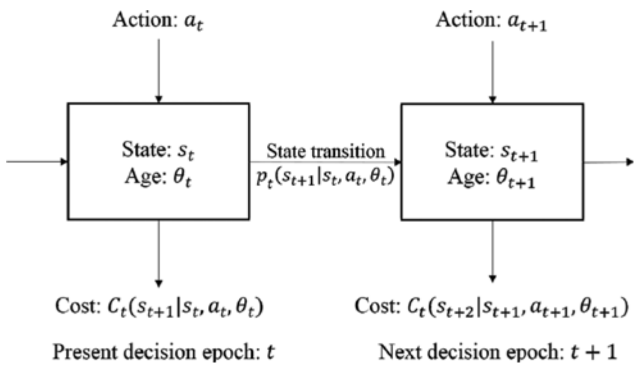

Fig. 1. Illustration of MDP.

cost. At the next decision epoch t+1, the process moves into a new state s_{t+1} and its corresponding cost occurs. The state transition probability of the process is $p_t(s_{t+1}|s_t, a_t, \theta_t)$. Fig. 1 illustrates the dynamic transition of MDP. Since it is a stochastic process, the cost should be evaluated as its expectation as follows:

$$E[C_t(s_{t+1}|s_t, a_t, \theta_t)]=C_t(s_t, a_t, \theta_t)=\Sigma_{s'\in S}p_t(s'|s_t, a_t, \theta_t)C_t(s'|s_t, a_t, \theta_t) \tag{1}$$

The optimal sequence of actions is referred to as optimal policy; $\pi^*=(a_1, a_2, \ldots, a_N)$, and the objective function is expressed as the cost-to-go function for the decision horizon N.

$$V_\pi=\Sigma_{t=1}^{N} C_t(s_t, a_t, \theta_t) \tag{2}$$

2. Structure of Transition Probability Matrix

A Markov transition probability matrix $P_t(a_t, \theta_t)$ is a matrix whose element of th row and th column denotes the transition probability $p_t(s_{t+1}=j|s_t=i, a_t, \theta_t)$. In the water main system, transition probability depends on the pipe deterioration model and the action taken at the corresponding decision epoch. Since the natural deterioration and action affect the transition probability independently, the transition probability matrix can be decomposed into deterioration part and action part, respectively.

Deterioration matrix $D(\theta_t)$ describes the deterioration model for a single pipe of age θ_t. Two assumptions are made to reduce the number of elements to be calculated [26]. First, it is assumed that the deterioration process can move from state i to state j only if j≥i, meaning that the natural improvement does not take place. Second, the pipeline can be deteriorated only one state at a time. When |S|=5, deterioration matrix can be expressed as follows:

$$D(\theta_t)=\begin{bmatrix} p(1|1,1,\theta_t) & p(2|1,1,\theta_t) & 0 & 0 & 0 \\ 0 & p(2|2,1,\theta_t) & p(3|2,1,\theta_t) & 0 & 0 \\ 0 & 0 & p(3|3,1,\theta_t) & p(4|3,1,\theta_t) & 0 \\ 0 & 0 & 0 & p(4|4,1,\theta_t) & p(5|4,1,\theta_t) \\ 0 & 0 & 0 & 0 & p(5|5,1,\theta_t) \end{bmatrix} \tag{3}$$

Action matrix $M(r_{a_t})$ describes the state transition for a single pipe of age θ_t when action a_t is taken. When the improvement performance is r_{a_t}, corresponding action matrix $M(r_{a_t})$ is given as follows:

$$M(r_{a_t})=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ r_{a_t} & 1-r_{a_t} & 0 & 0 & 0 \\ r_{a_t} & 0 & 1-r_{a_t} & 0 & 0 \\ r_{a_t} & 0 & 0 & 1-r_{a_t} & 0 \\ r_{a_t} & 0 & 0 & 0 & 1-r_{a_t} \end{bmatrix} \tag{4}$$

Since maintenance does not alter the state distribution, it is an identity matrix (r_1=0). We assume that the replacement recovers pipes completely and rehabilitation does with less efficiency. The rehabilitation performance r_2 has the value of 0.7 as discussed in [17]. Then left multiplication of action matrices by the deterioration matrix yield transition probability matrices; $P(1, \theta_t)=D(\theta_t)M(0)$, $P(2, \theta_t)=D(\theta_t)M(0.7)$, $P(3, \theta_t)=D(\theta_t)M(1)$.

DETERIORATION MODEL OF WATER PIPE

1. Deterioration Matrix Evaluation Using Survival Analysis

The pipe would deteriorate naturally if no improvement has been implemented. Deterioration matrix $D(\theta_t)$ can be evaluated using the deterioration model of a water pipe from survival analysis. Survival analysis estimates the survival function, which is the probability for a water pipe to survive over a particular time. The Weibull distribution is the most popular probability model used in survival analysis [3].

