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Hydrodynamics of gas-liquid and biophase-gas-liquid systems in stirred tanks of different scales

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Abstract−This research involves the influence of tank scale and, additionally, stirrer speed, the volumetric gas flow rate, the sucrose concentration in aqueous solution, and the yeast suspension concentration on the hydrodynamics of gas-liquid and gas-biophase-liquid systems. A stirred tank with internal diameters of T=0.288 m, and T=0.634 m was filled with a liquid to the height H=T. For measurements, two high-speed stirrers were used: a Rushton turbine stirrer (RT) and A 315 stirrer. The study was carried out for gas-liquid and biophase-gas-liquid systems, where the biophase was a suspension of Saccharomyces cerevisiae yeast, the gas phase was air, and the liquid phase was an aqueous solution of sucrose. The gas hold-up and power consumption depend on the scale of the tank. The experimental results were mathematically described. Eqs. (14)-(18) do not have equivalents in the literature.

Keywords: Stirred Tank, Gas Hold-up, Power Consumption, Residence Time of Gas Bubbles, Gas-liquid or Biophase-gas-liquid

INTRODUCTION

The production of multiphase systems is aimed at obtaining the most homogeneous system possible and maintaining this state for the time required during a specific process [1-3]. It is achieved, among others, by supplying mechanical energy to the system using an appropriate stirrer. Due to the versatility of design solutions of tanks with stirrers, they are commonly used for mixing single-, two- and three-phase systems in various industries. The main criterion for maintaining the appropriate hydrodynamic state in the stirred tank is the proper selection of a single stirrer or, if necessary, the number and configuration of stirrers placed on a single shaft [4]. Despite the large variety of stirrers, one stirrer more commonly used in single and multiphase studies is the Rushton turbine stirrer [5-12]. The main advantage of this stirrer is the production of very good gas dispersion in the entire volume of the stirred tank. The disadvantages include high energy consumption and high shear stress, which are unfavorable, especially if one of the phases is a biophase. To reduce energy consumption and shear stress, studies have started to use stirrers that produce a radial-axial circulation of the liquid, which are very commonly a modification of the turbine stirrer (e.g., Smith turbine stirrer) or new stirrers characterized, for example, by a much larger surface area of the stirrer blades [13-19].

Some of the more critical hydrodynamic quantities used in describing the mixing of two- and three-phase systems include the gas hold-up φ or the residence time of gas bubbles t_R. Another fundamental issue in liquid mixing operations is calculating the power consumption to provide the assumed hydrodynamic conditions in the stirred tank. There are several entries in the literature describing the influence of various parameters on the hydrodynamics of multiphase systems in a stirred tank [20-30]. Parameters affecting, for example, the gas hold-up or the power consumption can be divided into i) geometric parameters of the stirred tank - tank diameter, the height of liquid, presence or absence of baffles, location of stirred tank shaft, etc., [31-37]; ii) geometric parameters of the stirrer: type of stirrer, number of stirrers on the shaft, the diameter of the stirrer, number of stirrer blades, inclination or curvature of the stirrer blades, etc. [5,9-11,32,36,38-57]; iii) operating parameters: gas flow rate, stirrer speed, etc.; iv) physical parameters: density and viscosity of individual phases, the concentration of individual phases, surface tension [32,44,58-65].

Most often, the studies analyzed the effects of geometric and operational parameters on hydrodynamic quantities in gas-liquid or gas-solid-liquid systems. The results of measurements with different stirrers were worked out in the form of different dimensional correlations, e.g., $\varphi=f(P_G/V_L, w_{og})$, $\varphi=f(P_G/(\rho_L V_L), w_{og})$, $\varphi=f(n, V_g)$, and dimensionless correlations, e.g., $\varphi=f(Kg, We)$ or $P_G/P_O=f(Kg, Fr)$ obtaining different values of exponents at particular quantities. The effect of superficial gas velocity, stirrer speed, and cell volume on the hydrodynamics of the gas-liquid system was studied by Newell and Grano [66]. They found that assuming a constant value for the average bubble size, the bubble velocity increases with increasing superficial gas velocity. With increasing power consumption calculated per unit mass P/M, the bubble velocity decreased linearly until a critical value of P/M was reached. Above this value, the bubble velocity decreased slightly. With increasing cell volume, assuming constant values of superficial gas velocity and power consumption calculated per unit mass, the bubble velocity increased. The effect of stirrer type on power consumption and mixing time in the stirred tank was studied by Cabaret et al. [15] and Foucault et al. [14]. Caba-

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ret et al. [15] performed the study in a tank with an inner diameter of T=0.215 m, filled with aqueous corn syrup solution to a height of H=T. For mixing, they used eight different standard or modified turbine or turbine-disc stirrers producing axial, axial-radial, or radial liquid circulation. They performed experiments in the low Reynolds number range (Re<450) and found that stirrers producing radial flow are characterized by significantly higher power consumption and longer mixing times than stirrers producing axial flow. Foucault et al. [14] studied the effect of power and mixing time in a tank equipped with a new type of stirrer: Deflo, Sevin, or a hybrid combining Deflo and Sevin stirrers. They made the measurements for 80% and 90% corn syrup and xanthan gum in a stirred tank with an internal diameter T=0.36 m. They found that in all cases analyzed, the power number, in terms of laminar and transitional flow, decreases significantly with increasing Reynolds number. Cudak [51] analyzed the influence of such operational parameters as volumetric flow rate, stirrer speed, concentration (expressed as mass fraction c) of aqueous sucrose solution, and type of stirrer on the power consumption and gas hold-up in a 0.2 m^3 tank. She used high-speed stirrers for mixing: standard Rushton turbine and those with modified blade shape: Smith turbine (CD 6) or A 315. The results were elaborated in the form of a relationship considering the influence of gas flow number Kg, Froude number Fr and concentration of aqueous sucrose solution on the relative power consumption P_G/P_O (where P_O - power consumption for liquid; P_G - power consumption for the gas-liquid system):

$$\frac{P_G}{P_O}=a+\frac{b}{1+j(1+c)Kg^d}+i(1+c)\sqrt{\frac{Kg}{Fr}} \tag{1}$$

The values of the coefficients a, b, c, e, and the exponent d in the equation were determined for each stirrer tested. Eq. (1) is valid in the following range of variables: Kg∈<0.01; 0.1>; Fr∈<0.1; 0.8>; x∈<0.01; 0.1>.

