

# Predicting the fast transition conditions by the correlation of particle entrainment rate

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**Abstract**—A model for predicting the fast transition condition in a riser of a circulating fluidized bed was proposed using the correlation of particle entrainment rate. The saturation carrying capacity of Bai and Kato could be regarded as the particle entrainment rate at the fast transition condition. The correlation of Choi et al. on particle entrainment rate could be used as a tool to predict the fast transition condition. The effect of interparticle forces seemed to be negligible at the fast transition condition. The model was in fair agreement with the measured values at the fast transition condition.

**Keywords:** Saturation Carrying Capacity, Fast Transition Velocity, Fast Fluidization, Riser, Circulating Fluidized Bed

## INTRODUCTION

The flow regime of fast fluidization is divided into two regions, depending on the shape of the axial solid holdup distribution in the riser of the circulating fluidized bed (CFB) [1,2]. One is a region indicating an S-shaped axial profile of solid holdup in the riser - a dense phase in the lower part of the riser, a lean phase in the upper part of the riser, and a transition part between the dense and lean phases where the solid holdup decreases exponentially with increasing height. Particles are being accelerated in rise velocity in the dense phase, the acceleration to an equilibrium velocity is completed in the transition part, and particles are rising with a constant velocity in the lean phase. The other is a region indicating a lean upper part of the riser after the lower (transition) part that has an exponential decrease in solid holdup profile along the height without the presence of the dense phase. In this case, the particles are accelerated very quickly to the equilibrium velocity in the transition part.

At the boundary between two regions, the flow region shifts from the latter to the former when the solid flux increases or the gas

velocity decreases. Bai and Kato [1] defined the saturation carrying capacity (SCC) of gas as the solids flux at the boundary condition where the dense phase begins to form in the lower part of the riser. The SCC increases as the gas velocity increases. A solids flux greater than the SCC brings a dense phase, building up solids to the lower part of the riser. Namkung et al. [2] named this condition as the fast transition condition, and called the gas velocity at this condition as the fast transition velocity ( $U_{FT}$ ). There have been several studies to measure the properties of the condition and to provide correlations on it [1-8]. Table 1 summarizes correlations for predicting the fast transition conditions. The fast transition velocity was presented as a function of the solid flux and the gas and particle properties. The fast transition velocity ( $U_{FT}$ ) increased with increasing the SCC, i.e., another fast transition condition with the higher solids flux than before. The correlation determines the value of the remaining variable for the riser to reach the fast transition condition, given all other variables. The particles were considered to have a single size as a mean particle diameter.

According to Grace et al. [8], the correlation of Bi and Fan [5] should be applicable, but still should be improved. Bai and Kato

**Table 1. Correlations on fast transition conditions**

Authors	Correlations	Applicable range
Bi and Fan [5]	$\frac{U_{FT}}{\sqrt{gd_p}} = 21.6Ar^{0.105} \left( \frac{G_s^*}{\rho_g U_{FT}} \right)^{0.542}$	
Bai and Kato [1]	$\frac{G_s^* d_p}{\mu} = 0.125Fr^{1.85} Ar^{0.63} \left( \frac{\rho_p - \rho_g}{\rho_g} \right)^{-0.44}$ $Fr = U_{FT} / (gd_p)^{0.5}$	$4.7 < Ar < 1019, 41 < Fr < 226, 607 < (\rho_p - \rho_g) / \rho_g < 3607$
Namkung et al. [2]	$Re_{FT} = 0.395Ar^{0.572} \left( \frac{G_s^*}{\rho_g U_{FT}} \right)^{0.345}$ $Re_{FT} = d_p U_{FT} \rho_g / \mu$	$2.5 < Ar < 930, 17.0 < (G_s^* / (\rho_g U_{FT})) < 1040$