Survival analysis can be generalized from two states (failure or not) to n states when combined with semi-Markov process. A Markov process, whose waiting times are modeled as independent random variables, is referred to as a semi-Markov process. Subsequent analysis to evaluate the deterioration matrix is partially based on principles similar to those suggested in [22].

Let $\{T_1, T_2, \ldots, T_{|S|-1}\}$ be random variables representing the waiting time of states {1, 2, …, |S|−1} For instance, it takes T_i for the process to move from state i to i+1. When we define the random variable $T_{i\to k}$ as the sum of waiting times in states {i, i+1, …, k−1}, the cumulative waiting time between states i and k is

$$T_{i\to k}=\Sigma_{j=i}^{k-1}T_j \tag{5}$$

In general, the summation of two or more random variables can be calculated analytically by convolution integral. Probability density function (PDF) and survival function (SF) of $T_{i\to k}$ are denoted as $f_{i\to k}(T_{i\to k})$ and $S_{i\to k}(T_{i\to k})$, respectively. Then the transition probability of state i to state i+1 can be expressed as follows:

$$\Pr[s_{t+1}=i+1|s_t=i]=p(i+1|i, a_t=1, \theta_t)=\frac{f_{1\to i}(\theta_t)}{S_{1\to i}(\theta_t)-S_{1\to i-1}(\theta_t)} \tag{6}$$
for all i={1, 2, …, |S|−1}

Once the PDF and SF of waiting time T_i are established, each element of the deterioration matrix $D(\theta_t)$ can be calculated.

2. Weibull Distribution of Waiting Time

The waiting time T_i of state i is modeled to follow the Weibull probability distribution. Weibull model is a special case of the proportional hazards model which has two parameters and takes the following forms:

$$\begin{aligned} &\text{SF: } S_i(\theta_t)=\Pr[T_i\ge\theta_t]=\exp[-(\lambda_i\theta_t)^{\beta_i}] \\ &\text{PDF: } f_i(\theta_t)=\lambda_i\beta_i(\lambda_i\theta_t)^{\beta_i-1}\exp[-(\lambda_i\theta_t)^{\beta_i}] \end{aligned} \tag{7}$$

Parameters λ_i and β_i can be calculated by regression using the survival history of water main system (i.e., probability of being in the state i more than θ_t years). Survival time history of the pipe is recorded to find the parameters of Weibull model and evaluates the deterioration matrix.

DYNAMIC OPTIMIZATION

1. Finite Horizon Dynamic Programming

Dynamic programming is implemented to solve the MDP. The objective function is defined as Eqs. (1)-(2). The cost-to-go function for the pipe of age θ_k at the kth decision epoch, $V_k(s_k, \theta_k)$ is defined as

$$V_k(s_k, \theta_k)=\Sigma_{t=k}^{N}C(s_t, a_t, \theta_t) \tag{8}$$

Then we can find an optimal policy by working backward from N. The structure of a subproblem is

$$V_k(s_k, \theta_k)=\min_{a_k\in A}(C(s_k, a_k, \theta_k)+\Sigma_{s'\in S}p(s'|s_k, a_k, \theta_k)V_{k+1}(s', \theta_{k+1})) \tag{9}$$

This gives an optimal action a_k at decision epoch k. Let v_t be the column vector with the ith element being $V(s_t=i, \theta_t)$, and $c(a_t, \theta_t)$ is the column vector with the ith element being $C(s_t, a_t, \theta_t)$. Then the standard Bellman equation (Eq. (9)) can be expressed as the following vector-matrix form.

$$v_k(\theta_k)=\min_{a_k\in A}(c(a_k, \theta_k)+v_{k+1}(\theta_{k+1})P(a_k, \theta_k)) \tag{10}$$

2. Infinite Horizon Periodic Replacement

According to Eq. (6), deterioration matrices are time-varying. Hence, a deterioration matrix with an age index θ_t should be used in Eq. (10). Consider when the pipe is replaced at decision epoch k. According to Eq. (4), pipe age and state are initialized at the next decision epoch k+1; θ_{k+1}=1, s_{k+1}=1, [17]. To know the time index of the deterioration matrix at decision epoch k, the decision maker should have the replacement history. However, it is impossible to acquire such information since the optimal policy is evaluated from a_N to a_1, backwards. To avoid this algorithmic contradiction, two assumptions are made: periodic replacement and steady state assumptions.