The effect of the gas flow number Kg, Weber number We, Morton number Mo and concentration c of aqueous sucrose solution on the gas hold-up was worked out by Cudak [51] as follows:

$$\varphi=a\cdot Kg^b\cdot We^i\cdot(1+d\cdot c)^e\cdot Mo^g \tag{2}$$

Eq. (2) is valid in the following range of variables: Kg∈<0.01; 0.1>; We∈<780; 4,530>; Mo∈<3.65×10^{-10}; 6.32×10^{-11}>; c∈<0.01; 0.1>.

Karcz et al. [44] and Adamiak and Karcz [3] analyzed the effects of geometrical parameters of the stirred tank and stirrer and physical properties of the liquid phase on the gas hold-up and the power consumption. They made measurements in stirred tanks with working volumes of 0.02 m^3, 0.04 m^3, 0.2 m^3, and 0.4 m^3. Various sets of two high-speed stirrers were mounted on the stirred tank shaft: Rushton turbine, Smith turbine, turbine with three or six blades inclined at 45 or 90°, propeller, A 315 or HE3. The experiments were performed for air-liquid systems (different liquids: distilled water, aqueous glucose solution of 30% by weight, and three aqueous starch syrup solutions of 40%, 60%, and 70% by weight). The first two physical systems are characterized by their ability to coalesce, while aqueous starch syrup solutions are systems with limited ability to coalesce. The effects of specific power consumption and superficial gas velocity on the gas hold-up were developed as equations:

- for systems capable of coalescence (water and aqueous glucose solution with mass fraction x=30%)

$$\varphi=(0.36-6.67\cdot10^{-3}\cdot x)\left(\frac{P_g}{V_L}\right)^{(0.32-2\cdot10^{-3}\cdot x)}w_{og}^{(0.8-10^{-2}\cdot x)} \tag{3}$$

in the range of 0<x [%]<30; P_G/V_L<900 W/m^3; $w_{og}\le5.2\times10^{-3}$ m/s

- for systems with limited ability to coalescence (aqueous starch syrup solutions with a mass fraction x=40%; 60% or 70%)

$$\varphi=(0.299\cdot x-5.8)\left(\frac{P_g}{V_L}\right)^{\left(0.07\cdot exp\left(\frac{0.65}{10^{-2}\cdot x}\right)\right)}w_{og}^{1.1} \tag{4}$$

in the range of 40<x [%]<70; P_G/V_L<400 W/m^3; $w_{og}\le5.2\times10^{-3}$ m/s

Far fewer studies investigated the effects of physical parameters on hydrodynamic quantities in gas-liquid systems [3,44,60,61,67]. Khare and Niranjan [67] analyzed the effect of the viscosity of a non-Newtonian liquid (1% CMC) on the gas hold-up based on measurements made in a Rushton turbine stirrer. They presented the results of the measurements in the form of an equation:

$$\varphi=0.0196\cdot n^{0.985}\cdot w_{og}^{0.274}\cdot\left(\frac{\eta_L}{\eta_w}\right)^{-0.131} \tag{5}$$

Zhang et al. [61] analyzed the effects of the viscosity of four liquids (water and sugar solutions of 25, 50, and 60%) and the stirrer speed, and the superficial gas velocity on the gas hold-up. They performed the study in a stirred tank with six Rushton turbine stirrers. They developed the results in the form of the relationship:

$$\varphi=0.31\cdot n^{0.7}\cdot w_{og}^{0.52}\cdot\left(\frac{\eta_L}{\eta_w}\right)^{-0.19} \tag{6}$$

Garcia-Ochoa and Gomez [60] studied the effects of physical parameters (density of both phases, surface tension, viscosity) and geometrical parameters (stirrer diameter), and operational parameters (stirrer speed, superficial gas velocity) on the gas hold-up. Based on the study conducted, they developed the equation:

$$\frac{\varphi}{1-\varphi}=0.819\cdot\frac{w_{og}^{2/3}\cdot n^{2/5}\cdot d^{4/15}}{g^{1/3}}\cdot\left(\frac{\rho_L}{\sigma}\right)^{1/5}\cdot\left(\frac{\rho_L}{\rho_L-\rho_G}\right)\cdot\left(\frac{\rho_L}{\rho_G}\right)^{-1/15}\cdot\left(\frac{\eta_L}{\eta_w}\right)^{-1/4} \tag{7}$$

In the case of gas-solid-liquid three-phase systems, many works can be found in the literature where the solid phase is an inanimate phase [68-73]. At the same time, there are very few works where a biophase such as yeast is present in the system in addition to the gas and liquid. In this case, the works are mainly concerned with power consumption, mixing time, and volumetric mass transfer coefficient. The modeling of the effect of superficial gas velocity on the flow hydrodynamics and mass transfer processes in the gas-liquid system in a bioreactor was done by Devi and Kumar [74]. Numerical simulation was performed in a bioreactor with two Rushton or CD 6 turbine stirrers. They found that the values of the dissipation rate increase with increasing superficial gas velocity. Significantly higher values of relative power consumption P_g/P_o were obtained for CD 6 stirred tank than for Rushton turbine stirrers. However, Devi and Kumar found no significant effect of stirrer type on the value of average volumetric mass transfer coefficient k_La. Xia et al. [75] modeled the flow dynamics in a bioreactor with different stirrer combinations. They compared the simulation results with experimental results obtained in similarly equipped bioreactors.

The biophase in all cases was a suspension of the yeast *Streptomyces avermitilis*. Measurements and numerical simulations were performed in bioreactors in which three high-speed stirrers were mounted on a standard shaft. Comparing the results of experimental studies and numerical simulations, they found that the most favorable conditions were provided by a system of three stirrers: two downward-pumping modified propeller stirrers and a turbine stirrer with six curved blades (down stirrer).