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[1] presented the correlation based on their data and the data in the literature. However, detailed information on the data of Bi and Fan [5] and Bai and Kato [1] was unavailable. Bai and Kato [1] used the data measured by Takeuchi et al. [3] and Li et al. [4] and the data they extracted from the axial solids distribution of the riser reported in the literature. However, it was not clear on what basis the extracted data were determined. The correlation of Bai and Kato [1] was in good agreement with the fast transition conditions measured by Namkung et al. [2] using fluid catalytic cracking (FCC) particles as bed material, but not for the case of sand bigger in size and density than FCC. Furthermore, the predicted SCC of sand was greater than that of FCC, contradicting the measured ones. Namkung et al. [2] applied for sand the correlation of Yang [9], which represented the choking condition. However, the correlation of Yang [9] excessively overestimated the SCC, while underestimating the fast transition velocity. The predicted fast transition velocity was rather close to the transition velocity between turbulent and fast fluidized bed. The same results were obtained in the evaluation of this study, and therefore the correlation of Yang [9] was excluded from considering the correlation for fast transition conditions. Namkung et al. [2] proposed an improved correlation for predicting fast transition conditions by adding their data to the ones used by Bai and Kato [1]. Smolders and Baeyens [7] measured the fast transition velocity at room temperature and pressure using sand and FCC particles as bed materials. The correlation of Bai and Kato [1] predicted the SCC 100% greater than that measured by Smolders and Baeyens [7]. The correlation of Bi and Fan [5] was as good as that of Bai and Kato [1].

The correlations presented in the literature for predicting the fast transition conditions are empirical formulas based on the data measured in risers of CFBs at atmospheric temperature and pressure conditions with air as a gas. Therefore, each correlation is valid in the experimental range where the data were obtained, but unsure for the condition out of the range in solid and gas properties and riser size. More studies are still needed to develop the correlation applicable in wide range.

Particle acceleration in the lower zone of the riser at the fast transition condition is very similar to particle acceleration in the splash zone (the lower part of the freeboard) of the low velocity fluidized bed that uses multi-sizes particles as bed material in which some are entrained out but others not [10,11]. The solid holdup decreases exponentially with increasing the freeboard height in the splash zone. The behavior of the entrained small particles in the not entrained large particle bed must be significantly different from that in the dense bed of the fast bed where all particles are accelerated and carried out. The small particles in the large particle bed will not accelerate well. Only the bubble carries the small particles as a part of bed particle mixture of the wake to the bed surface to be ejected there to the freeboard. In this case, the entrained particles are accelerated to the equilibrium rise velocity in the splash zone of the freeboard for a short time. With respect to the acceleration of the entrained particles, it is the same phenomenon that occurs in the lower part of the riser at the fast transition condition. Eventually, the entrainment rate at the same condition as the fast transition condition corresponds to the SCC of the riser. In the bubbling bed, the particles are accelerated by both jet flows of

bubble eruption that generates the cluster flux and of interstitial gas. At the fast transition condition, the particles are accelerated by the gas jets of the distributor. Thus, it can be considered that the gas jet of the distributor replaces the roles of the bubble and interstitial gas.

The correlation of Choi et al. [12-15] on particle entrainment rate was formulated for the well-mixed fluidized bed where multi-size particles are bed material. This study aimed at testing the possibility whether the correlation provided an effective way in predicting the fast transition condition.

## MODEL

The correlation of Choi et al. [12-15], including the effects of temperature, pressure, particle characteristics, and size of the fluidized bed in the relatively wide range, was used as the base correlation.

$$K_i^* = K_{ih}^* + K_{i\infty}^* \quad (1a)$$

where

$$K_{ih}^* d_p / \mu = C_d Re_p \exp(-9.12 - 0.0153a(H_t - H_b)) \quad (1b)$$

$$K_{i\infty}^* d_p / \mu = Ar^{0.5} \exp(6.92 - 2.39F_g^{0.303} - 13.1/F_d^{0.902}) \quad (1c)$$

with

$$ad_p = \exp\left(-11.2 + 210 \frac{d_p}{D_t - d_p}\right) \left(\frac{d_p \rho_g (U - U_{mf})}{\mu}\right)^{-0.492} \quad (2)$$

$$\left(\frac{\rho_p g d_p}{\rho_g (U - U_{mf})^2}\right)^{0.725} \left(\frac{\rho_p - \rho_g}{\rho_g}\right)^{0.731} C_d^{-1.47}$$

$$Ar = g d_p^3 \rho_g (\rho_p - \rho_g) / \mu^2 \quad (3)$$

$$F_g = 2g d_p (\rho_p - \rho_g) / 3 \quad (\text{SI units throughout}) \quad (4a)$$