Since the replacement initializes the time index of a deterioration matrix and does not alter the deterioration model, this problem can be considered as a periodic replacement problem. The policy structure is exactly repeated after each replacement period passes through. To obtain the optimal replacement period, it is assumed that a pipe is replaced periodically without the terminal time. This steady state assumption has been discussed in the literature [6,17, 24], and the horizon in Eq. (2) becomes infinity. γ is the discount factor for the convergence of Eq. (11).

$$V_\pi=\Sigma_{t=1}^{\infty}\gamma^{t-1}C_t(s_t, a_t, \theta_t) \tag{11}$$

Let the first replacement period take place at h and the remaining replacement period τ. Note that h takes a smaller value than τ. Then the pipe would be replaced at decision epoch h+kτ(k=0, 1, 2, …, ∞). Define the cost-to-go function V_η^h during the decision epoch within the first period $T_h=\{1, 2, \ldots, h\}$. The replacement action at the decision epoch h is also included in V_η^h.

$$V_\eta^h = \min_\eta\left(\sum_{t=1}^{h-1}\gamma^{t-1}C_t(s_t, a_t, \theta_t)\right)+\gamma^{h-1}C_h(s_h, a_h=3, \theta_h)$$
$$\eta=\{a_1, a_2, \ldots, a_{h-1}, a_h=3\} \quad a_i\in A\backslash\{3\} \quad 3\text{: Replacement} \tag{12}$$

The optimal policy η^* is the action sequence of maintenance and rehabilitation that minimizes the cost-to-go function V_η^h within the first period, and can be found by using DP (Eq. (10)). The *k*th remaining replacement of the decision horizon is $T_{\tau,k}=\{h+k\tau+1, h+k\tau+2, \ldots, h+(k+1)\tau\}$ and a corresponding cost-to-go function is $V_{\rho,k}^\tau$. The replacement action at the decision epoch $h+(k+1)\tau$ is also included in $V_{\rho,k}^\tau$.

$$V_{\rho,k}^\tau = \min_\rho\left(\sum_{t=h+k\tau+1}^{h+(k+1)\tau-1}\gamma^{t-1}C_t(s_t, a_t, \theta_t)\right)$$
$$+\gamma^{h+(k+1)\tau-1}C_{h+(k+1)\tau}(s_{h+(k+1)\tau}, a_{h+(k+1)\tau}=3, \theta_{h+(k+1)\tau}) \tag{13}$$
$$\rho_k=\{a_{h+k\tau+1}, a_{h+k\tau+2}, \ldots, a_{h+(k+1)\tau}=3\} \quad a_i\in A\backslash\{3\} \quad 3\text{: Replacement}$$
$$k=\{0, 1, \ldots, \infty\}$$

The optimal policy ρ_k^* minimizes the total cost-to-go function $V_{\rho,k}^\tau$ of the remaining periods. Since the pipe is replaced at decision epoch $h+k\tau$, both the pipe age and state at decision epoch $h+k\tau+1$ ($k=0, 1, 2, \ldots, \infty$) become 1. Since the optimal policy for the remaining periods is the same regardless of k, the time index of ρ_k^* can be omitted; $\rho_k^*=\rho^*$. This scheme is illustrated in Fig. 2.

The cost-to-go function can be reduced with the periodicity as $V_{\rho,k}=V_{\rho,0}\gamma^{(k-1)\tau}$, and the optimal policy π^* is equal to $\{\eta^*, \rho^*, \rho^*, \ldots\}$. The total cost over the entire horizon under policy π, V_π can be written with V_η^h and $V_{\rho,k}^\tau$ as follows:

$$V_\pi=\sum_{t=1}^{\infty}\gamma^{t-1}C_t(s_t, a_t, \theta_t)=V_\eta^h+\sum_{k=0}^{\infty}\gamma^h V_{\rho,k}^\tau$$
$$=V_\eta^h+\gamma^h\Sigma_{k=0}^{\infty}V_{\rho,0}\gamma^{(k-1)\tau}=V_\eta^h+\frac{\gamma^h}{1-\gamma^\tau}V_{\rho,0}^\tau \tag{14}$$

The goal is to find the optimal replacement period h^* and τ^*, optimal policies of subproblems η^* and ρ^*. The optimal policies η^* and ρ^* can be evaluated by applying Eq. (10) with the corresponding decision horizon. The total cost-to-go function can be calculated with Eq. (14). Then the optimal replacement period h^* and τ^* are found by exhaustively searching for the whole h and τ spaces. The final result of the proposed algorithm is the optimal policy, π^*. Fig. 3 summarizes the dynamic optimization scheme.