Most studies of the influence of different parameters on the hydrodynamics of two- and three-phase systems were performed in stirred tanks (bioreactors) of a specific volume. Detailed studies of hydrodynamic parameters in tanks of different volumes have been the subject of a few studies [13,31,32,37,38,40,67,69-71,76]. Vrábel et al. [13] analyzed the influence of selected parameters (stirrer type, stirrer speed, gas flow rate, among others) on mixing time and power in bioreactors of different scales. They found that mixing time decreases with increasing power consumption related to the unit mass P/M. The influence of unit power consumption P/M on mixing time t_m depends on the type of stirrer used. The authors observed more considerable differences in time for a Rushton turbine stirrer than for a system with a Scab stirrer. Assuming a constant value of power consumption related to unit mass P/M, the mixing time increases up to tenfold with increasing bioreactor scale. The effect of the apparatus scale on mixing time decreases significantly with increasing unit power consumption P/M. Dohi et al. [69-71] investigated the effect of stirrer type and tank scale on hydrodynamic parameters such as mixing time, gas hold-up, critical stirrer speed, and power consumption. They studied the mixing time, gas hold-up, and critical stirrer speed, for a gas-solid-liquid system, in a stirred tank with different diameters ranging from 0.2 m to 0.8 m, in which a set of three stirrers was mounted on the shaft (two four-pitched blade downflow disc turbines and Pfaudler type impeller). On the other hand, they additionally measured the power consumption for a system in which single Maxblend or Fullzone stirrers were mounted on the shaft. Glass beads and polymer particles were used as the solid phase. The solid concentration was in the range $X \in 0$-20%. Tap water, methanol, and glycerin solution were used as the liquid phase, and the gas phase was air. They found that increasing the gas flow rate in the stirred tank reduced the mixing intensity and resulted in higher stirrer speed rates required to obtain a homogeneous mixture. At a given stirrer speed, the power consumption for the stirred tank with the Maxblend stirrer was half that of the stirred tank with the Full zone stirrer. They found a more significant decrease in power consumption due to adding gas in the stirred tank with three stirrers on the shaft than in the system with Maxblend or Fullzone stirrers.

Khare and Niranjan [31,38,40,67] analyzed the effect of stirred tank diameter on the gas hold-up in gas-liquid systems. They used two highly viscous liquids: CMC and Castor oil. Based on tests performed in a stirred tank with two diameters of 0.3 and 0.6 m, they found that the gas hold-up strongly depends on the scale of the tank. The values of the gas hold-up in the stirred tank with a diameter of T=0.3 m are much smaller than those obtained for the stirred tank with a two-fold larger diameter.

Karcz and Siciarz [32] proposed a k_D coefficient considering the effect of the tank scale on the gas hold-up. Measurements for the air-water system were performed in stirred tanks with a working volume of 0.02 m^3, 0.04 m^3, 0.2 m^3, and 0.4 m^3. Various sets of two high-speed stirrers were mounted on the stirred tank shaft: Rushton turbine, Smith turbine, turbine with three or six blades inclined at 45 or 90°, propeller, A 315 or HE3. The results of measurements were developed in the form of a relationship:

$$\varphi = a_1(k_D)\cdot\left(\frac{P_G}{V_L}\right)^{a_2(k_D)}\cdot w_{og}^{a_3(k_D)} \tag{8}$$

where $k_D=\left(\frac{D_{0.634}}{D_{0.288}}\right)\in\langle 1;\, 2.2\rangle$; $a_1(k_D)=x_1\cdot k_D+x_2$; $a_2(k_D)=x_3\cdot k_D+x_4$; $a_3(k_D)=x_5\cdot k_D+x_6$; - scale impact functions.

An analogous equation, extended by the effect of sucrose aqueous solution concentration, was developed by Cudak [37]:

$$\varphi = a_1(k_D)\cdot\left(\frac{P_G}{V_L}\right)^{a_2(k_D)}\cdot w_{og}^{a_3(k_D)}\cdot(1+c)^{a_4(k_D)} \tag{9}$$

The effect of the tank scale on the gas hold-up φ was analyzed by testing air-water sucrose solution systems with concentration $c \in$ <0.01; 0,1> made in tanks with working volumes of 0.02 m^3 and 0.2 m^3. For mixing, she used high-speed stirrers: a standard Rushton turbine and a modified blade shape, A 315.

The study presented in this paper is aimed at determining the effect of the scale of the apparatus and, additionally, the stirrer speed n, the volumetric gas flow rate Q_{GV}, the sucrose concentration c in aqueous solution, and the yeast suspension concentration y_s on the hydrodynamics of gas-liquid and gas-biophase-liquid systems. The effect of the tank scale on the gas hold-up was analyzed based on measurements obtained in two stirred tanks differing by ten times the volume of liquid in the tank.

EXPERIMENTAL SETUP

The gas hold-up, the power consumption, and the average residence time were measured in stirred tanks with a liquid height of H=T and two internal diameters of T=0.288 m and D=0.634 m. The study was performed in tanks with liquid volumes of V_L= 0.02 m^3 and V_L=0.2 m^3. Four standard B=0.1T baffles were placed in each tank. Two high-speed stirrers differing in the type of circulation generated and the amount of shear stress produced were used for the measurements: a Rushton turbine (RT) or A 315. The A 315 stirrer, characterized by axial-radial fluid circulation, produces low shear stresses due to its large surface area and the shape of the stirrer blades, which is advantageous for biological systems. On the other hand, the Rushton turbine stirrer, with radial-axial fluid circulation and relatively high shear stresses, was chosen because of its wide application in many processes. Detailed parameters of the stirred tank and stirrers are shown in Fig. 1 and summarized in Tables 1 and 2.

The study was carried out for gas-liquid and biophase-gas-liquid systems, where the biophase was a suspension of *Saccharomyces cerevisiae* yeast, the gas phase was air, and the liquid phase was an aqueous solution of sucrose. The study was carried out for four aqueous solutions of sucrose, one concentration of yeast suspension, several volumetric gas flow rates, and several stirrer speeds for each series of measurements. The detailed scope of the con-

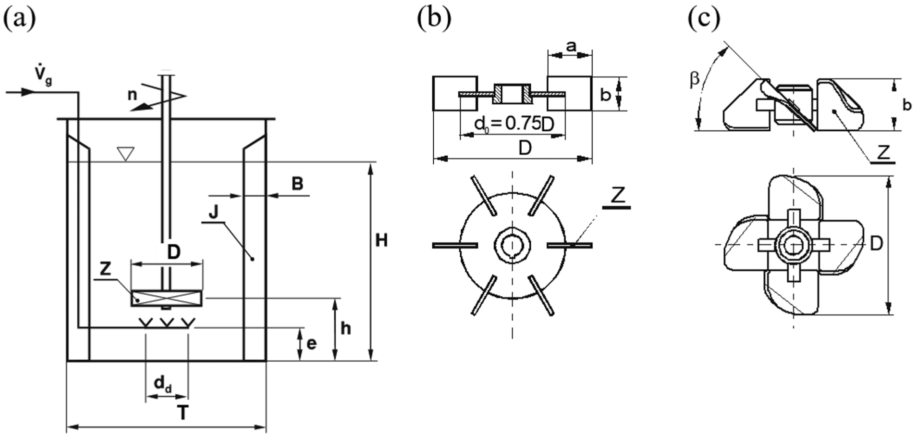

Fig. 1. Geometrical parameters of the: (a) stirred tank, (b) Rushton turbine stirrer (RT), (c) A 315 stirrer.