$$F_d = C_d \rho_g U^2 / 2 \quad (\text{SI units throughout}) \quad (4b)$$

$$C_d = 24 / Re_p \quad \text{for } Re_p \leq 5.8 \quad (5a)$$

$$C_d = 10 / Re_p^{0.5} \quad \text{for } 5.8 < Re_p \leq 540 \quad (5b)$$

$$C_d = 0.43 \quad \text{for } 540 < Re_p \quad (5c)$$

$$Re_p = d_p U \rho_g / \mu \quad (6)$$

Solid particles are ejected from the bed surface into the freeboard mainly by bubble eruption. They have various different initial rise velocities at the ejection. Mostly in the splash zone just above the bed surface, the solid particle either rises or falls to get to equilibrium in rise velocity. The process depends on the gas velocity and its own size and initial rise velocity. As the freeboard height increases, the entrainment rate of particles is regarded to level off in the equilibrium after decreasing exponentially [10]. Choi et al. [12] considered the entrainment flux of particles in size  $i$  ( $K_i^*$ ) at the freeboard gas exit to consist of a cluster flux ( $K_{ih}^*$ ) and a dispersed noncluster flux ( $K_{i\infty}^*$ ) as suggested by Hazlett and Bergougnou [16]. The trend of cluster flux with freeboard height was described as an exponential decrease. However, the dispersed noncluster flux remained constant with the freeboard height, which corresponded to the entrainment rate above the transport disengaging height. The decay constant of the cluster flux decreasing exponentially with an increase of the freeboard height was considered linearly proportional to that of the axial solid holdup profile

[13].

In Eq. (1b),  $H_b$  is the bed height of the dense phase and the term  $(H_t - H_b)$  has the meaning of freeboard height. At the fast transition condition,  $H_b$  was regarded as zero and the riser height becomes the freeboard height. The bed and entrained particles were considered having the same apparent density for simplification. The minimum fluidization velocity ( $U_{mf}$ ) was determined from the correlation of Wen and Yu [17]. The correlation of Choi et al. [12] was valid in the following range: particle diameter of 21–710  $\mu\text{m}$ , particle density of 2,400–6,158  $\text{kg/m}^3$ , gas velocity of 0.15–2.8 m/s, temperature of 12–600  $^\circ\text{C}$ , pressure of 101–3,200 kPa, column diameter of 0.1–0.91 m, and column height of 1.97–9.1 m. However, the correlation excluded the effect of the interparticle forces.

When the particle size is small enough to concern interparticle forces, the correlation of Choi et al. [12] should be corrected toward including the effect of them. Ma and Kato [18] and Li and Kato [19] investigated the entrainment of group C particles (adhesion particles) from a fluidized bed of fine-coarse particle mixtures at the atmospheric pressure and temperature with air as the fluidizing gas. A correlation on the critical particle diameter ( $d_{crit}$ ), i.e., the maximum particle diameter at which the sum of the interparticle adhesion forces had a dominant influence on particle entrainment, was proposed by Ma and Kato [18].

$$d_{crit} = 0.101 / (g \rho_p^{0.731}) \quad (7)$$

The critical diameter corresponds approximately to the boundary between group A and C powders [19,20]. Thus the group of particles smaller than  $d_{crit}$  in diameter can be classified as a group C powder. Li and Kato [19] investigated the effect of the diameter of bed particles on the entrainment rate of group C particles. When both bed and entrained particles were same in apparent density and smaller than 60  $\mu\text{m}$  in diameter, the entrainment rate of particle in the range of diameter  $d_p < 60 \mu\text{m}$  at a gas velocity turned out to be constant as the entrainment rate for diameter of 60  $\mu\text{m}$ . The 60  $\mu\text{m}$  was the particle diameter at the boundary between group A and C powders and virtually the critical particle diameter (calculated  $d_{crit} = 53.1 \mu\text{m}$ ). It also can be confirmed in their correlation on entrainment rate of group C particles (adhesion particles) that the term of particle diameter disappears at the condition. In the present model with reference to the findings of Li and Kato [19], the entrainment rate of group C particles (adhesion particles) was calculated from Eqs. (1) to (6) by considering the particle size as follows.