SIMULATION RESULTS

1. Case Study

An illustrative example is solved to validate the proposed algorithm given that there are five states (|S|=5), and the decision horizon is 100 years (N=100). Parameters of Weibull distribution model are arbitrarily chosen, which makes the pipe deteriorate from state 1 to state 5 within the decision horizon. In this example, the parameters in Kleiner [22] are used, and the discount factor γ is 0.99. The expectation costs of failure, rehabilitation and replacement are assumed to have the following values:

$$C_t(s_{t+1}|s_t, 1, \theta_t)=\begin{bmatrix}0 & 0 & 0 & 0 & 200\\ 0 & 0 & 0 & 0 & 200\\ 0 & 0 & 0 & 0 & 200\\ 0 & 0 & 0 & 0 & 200\\ 0 & 0 & 0 & 0 & 200\end{bmatrix}$$

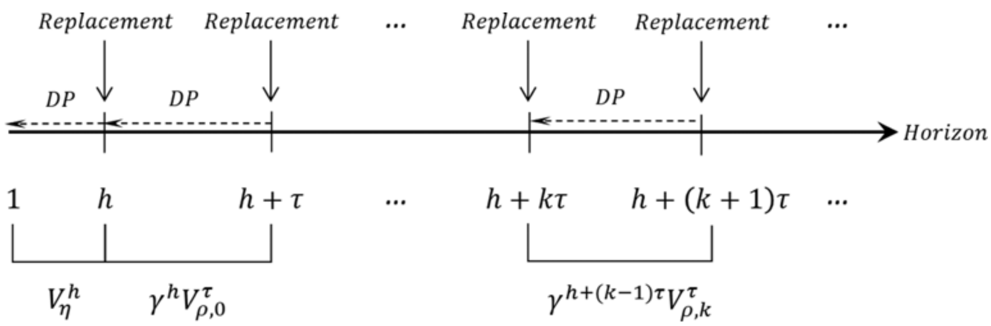

Fig. 2. Illustration of the infinite horizon periodic replacement problem.

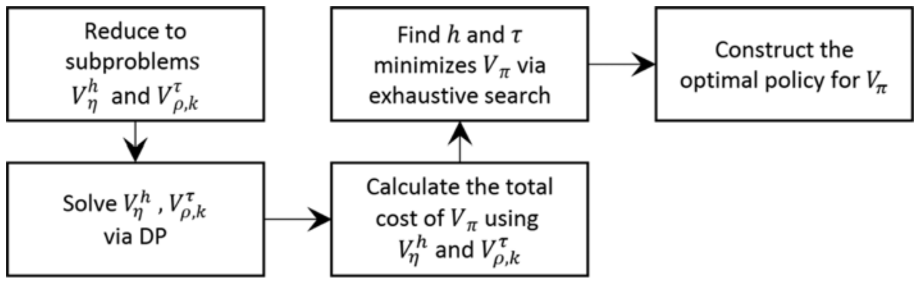

Fig. 3. Dynamic optimization flow diagram.

$$C_t(s_{t+1}|s_t, 2, \theta_t)=\begin{bmatrix} 70 & 70 & 70 & 70 & 200 \\ 70 & 70 & 70 & 70 & 200 \\ 70 & 70 & 70 & 70 & 200 \\ 70 & 70 & 70 & 70 & 200 \\ 70 & 70 & 70 & 70 & 200 \end{bmatrix}$$

$$C_t(s_{t+1}|s_t, 3, \theta_t)=\begin{bmatrix} 100 & 100 & 100 & 100 & 200 \\ 100 & 100 & 100 & 100 & 200 \\ 100 & 100 & 100 & 100 & 200 \\ 100 & 100 & 100 & 100 & 200 \\ 100 & 100 & 100 & 100 & 200 \end{bmatrix} \quad \text{for} \quad \forall t$$

Rehabilitation performance and cost are assumed to be time-invariant, and all the data are dimensionless.

2. Optimal Policy from the Proposed Algorithm

The proposed decision framework with the example data was implemented in MATLAB R2013a. Calculated optimal policy has the form of 5×100 matrix; *i*th row is the 100 year plan of the pipe whose initial state is estimated to be i. Fig. 4 shows the variation of the total cost of V_π as V_η^h of subproblem changes when the initial state is given. All the plots have a minimum point, and it is clear that those points are the optimal replacement periods for the corresponding initial state.

Table 1 shows the optimal replacement period and the objective value. For example, when the initial state is estimated to be 4, the first replacement after 21 years and a 45-year periodic replacement plan yield the minimum cost within the decision horizon. The proper rehabilitation plan is also evaluated by the proposed algorithm.

Nevertheless, a 100 year plan cannot be applied directly to a real system. In practice, predetermined regular inspection is performed in parallel and obtains the properties of the pipe so that the initial state is updated. Based on this, the optimal schedule can be newly established regularly. It is a similar method to receding horizon control, where an optimization problem is solved at each step to determine an optimal policy in a repeated manner. Fig. 5 summarizes the whole decision process for practical application.