Table 1. Geometrical parameters of stirred tanks

No.	Geometrical parameters of stirred tanks	Parameter values
1.	Inner tank diameter	T=0.288 m; 0.634 m
2.	Liquid height in the tank	H=T
3.	Number of baffles	J=4
4.	Width of the baffle	B=0.1T
5.	Number of stirrers	i=1
6.	The distance of the stirrer from the bottom	h=0.33 H
7.	Gas sparger off-bottom clearance	e=0.5h
8.	Gas sparger diameter	d_d=0.7D

Table 2. Geometrical parameters of stirrers

No.	Stirrer	D/T	a/D	b/D	Z	β
1.	Rushton turbine (RT)	0.33	0.25	0.2	6	-
2.	A 315	0.33		0.34	4	45

ducted research is presented in Table 3.

The properties of the system changed in the following ranges: density ρ [kg/m^3]$\in$<1,000; 1,041>, surface tension σ [N/m]$\in$<0.072; 0.086>; dynamic viscosity coefficient of the liquid phase η_L [Pas]$\in$ <1×10^{-3}; 1.33×10^{-3}>; dynamic viscosity coefficient for the biophase-liquid system was calculated from the following equation:

$$\eta_{b-L}=\mathrm{K}\cdot\gamma^{m-1}=\mathrm{K}\cdot(\mathrm{B}\cdot\mathrm{n})^{m-1} \tag{10}$$

where the consistency constant K$\in$<0.0022; 0.012>; flow index m$\in$<0.71; 0.97>; B=11.5 (for Rushton turbine stirrer, RT); B=12.57 (for A 315 stirrer) [1].

The gas hold-up was calculated from the equation

$$\varphi=\frac{\mathrm{h}_{b-G-L}}{\mathrm{h}_{b-G-L}+\mathrm{H}} \tag{11}$$

where h_{b-G-L} is the difference between the height of the level of the gas-liquid (or biophase-gas-liquid mixture) and the height of the level of the liquid (or biophase-liquid mixture), m; H is the height of the liquid column (or biophase-liquid mixture), m. Each experimental point was determined as the mean of the ten values h_{b-G-L} (h_{G-L}) read from the scale located at the wall of the stirred tank. The power consumption was measured by the strain gauge method

Table 3. Range of the studies

No.	Range of the studies	T=0.288 m	T=0.634 m
1.	Sucrose concentration, c, %	1; 2.5; 5; 10	
2.	Concentration of yeast suspension, y_s, %	0; 1	
3.	Gas flow rate, V_g, m^3/s	<1.67×10^{-4}; 5×10^{-4}>	<5.56×10^{-4}; 2.78×10^{-3}>
4.	Volumetric gas flow rate, Q_{GV}, vvm (m^3/min)/m^3	<0.5; 1.5>	<0.16; 0.83>
5.	Stirrer speed, n 1/s	<7.33; 13.33>	<2.5; 6>
6.	Gas flow number, Kg	<0.014; 0.071>	<0.010; 0.099>
7.	Weber number, We	<590; 2,156>	<674; 4,571>
8.	Specific power consumption, P_G/V_L (W/m^3)	<238; 4,710>	<60; 2,088>

[77]. The strain gauge method uses the twist of the stirrer shaft due to the liquid's resistance during mixing. This deformation is proportional to the change in the resistance of the strain gauges caused by the change in the length of the wire. The deformation of the torsiometer is converted into changes in voltage. Next, these signals are amplified and recorded. The power consumption is calculated from the equation,

$$P = 2\pi M_u n = 2\pi k L_{mean} n \tag{12}$$

where L_{mean} - mean elongation of strain gauges; M_u - the torque, Nm; k - proportionality coefficient; n - stirrer speed, 1/s.

The average residence time t_R of gas bubbles in the system was calculated from the relation [78]

$$t_R = \frac{V_L \varphi}{V_G(1-\varphi)} \tag{13}$$

where: V_L - a volume of liquid (or biophase-liquid mixture), m^3; V_G - gas flow rate, m^3/s; φ - gas hold-up.

RESULTS AND DISCUSSION

The analysis of the influence of the scale of the tank k_D, stirrer speed n, volumetric gas flow rate Q_{GV}, type of stirrer, sucrose concentration *c* in aqueous solution, and yeast suspension concentration y_s on the gas hold-up φ and residence time t_R of gas bubbles in gas-liquid and gas-biophase-liquid systems was performed based on about 3000 measurement points.

Due to the different scales of the tanks, the range of gas flow rates V_G through the stirred tank and the stirrer speed n at which the measurements were made varied considerably. Measurements were made for nine gas flow rates through the stirred tank: four (V_g= 0.000167 m^3/s; 0.000278 m^3/s; 0.000389 m^3/s; 0.0005 m^3/s) for the stirred tank with diameter T=0.288 m and five (V_g=0.000556 m^3/s; 0.001111 m^3/s; 0.001667 m^3/s; 0.002222 m^3/s; 0.002778 m^3/s) for the stirred tank with diameter T=0.634 m. In turn, such stirrer speed at which good gas dispersion in the liquid was observed was taken as the smallest in both stirred tanks. On the other hand, the highest one was there, at which surface aeration of the liquid in the tank did not occur yet. To determine at what stirrer speed rates the values of the gas hold-up will be comparable, regardless of the scale of the tank, it is necessary to select a suitable scale-up criterion. According to Taterson [79], the peripheral velocity of the end of the stirrer blades, the specific power consumption, the conventional linear gas velocity, or the volumetric gas flow rate Q_{GV} can be selected as the scale-up criterion depending on the assumed parameters that should be maintained in both apparatuses. In this study, the volumetric gas flow rate Q_{GV} (the ratio of the volume flow rate of liquid through the stirred tank in m^3/min to the volume of liquid in the tank in m^3, vvm) and the specific power consumption P_G/V_L were chosen as a criterion for determining the impact of scale-up.