$$d_p = d_{crit} \text{ for } d_p < d_{crit} \quad (8)$$

Li and Kato [19] also found that when the size ( $d_{pm}$ ) of the bed particles was greater than  $d_{crit}$  and the entrained particles ( $d_p$ ) were smaller than  $d_{crit}$ , the entrained particles adhered to the bed particles and thus the entrainment rate decreased. The entrainment rate decreased with the size of entrained particles due to enhanced adhesion. However, the bed particle size had no effect on the entrainment rate when  $d_p > d_{crit}$  or  $d_{pm} < d_{crit}$ . The entrainment rate increased as the size of entrained particles decreased in the range  $d_p > d_{crit}$  covered by the correlation of Choi et al. [12]. But the particle size was out of the range of group C powder. The condition  $d_{pm} < d_{crit}$  was included in the case resulting in Eq. (8). The apparent density

of bed particles had little effect on the entrainment rate of group C particles. To reflect the effect of bed particle size on the entrainment rate of group C particles, Li and Kato [19] proposed the following correlation on the correction factor ( $C_{ps}$ ). The  $C_{ps}$  represents the ratio of the actual entrainment rate to the entrainment rate at the absence of the effect of bed particle size. In the present study, the entrainment rate at no effect of bed particle size could be calculated by Eqs. (1) to (6) at the condition of Eq. (8).

$$C_{ps} = 1 \text{ for } d_{pm} \leq 60 \mu\text{m} \quad (9a)$$

$$C_{ps} = ((200 - d_{pm})/150)^\alpha + ((d_{pm} - 60)/150)^\alpha (d_p/d_{crit})^{1.4} \quad (9b)$$

$$C_{ps} = (d_p/d_{crit})^{1.4} \text{ for } 60 \mu\text{m} < d_{pm} \leq 200 \mu\text{m} \quad (9c)$$

$$\alpha = (d_{crit}/d_p)^{0.3} \quad (10)$$

Therefore, Eq. (1a) was modified with the correction factor ( $C_{ps}$ ) for calculating the entrainment rate of group C particles ( $K_i^{**}$ ).

$$K_i^{**} = C_{ps} (K_{di}^* + K_{eo}^*) \text{ at } d_p = d_{crit} \text{ for group C particles } (d_p < d_{crit}) \quad (11)$$

In calculating the fast transition condition, particles are considered to have a single size a mean particle diameter and the same apparent density as usual. As a result, when the effect of the interparticle forces on the entrainment rate is neglected ( $d_p > d_{crit}$ ), the fast transition velocity ( $U_{FT}$ ) in the riser is defined as the gas velocity at which the solid flux is equal to the particle entrainment rate ( $K_i^*$ ).

$$U_{FT} = U \text{ at } G_s = G_s^* = K_i^* \quad (12)$$

Similarly, the  $U_{FT}$  for group C particles (adhesion particles) is defined as the gas velocity at which the solid flux is equal to the corrected particle entrainment rate ( $K_i^{**}$ ). Considering particles to have a single size a mean particle diameter and according to Eq. (9a),  $C_{ps} = 1$ .

$$U_{FT} = U \text{ at } G_s = G_s^* = K_i^{**} \quad (13)$$

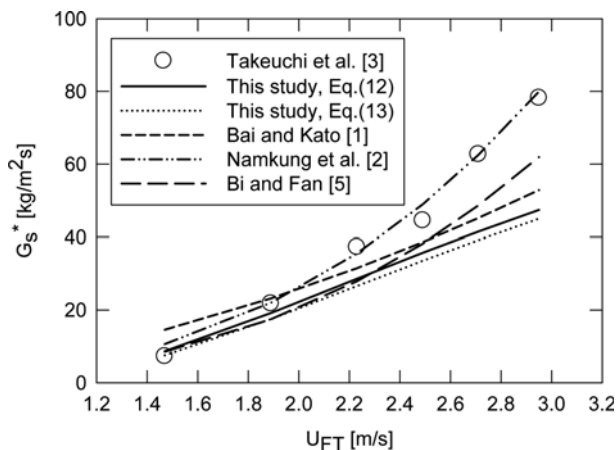
## RESULTS AND DISCUSSION

In this study, we used the limited data of Namkung et al. [2], Takeuchi et al. [3], Li et al. [4], Jiang et al. [6], and Smolders and Baeyens [7] as the ones obtained explicitly for the purpose of measuring the fast transition conditions for comparing this model with correlations [1,2,5] on fast transition conditions. Table 2 summarizes the size of the riser and the particle properties used in experiment. All tests were at ambient conditions in temperature and pressure using air as gas. The apparent density of particles of Takeuchi et al. [3] was considered as 1,050  $\text{kg/m}^3$  as given in the study of Bi and Fan [5]. The mean diameter of particles used in studies of Takeuchi et al. [3] and Li et al. [4] was smaller than the critical diameter calculated by Eq. (7). In these cases, Eqs. (12) and (13) were tested, respectively. If the particle size was greater than  $d_{crit}$ , the Eq. (12) was applied to define the fast transition condition.