Table 1. Optimal replacement period

Initial state	State1	State2	State3	State4	State5
h^* (year)	44	44	21	21	1
τ^* (year)	45	45	45	45	45
V^*	219.42	221.02	259.43	259.43	317.19

3. Sensitivity Analysis of the Replacement Period

The optimal replacement period is observed by varying the ratio of failure/replacement and rehabilitation/replacement. Fig. 6 shows the result of sensitivity analysis of the replacement period. Failure/replacement cost ratio varies from 1 to 10, and rehabilitation/replacement cost ratio varies from 0.1 to 3. Replacement period changes dramatically by rehabilitation cost variation, while failure cost variation has a smaller effect. It can be concluded that the sensitivity of rehabilitation cost is much higher than that of the failure cost.

This gives some advantage and convenience to the decision maker. The failure cost is an abstract concept and hard to estimate because of various overhead costs such as the residential inconvenience cost, while rehabilitation cost is easy to estimate. The result of the sensitivity analysis implies that the inaccurate value of the failure cost does not have much effect on the result.

4. Comparison with other Policies via Monte Carlo Simulation

Monte Carlo simulation is widely used to solve an optimization

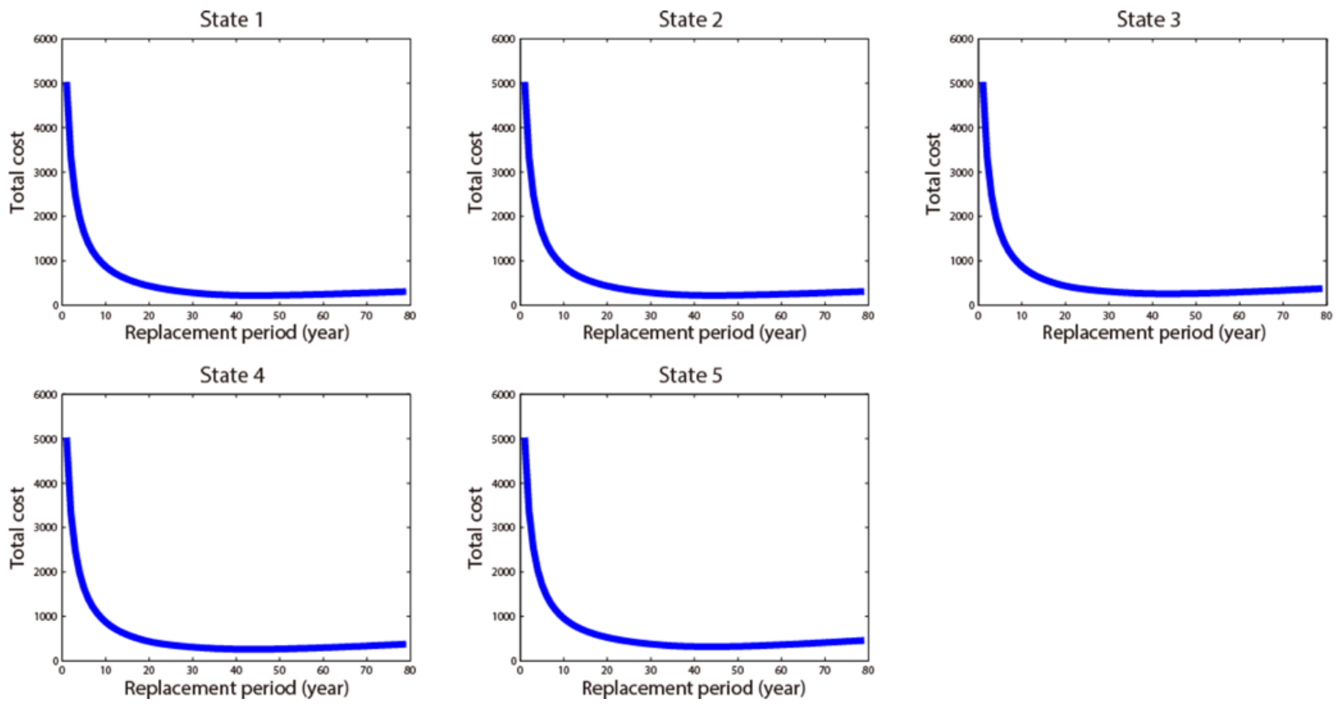

Fig. 4. The total cost-to-go function versus replacement period for each initial state.