Based on the obtained results, assuming a constant value of the volumetric gas flow rate Q_{GV}, it was found that to obtain the same values of the gas hold-up φ in both stirred tanks, it is necessary to increase even more than two times the stirrer speed in the small tank, e.g., for T=0.634 m φ=4% was obtained for stirrer speed n= 4.5 1/s, while in the stirred tank with diameter T=0.288 m only at stirrer speed n≅10 1/s.

The relationship φ=f(n) for different systems is shown in Fig. 2. In all cases analyzed, regardless of the scale of the tank, the gas hold-up φ increases with increasing stirrer speed and with increasing values of the volumetric gas flow rate Q_{GV}. However, to varying degrees. A significantly greater effect of both the stirrer speed

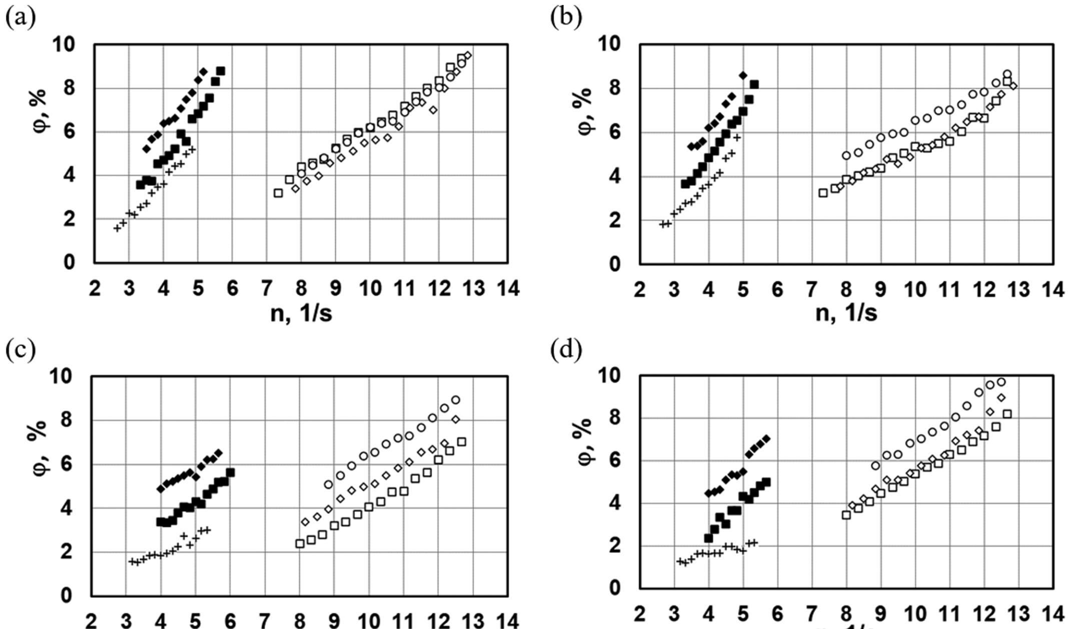

Fig. 2. The dependence φ=f(n); (a) RT; c=10%; y_s=0%; (b) RT; c=10%; y_s=1%; (c) A 315; c=10%; y_s=0%; (d) A 315; c=10%; y_s=1%; +, x, □ - T=0.288 m; ■, ◆, ▲, ● - T=0.634 m; + - Q_{GV}=0.17 vvm; ■, □ - Q_{GV}=0.5 vvm; ◆, ◇ - Q_{GV}=0.83 vvm; ● - Q_{GV}=1.5 vvm.

and the volumetric gas flow rate Q_{GV} on the gas hold-up φ was observed in a stirred tank with a diameter T=0.634 m than in a tank with a diameter T=0.288 m.

As the stirrer speed increases, the gas hold-up increases up to three times. The effect of stirrer speed on the gas hold-up depends on the other quantities (type of stirrer, volumetric gas flow rate Q_{GV}, sucrose concentration, yeast concentration, and tank scale). For a stirred tank with a Rushton turbine stirrer, a greater (up to three-fold) effect of stirrer speed on the gas hold-up was found. A slightly smaller effect of stirrer speed on the gas hold-up was obtained in the stirred tank with the A315 stirrer. In most cases, the effect of stirrer speed on the gas hold-up decreased with increasing value of the volumetric gas flow rate Q_{GV} (with increasing gas flow rate through the stirred tank), e.g. increasing the speed from n=9 1/s to n=12 1/s in a tank with diameter T=0.288 m for Q_{GV}= 0.5 $(m^3/min)/m^3$=0.5 vvm (which corresponds to $w_{og}=2.56\times10^{-3}$ m/s for T=0.288 m and $w_{og}=5.28\times10^{-3}$ m/s for T=0.634 m) causes an increase in the gas hold-up by a little more than two times, and for Q_{GV}=0.83 vvm (which corresponds to $w_{og}=4.27\times10^{-3}$ m/s for T=0.288 m and $w_{og}=8.80\times10^{-3}$ m/s for T=0.634 m) only 1.5 times; in the case of a stirred tank with diameter T=0.634 m, increasing the speed from n=4 1/s to n=6 1/s for Q_{GV}=0.5 vvm causes an increase in the gas hold-up by almost two times, and for Q_{GV}=0.83 vvm slightly less, i.e., about 1.4 times. The influence of the stirrer speed on the gas hold-up decreased when yeast was added to the system.

Increasing the volumetric gas flow rate Q_{GV} resulted in an increase in the gas hold-up φ from about 1.2 to 3 times depending, however, on the tank scale, the stirrer speed, the sucrose concentration, and the presence of yeast suspension in the system. The most significant effect of the volumetric gas flow rate Q_{GV} on the gas hold-up was found for a stirred tank with a diameter of T= 0.634 m. This influence decreased with increasing stirrer speed and increasing sucrose concentration in the two- and three-phase system. For example, for T=0.634 m and c=1% with an increase in volumetric gas flow rate Q_{GV} from 0.17 to 0.83, the gas hold-up increased by almost three times and for c=10% by only slightly more than 1.5 times. However, for T=0.288 m, the effect of the volumetric gas flow rate Q_{GV} (from 0.5 to 1.5) with increasing sucrose concentration in two- and three-phase systems was much smaller and was about 1.1 and 1.5 for c=1% and c=10%, respectively.