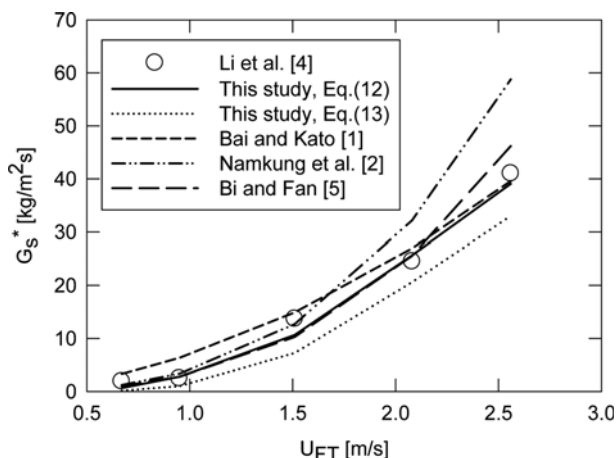
Fig. 1 compares the present model and correlations [1,2,5] with the relationship measured by Takeuchi et al. [3] between gas velocity and solids flux of saturation carrying capacity at the fast transition condition. The saturation carrying capacity increased with the fast transition velocity, as the particle entrainment rate increased

**Table 2. Riser size and particle properties for the data on fast transition conditions reported in literature**

Authors	Riser diameter [m]	Riser height [m]	Particles	Particle diameter [mm]	Apparent particle density [kg/m <sup>3</sup> ]	Critical particle diameter [mm]
Takeuchi et al. [3]	0.1	5.5	FCC	0.057	1050 [5]	0.0638
Li et al. [4]	0.09	10	FCC	0.054	929.5	0.0697
Jiang et al. [6]	0.102	6.32	FCC	0.089	1153	0.0596
			Polyethylene	0.325	660	0.0895
Namkung et al. [2]	0.1	5.3	FCC	0.065	1720	0.0445
			Sand	0.125	3055	0.0292
Smolders and Baeyens [7]	0.1	6.47	Sand	0.090	2600	0.0329
			FCC	0.070	2700	0.0320

**Fig. 1. Comparison of correlations with transition conditions measured by Takeuchi et al. [3].**

with the gas velocity. As shown in the figure, the correlation of Namkung et al. [2] is in good agreement with the measured data. Other correlations [1,5], including this model, predicted saturation carrying capacity to be somewhat less than the measured value. On the other hand, Eq. (12) is better than Eq. (13) although the particle size is smaller than the critical diameter. The terminal velocity [21]

**Fig. 2. Comparison of correlations with transition conditions measured by Li et al. [4].**

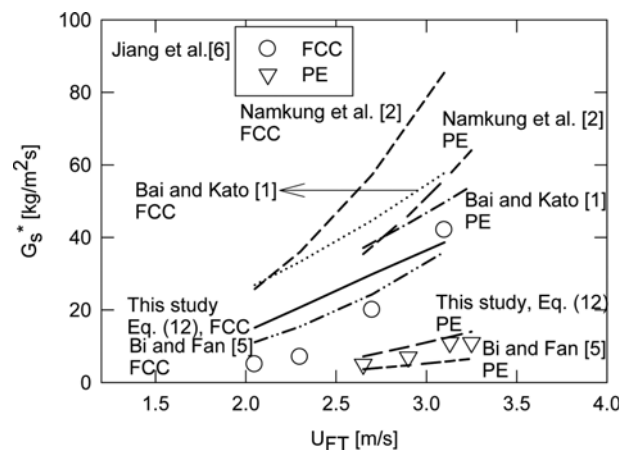
was calculated by

$$U_t = \left\{ \frac{4gd_p(\rho_p - \rho_g)}{3\rho_g C_d} \right\}^{1/2} \quad (14)$$

Fig. 2 depicts the present model and correlations with the fast transition conditions measured by Li et al. [4]. Correlations of Bi and Fan [5] and Bai and Kato [1], and the present model by Eq. (12) agreed well with the measured values. The correlation of Namkung et al. [2] and the present model by Eq. (13) fitted the data fairly well. On the other hand, Eq. (12) was better again than Eq. (13) to fit the measured data, although the particle size was smaller than the critical diameter. From the results of Figs. 1 and 2 on the present model, the effect of interparticle forces at the fast transition condition seemed to be negligible, compared to hydrodynamic forces.