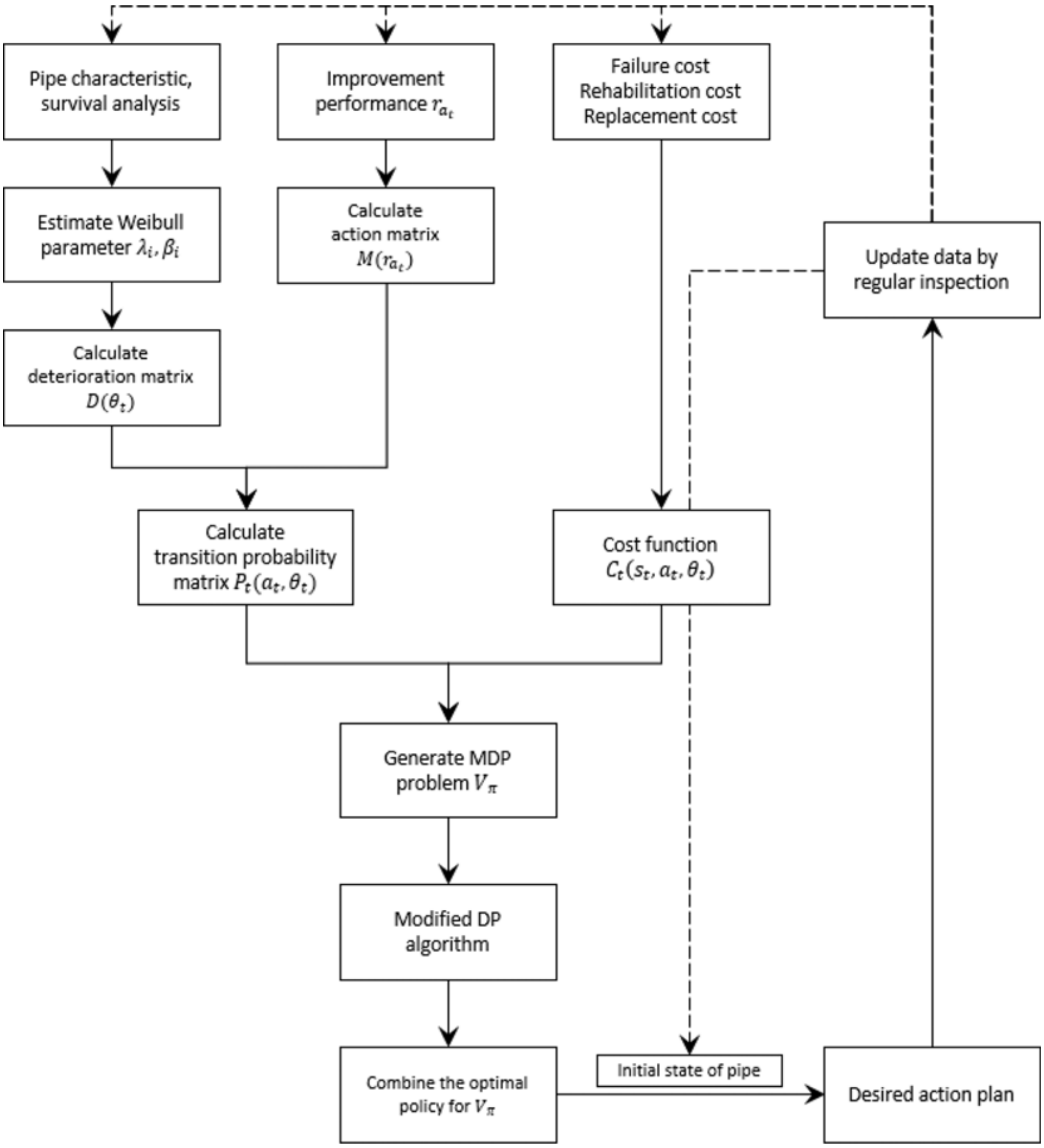

Fig. 5. Decision process flow diagram.