The effects of tank scale, stirrer speed n, volumetric gas flow rate Q_{GV}, the concentration of aqueous sucrose solution c, and relative viscosity on the gas hold-up for gas-biophase-liquid systems were developed as the relationship (equation developed using Statistica 13.3):

$$\varphi = x_1 \cdot n^{x_2} \cdot Q_{GX}^{x_3} \cdot (1+c)^{x_4} \cdot \left(\frac{\eta}{\eta_w}\right)^{x_5} \tag{14}$$

The functions x_1, x_2, x_3, x_4, x_5 in Eq. (14) are listed in Table 4.

An analogous equation for the gas-liquid system is presented by Cudak [37]. In this equation, the effect of viscosity on the gas hold-up was not considered due to the minimal differences in lightness between the measurement series.

No data in the literature compares the test results presented in this paper with those obtained for other gas-liquid biophase systems. Therefore, the results obtained for the Rushton turbine stirrer, developed separately for each stirred tank, in the form of the standard relationship φ=f(n, w_{og}, …)

Table 4. Functions x_1, x_2, x_3, x_4, x_5 in Eq. (14)

Stirrer	RT	A 315
x_1	$-1.686\cdot10^{-3}\cdot k_D+5.721\cdot10^{-3}$	$-4.574\cdot10^{-3}\cdot k_D+1.050\cdot10^{-2}$
x_2	$-0.309\cdot k_D+2.182$	$0.362\cdot k_D+1.108$
x_3	$-0.142\cdot k_D+0.4452$	$-0.334\cdot k_D+0.991$
x_4	$1.609\cdot k_D+0.695$	$-5.164\cdot k_D+9.588$
x_5	$-0.331\cdot k_D+0.240$	$0.768\cdot k_D-1.065$
±D	6%	7%

Range: $k_D=\left(\frac{D_{0.634}}{D_{0.288}}\right)\in\langle 1\div2.2\rangle$; $c\in\langle0.01;\ 0.1\rangle$;

for T=0.288 m: $n[1/s]\in\langle7.33;\ 13.33\rangle$; $Q_{GV}[vvm]\in\langle0.5;\ 1.5\rangle$

for T=0.634 m: $n[1/s]\in\langle2.5;\ 6\rangle$; $Q_{GV}[vvm]\in\langle0.16;\ 0.83\rangle$.

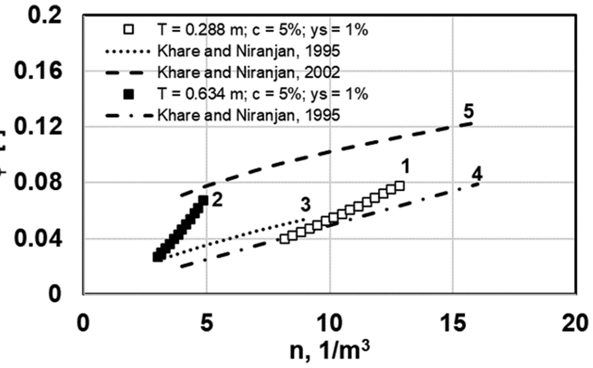

Fig. 3. The dependence φ=f(n); RT; w_{og}=0.006 m/s; 1 - T=0.288 m; air-5% aqueous solution of sucrose y-1% yeast suspension; 2 - T=0.634 m; powietrze-5% aqueous solution of sucrose −1% yeast suspension; 3 - T=0.6 m; air -1%CMC; 4 - T=0.3 m; air-1%CMC; 5 - T=0.3 m; air-Castor oil.

- for T=0.288 m

$$\varphi = 4.049\cdot10^{-3}\cdot n^{1.50}\cdot w_{og}^{0.13}\cdot(1+c)^{4.26}\cdot\left(\frac{\eta}{\eta_w}\right)^{-0.47} \tag{15}$$

- for T=0.634 m

$$\varphi = 1.603\cdot10^{-2}\cdot n^{1.87}\cdot w_{og}^{0.30}\cdot(1+c)^{2.28}\cdot\left(\frac{\eta}{\eta_w}\right)^{-0.10} \tag{16}$$

were compared with those available in the literature for gas-liquid systems and are shown in Fig. 3 (equations developed using Statistica 13.3).

The power consumption results obtained in both tanks were also analyzed in terms of the scale of the apparatus. The relationship P_G/V_L=f(n) is shown in Figs. 4-5. The effect of volumetric gas flow rate Q_{GV} on the specific power consumption was only observed for the stirred tank with the Rushton turbine stirrer. The specific power consumption increased with increasing stirrer speed n. However, increasing sucrose concentration in the system and adding yeast to the system slightly affected the value of unit power consumption. In the case of a stirred tank with a Rushton stirrer,