Fig. 3 illustrates the present model and correlations compared with the fast transition conditions measured by Jiang et al. [6]. They used FCC and polyethylene (PE) particles as bed materials. The present model (Eq. (12)) and the correlation of Bi and Fan [5] agreed well with the measured data. However, the correlations of Namkung et al. [2] and Bai and Kato [1] overestimated the saturation carrying capacity significantly.

Fig. 4 compares the present model (Eq. (12)) and correlations [1,2,5] with the relationship measured by Namkung et al. [2] be-

**Fig. 3. Comparison of correlations with transition conditions measured by Jiang et al. [6].**

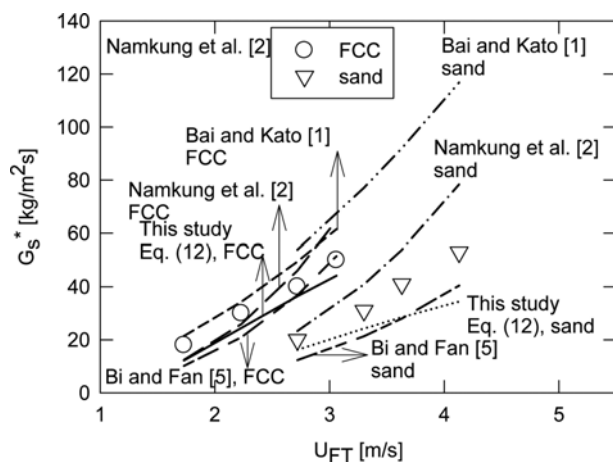


Fig. 4. Comparison of correlations with transition conditions measured by Namkung et al. [2].

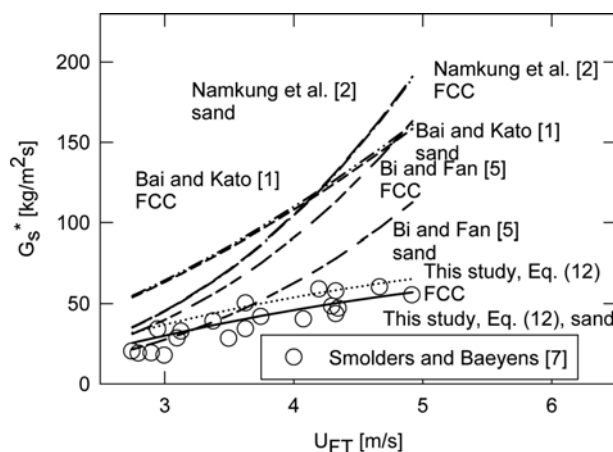


Fig. 5. Comparison of correlations with transition conditions measured by Smolders and Baeyens [7].

tween fast transition velocity and saturation carrying capacity. As shown in the figure, correlations of Bi and Fan [5] and Namkung et al. [2] and the present model are in good agreement with the measured values, except the correlation of Bai and Kato [1]. The correlation of Bai and Kato [1] shows that the saturation carrying capacity of the sand is greater than that of FCC, which is contrary to the experimental results.

Fig. 5 illustrates the correlations with the fast transition conditions measured by Smolders and Baeyens [7]. They used FCC and sand as solid particles, but did not distinguish the measured data by particle. As shown in the figure, all correlations [1,2,5] except this model (Eq. (12)) predicted a much greater value than the measured saturation carrying capacity at the fast transition condition. The saturation carrying capacities for sand and FCC were predicted to be almost same by correlations of Bai and Kato [1] and Namkung et al. [2]. The correlation of Bi and Fan [5] began to show the appreciable deviation at the gas velocity greater than 3 m/s for FCC and 4 m/s for sand. The present model was successful in fitting the measured trend.