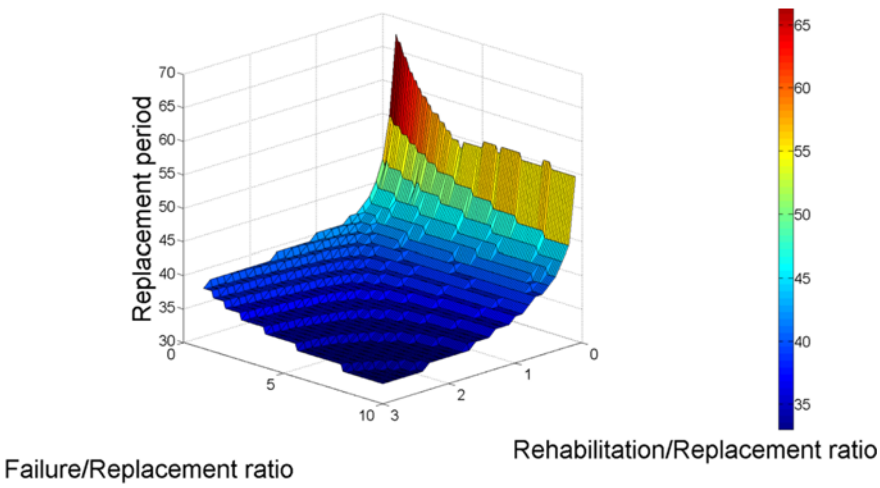

Fig. 6. The optimal replacement period versus failure and replacement cost ratio, and failure and rehabilitation cost ratio.

Table 2. Policy comparison

Initial state	State1	State2	State3	State4	State5
Proposed	248	240	313	314	382
Heuristics 15	315	315	415	471	414
Heuristics 30	323	324	440	477	418
Myopic	594	594	594	594	693

problem or to observe the scenario of nonconvex, sequential stochastic process. It generates a set of random samples to obtain numerical results and observe a stochastic dynamics. Total cost and state variation would be simulated under each policy, and the global optimal policy can be obtained by enumerating all the possible cases. However, the number of cases in this problem is $|A|^{N\times|S|}$, which is too huge to enumerate. Nevertheless, it is useful to observe the state variations over the decision horizon and the costs when the action set is implemented.

To prove the validity of the proposed algorithm, several heuristics and optimal policies are compared to the proposed method using Monte Carlo simulation. The map of state transition probability $p_t(s_{t+1}|s_t, a_t, \theta_t)$ is revealed when the action set is given. Monte Carlo simulation creates the path regarding the transition probability matrices from decision epoch 1 through 100. Each experiment was repeated 5000 times to reduce the effect of randomness. The discount factor has been omitted in policy comparison to provide the total cost incurred over the simulation horizon.

Monte Carlo simulation results of several popular policies are compared to the policy obtained by the proposed algorithm and shown in Table 2. Heuristics 15 and 30 replace the pipe every 15 and 30 years, respectively, regardless of its initial state. The myopic policy is to replace the pipe when a failure occurs. Simulation results show that the proposed algorithm leads to the smallest value in all initial states.

CONCLUSION

We have formulated and suggested a dynamic optimization method to solve the periodic replacement problem of a water pipe. The usage of pipe characteristics such as geometry, pH, soil type, weather, and population to determine the Weibull distribution parameters and transition probability matrices was introduced.

Due to the time index initialization of the transition probability matrix when replacement takes place, the conventional DP algorithm cannot be implemented. Hence, a modified method was proposed to obtain an optimal policy: (1) divide the problem into infinite horizon periodic replacement subproblems, (2) apply DP to each subproblem to obtain the optimal maintenance and rehabilitation schedule, and (3) and reassemble them.

Sensitivity analysis showed that the rehabilitation cost affects the replacement period more than the failure cost. Calculated optimal policy was compared with several simple policies (heuristics, myopic policy) using Monte Carlo simulation. Through simulation results, we concluded that the optimal policy from the proposed algorithm can reduce the expense compared to the existing policies.

Moreover, state-specific scheduling plan would be one of the major differences compared to existing policies. It considers the effect of current state, property, cost, decision horizon comprehensively, while heuristic and myopic policies do not. Hence, the policy can be altered flexibly under frequently varying circumstances. By regular inspections, the states of pipe are updated, and an optimal policy can be regularly derived.

ACKNOWLEDGEMENTS

This research is supported by the Korea Ministry of Environment as a Project for Developing Eco-Innovation Technologies (GT-11-G-02-001-3) and the Korea Ministry of Trade, Industry and Energy (MOTIE) as a grant entitled: Development of standardization/certification technologies and whole chain-integrated modules for CCS commercialization (2012100201687).