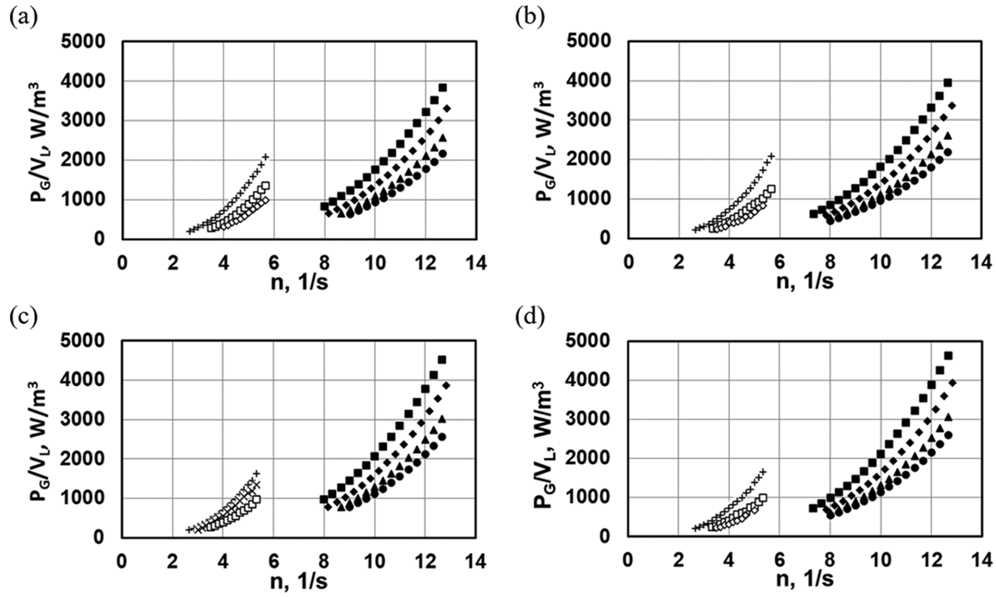

Fig. 4. The dependence P_G/V_L=f(n); RT; (a) c=1%; y_s=0%; (b) c=10%; y_s=0%; (c) c=1%; y_s=1%; (d) c=10%; y_s=1%; ■, ◆, ▲, ● - T=0.288 m; +, x, □ - T=0.634 m; + - Q_{GV}=0.17 vvm; x - Q_{GV}=0.33 vvm; ■, □ - Q_{GV}=0.5 vvm; ◆, ◇ - Q_{GV}=0.83 vvm; ▲ - Q_{GV}=1.17 vvm; ● - Q_{GV}=1.5 vvm.

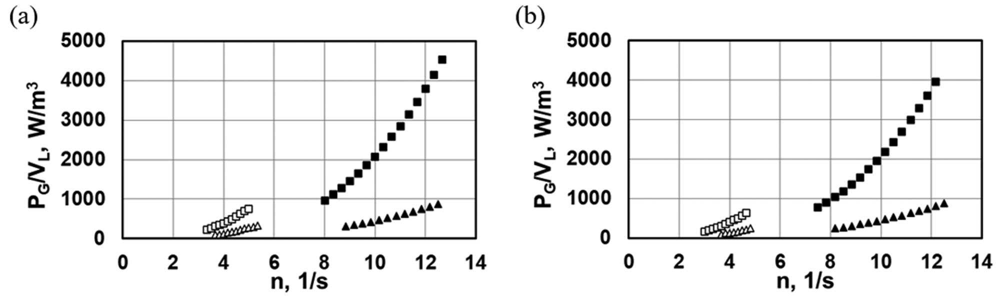

Fig. 5. The dependence P_g/V_L=f(n); Q_{GV}=0.5 vvm; (a) c=2.5%; y_s=1%; (b) c=5%; y_s=1%; full - T=0.288 m; empty - T=0.634 m; square - RT; triangle - A 315.

assuming Q_{GV}=const=0.5 vvm, it was found that the specific value of P_G/V_L in both stirred tanks was achieved at about 1.6 times higher stirred tank speed (gas-liquid system) and at about 1.5 times higher stirred speed (gas-biophase-liquid system) in a small tank.

A more significant effect of stirrer speed n on specific power consumption P_g/V_L was found in the tank with a Rushton turbine stirrer than in the tank with an A315 stirrer (Fig. 5). The effect of stirrer type on specific power consumption increased with increasing stirrer speed, e.g., for T=0.288 m at lower stirrer speed the values of specific power consumption were about 3.5 times higher for the stirred tank with Rushton stirrer compared to the values obtained for the stirred tank with A315 stirrer, and for higher stirrer speed this difference increased to five times. This influence decreased with an increase in the stirred tank diameter.

For tanks differing tenfold in liquid volume, the relationship φ=(P_G/V_L) is shown in Fig. 6. The gas hold-up increased with increasing specific power consumption. Assuming a constant value of P_G/V_L, the gas hold-up for a tank with diameter T=0.634 was about 1.5 times higher (for a stirred tank with an A315 stirrer) and 2.5-3 times higher (for a stirred tank with a Rushton turbine stirrer) compared to the values obtained for a tank with diameter T=0.288 m. On the other hand, to obtain an equal gas hold-up, assuming a constant value of the volumetric gas flow rate Q_{GV}, the unit power consumption in the small tank (T=0.288 m) was from 1.5 to even more

Table 5. Functions x_1, x_2, x_3, x_4, x_5 in Eq. (17)

Stirrer	RT	A 315
x_1	$-1.152\cdot10^{-3}\cdot k_D+5.268\cdot10^{-3}$	$4.952\cdot10^{-3}\cdot k_D+1.217\cdot10^{-2}$
x_2	$6.256\cdot10^{-3}\cdot k_D+0.386$	$0.187\cdot k_D+0.176$
x_3	$-0.176\cdot k_D+0.509$	$-0.222\cdot k_D+0.778$
x_4	$-1.490\cdot k_D+5.168$	$-0.472\cdot k_D+2.976$
x_5	$-10.826\cdot k_D+19.850$	$20.775\cdot k_D-32.219$
±D	11%	4%

Range: $k_D \in \langle 1 \div 2.2\rangle$; $c \in \langle 0.01; 0.1\rangle$; $y_s \in \langle 0; 0.01\rangle$

for T=0.288 m: $Q_{GV}[\text{vvm}] \in \langle 0.5; 1.5\rangle$; $\frac{P_G}{V_L}[\text{W/m}^3] \in \langle 0; 0.01\rangle$

for T=0.634 m: $Q_{GV}[\text{vvm}] \in \langle 0.16; 0.83\rangle$; $\frac{P_G}{V_L}[\text{W/m}^3] \in \langle 60; 2{,}088\rangle$.