At the fast transition condition, the entrainment rate  $K_i^*(=G_s^*)$

of Eq. (1a) was dominated by  $K_{io}^*$ . In Eq. (1c), the slope of the change in  $K_{io}^*$  because of increasing gas velocity is different.  $K_{io}^*$  increases with the gas velocity. However, the increasing slope of  $K_{io}^*$  decreases with increasing the gas velocity. That is consistent with the trend of experimental results [10,22]. Figs. 1 to 5 also show that the apparent exponent ( $n$ ) of the gas velocity ( $U_{FT}$ ) for  $G_s^*$  varies widely in relationship,  $G_s^* \propto U_{FT}^n$ . However, the exponent of the gas velocity in earlier correlations for  $G_s^*$  is fixed as shown in Table 1. That kind of difference as an example seems to make the present model favorable to fit the experimental data.

As a result, this model (Eq. (12)) is acceptable as a tool to predict the fast transition velocity and the saturation carrying capacity at room temperature and atmospheric pressure. We also realized that the effect of interparticle forces at the fast transition condition was negligible. Furthermore, the application of this model was claimed to be extendable for an approximation within the valid range of the correlation of Choi et al. [12].

## CONCLUSIONS

A model (Eq. (12)) based on correlations of Choi et al. [12] on particle entrainment rate was proposed successfully to predict the fast transition velocity and saturation carrying capacity in the riser of a circulating fluidized bed. The saturation carrying capacity of Bai and Kato [1] could be regarded as the particle entrainment rate at the fast transition condition. The effect of interparticle forces at the fast transition condition seemed to be negligible.

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## NOMENCLATURE

$a$	: decay constant [1/m]
$Ar$	: Archimedes number, $\rho_g d_p^3 (\rho_p - \rho_g) g / \mu^2$ [-]
$C_d$	: drag coefficient on the particle surface based on the superficial gas velocity [-]
$C_{ps}$	: particle size coefficient [-]
$d_{crit}$	: critical particle diameter, i.e. the maximum particle diameter at which the sum of interparticle adhesion forces influence dominantly in particle entrainment [m]
$d_{pm}$	: mean diameter of bed particles [ $\mu$ m]
$d_p$	: particle diameter [m]
$D_t$	: column diameter [m]
$F_d$	: drag force on the particle per projection area [Pa]
$F_g$	: gravity force minus buoyancy force per projection area of particle [Pa]
$Fr$	: Froude number, $U_{FT} / (gd_p)^{0.5}$ [-]
$g$	: gravitational acceleration, 9.8 [ $m/s^2$ ]
$G_s$	: solid flux [ $kg/m^2s$ ]
$G_s^*$	: saturation carrying capacity [ $kg/m^2s$ ]
$H_b$	: bed height [m]

- $H_t$  : column height [m]  
 $K_i^*$  : entrainment rate of particles in size  $i$  [ $\text{kg}/(\text{m}^2 \text{ s})$ ]  
 $K_i^{*+}$  : entrainment rate of particles in size  $i$ , corrected by  $C_{ps}$  [ $\text{kg}/(\text{m}^2 \text{ s})$ ]  
 $K_{th}^*$  : cluster flux of entrained particles in size  $i$  [ $\text{kg}/(\text{m}^2 \text{ s})$ ]  
 $K_{tco}^*$  : dispersed noncluster flux of entrained particles in size  $i$  or elutriation rate constant of particles in size  $i$  above transport disengaging height [ $\text{kg}/(\text{m}^2 \text{ s})$ ]  
 $n$  : exponent [-]  
 $Re_{FT}$  : particle Reynolds number at fast transition velocity,  $d_p U_{FT} \rho_g / \mu$  [-]  
 $Re_p$  : particle Reynolds number,  $d_p U \rho_g / \mu$  [-]  
 $U$  : superficial gas velocity [m/s]  
 $U_{FT}$  : fast transition velocity [m/s]  
 $U_{mf}$  : minimum fluidizing velocity of bed particle [m/s]  
 $U_t$  : terminal velocity of particle [m/s]

### Greeks

- $\alpha$  : exponent defined by Eq. (10) [-]  
 $\mu$  : gas viscosity [Pa s]  
 $\rho_g$  : gas density [ $\text{kg}/\text{m}^3$ ]  
 $\rho_p$  : particle density [ $\text{kg}/\text{m}^3$ ]

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