NOMENCLATURE

A : finite set of actions
$C_t(s_t, a_t, \theta_t)$: expectation of the cost function at decision epoch t, state s_t, action a_t and age θ_t
$C_t(s_{t+1}|s_t, a_t, \theta_t)$: cost function at decision epoch t, state s_t, action a_t, age θ_t and next state s_{t+1}
$D(\theta_t)$: deterioration matrix at decision epoch θ_t
$M(r_{a_t})$: action matrix with improvement performance r_{a_t}
N : decision horizon
$P_t(a_t, \theta_t)$: transition probability matrix with action a_t, age θ_t
T : set of decision epoch
T_i : random variable of waiting time in state i
$T_{i\to k}$: random variable of cumulative waiting time between states i and k
T_h : set of decision epoch of the first period
$T_{\tau,k}$: set of decision epoch of the remainder period
S : finite set of states
$S_{i\to k}(T_{i\to k})$: survival function (SF) of cumulative waiting time $T_{i\to k}$
$V_k(s_k, \theta_k)$: value function at decision epoch k, state s_k and age θ_k
V^h_η : value function of the first period T_h given that action set is η
V_{π^*} : optimal value function with optimal policy π^*
$V^\tau_{\rho,k}$: value function of the remainder period T_r given that action set is ρ
a_t : action at decision epoch t
$f_{i\to k}(T_{i\to k})$: probability density function (PDF) of cumulative waiting time $T_{i\to k}$
$p_t(s_{t+1}|s_t, a_t, \theta_t)$: transition probability at decision epoch t, state s_t, action a_t and age θ_t
r_{a_t} : improvement performance of action a_t
s_t : state at decision epoch t
v_k : value function expressed by a vector-matrix form.
β_i : Weibull parameter
γ : discount factor
η, η^* : policy of the first period. Asterisk stands for the optimal policy
θ_t : pipe age at decision epoch t
λ_i : Weibull parameter
π, π^* : policy of Bellman equation. Asterisk stands for the optimal policy
ρ, ρ^* : policy of the remainder period. Asterisk stands for the opti-

mal policy

τ, τ^* : replacement period. Asterisk stands for the optimal.

REFERENCES

1. S. A. Andreou, D. H. Marks and R. M. Clark, *Adv. Water Res.*, **10**(1), 11 (1987).
2. S. A. Andreou, D. H. Marks and R. M. Clark, *Adv. Water Res.*, **10**(1), 2 (1987).
3. A. Martins, J. P. Leitão and C. Amado, *J. Infrastructure Systems*, **19**(4), 442 (2013).
4. Y. Le Gat and P. Eisenbeis, *Urban Water*, **2**(3), 173 (2000).
5. G. V. Loganathan, S. Park and H. Sherali, *J. Water Resources Planning and Management*, **128**(4), 271 (2002).
6. Y. Kleiner, B. J. Adams and J. S. Rogers, *Water Resour. Res.*, **34**(8), 2039 (1998).
7. Y. Kleiner, B. J. Adams and J. S. Rogers, *Water Resour. Res.*, **34**(8), 2053 (1998).
8. G. C. Dandy and M. O. Engelhardt, *J. Water Resources Planning and Management*, **132**(2), 79 (2006).
9. A. Nafi, C. Werey and P. Llerena, *Canadian J. Civil Engineering*, **35**(1), 87 (2008).
10. T. D. Prasad and N. S. Park, *J. Water Resources Planning and Management*, **130**(1), 73 (2004).
11. K. Boston and P. Bettinger, *Forest Science*, **45**(2), 292 (1999).
12. G. Buxey, *J. Operational Research Society*, 563 (1979).
13. D. Halhal, G. A. Walters, D. Ouazar and D. A. Savic, *J. Water Resources Planning and Management*, **123**(3), 137 (1997).
14. D. Li and Y. Y. Haimes, *Water Resour. Res.*, **28**(4), 1053 (1992).
15. M. A. Cesare, C. Santamarina, C. Turkstra and E. H. Vanmarcke, *J. of Transportation Engineering*, **118**(6), 820 (1992).
16. H. S. Baik, H. S. Jeong and D. M. Abraham, *J. Water Resources Planning and Management*, **132**(1), 15 (2006).
17. F. Guignier and S. Madanat, *J. Infrastructure Systems*, **5**(4), 124 (1999).
18. S. Madanat, R. Mishalani and W. H. W. Ibrahim, *J. Infrastructure Systems*, **1**(2), 120 (1995).
19. S. Madanat and W. H. W. Ibrahim, *J. Transportation Engineering*, **121**(3), 267 (1995).
20. T. Micevski, G. Kuczera and P. Coombes, *J. Infrastructure Systems*, **8**(2), 49 (2002).
21. R. Wirahadikusumah, D. Abraham and T. Iseley, *J. Infrastructure Systems*, **7**(2), 77 (2001).
22. Y. Kleiner, *J. Infrastructure Systems*, **7**(4), 136 (2001).
23. A. J. Draper, M. W. Jenkins, K. W. Kirby, J. R. Lund and R. E. Howitt, *J. Water Resources Planning and Management*, **129**(3), 155 (2003).
24. K. Golabi, R. B. Kulkarni and G. B. Way, *Interfaces*, **12**(6), 5 (1982).
25. S. Madanat and M. Ben-Akiva, *Transportation Science*, **28**(1), 55 (1994).
26. V. Kathuls and R. McKim, *Sewer deterioration prediction*, In Proc., Infra 99 Int. Convention (1999).