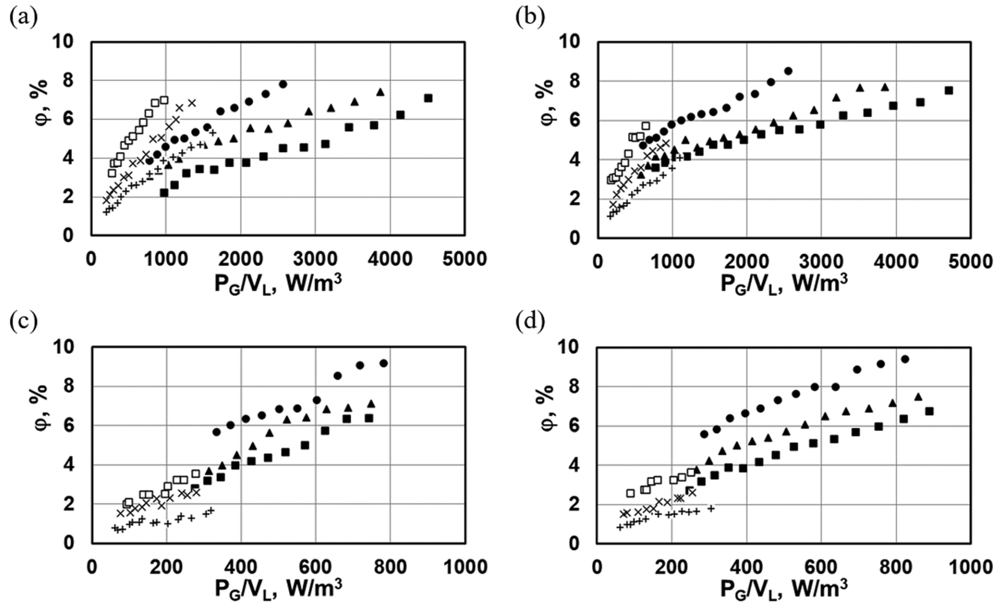

Fig. 6. The dependence φ=f(P_G/V_L); (a) RT, c=1%, y_s=1%; (b) RT, c=5%, y_s=1%; (c) A 315, c=1%, y_s=1%; (d) A 315, c=5%, y_s=1%; ■, ◆, ▲, ● - T=0.288 m; +, x, □ - T=0.634 m; + - Q_{GV}=0.17 vvm; x - Q_{GV}=0.33 vvm; ■, □ - Q_{GV}=0.5 vvm; ▲ - Q_{GV}=0.83 vvm; ● - Q_{GV}= 1.5 vvm.

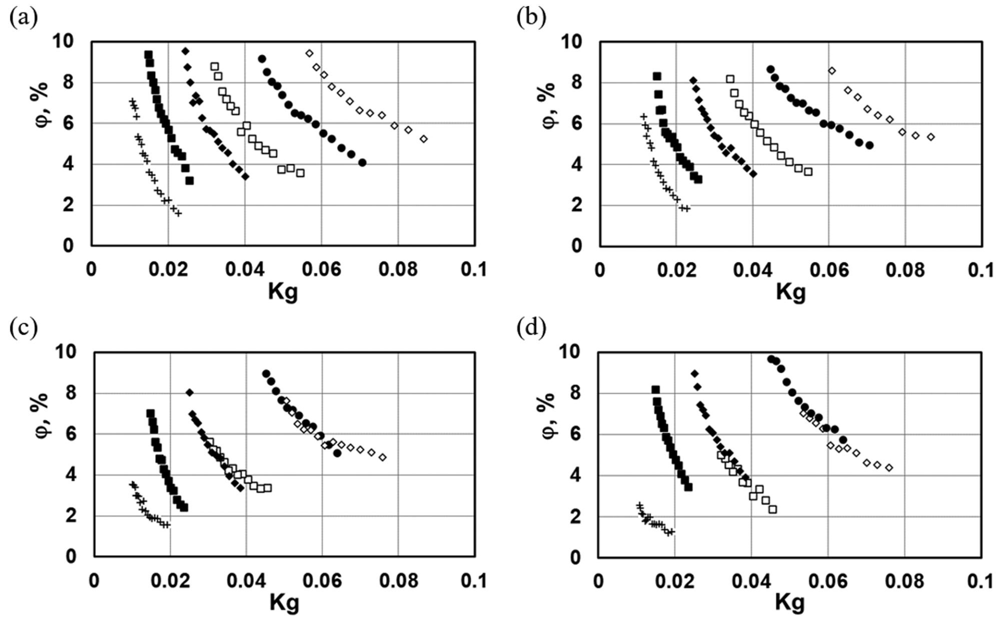

Fig. 7. The dependence φ=f(Kg); (a) RT, c=10%; y_s=0%; (b) RT, c=10%; y_s=1%; (c) A315, c=10%; y_s=0%; (d) A 315 c=10%; y_s=1%; ■, ◆, ▲, ● - T=0.288 m; +, x, □ - T=0.634 m; + - Q_{GV}=0.17 vvm; ■, □ - Q_{GV}=0.5 vvm; ◆, ◇ - Q_{GV}=0.83 vvm; ● - Q_{GV}=1.5 vvm.

than 4.5 times greater than in the large tank (T=0.634 m).

The effects of tank scale, specific power consumption P_G/V_L, volumetric gas flow rate Q_{GV}, the concentration of aqueous sucrose solution c, and concentration of yeast suspension y_s on the gas hold-up for a gas-liquid and gas-biophase-liquid systems were developed as the relationship (equation developed using Statistica 13.3):

$$\varphi = x_1 \cdot \left(\frac{P_G}{V_L}\right)^{x_2} \cdot Q_{GV}^{x_3} \cdot (1+c)^{x_4} \cdot (1+x_5 \cdot y_s) \tag{17}$$

The functions x_1, x_2, x_3, x_4, x_5 in Eq. (17) are listed in Table 5.

The relationship φ=f(Kg) for different systems is shown in Fig. 7. In all analyzed systems, the gas hold-up φ decreased with increasing gas flow number Kg. With an increase in the gas flow number, the gas hold-up decreased up to almost four times (for lower values of the volumetric gas flow rate Q_{GV}) and about 1.5 times for higher values of the volumetric gas flow rate Q_{GV}.

Comparable values of the gas hold-up obtained in two tanks of different scales can be obtained at different values of the gas flow

Table 6. Functions x_1, x_2, x_3, x_4, x_5 in Eq. (18)

Stirrer	RT	A 315
x_1	$4.192\cdot 10^{-5}\cdot k_D+4.914\cdot 10^{-5}$	$-1.068\cdot 10^{-4}\cdot k_D+3.145\cdot 10^{-4}$
x_2	$-0.181\cdot k_D+0.517$	$-0.226\cdot k_D+0.783$
x_3	$-0.030\cdot k_D+0.946$	$0.125\cdot k_D+0.746$
x_4	$-1.490\cdot k_D+5.168$	$-0.472\cdot k_D+2.976$
x_5	$-10.826\cdot k_D+19.850$	$20.775\cdot k_D-32.219$
±D	8%	8%

Range: $k_D \in \langle 1 \div 2.2\rangle$; $c \in \langle 0.01; 0.1\rangle$; $y_s \in \langle 0; 0.01\rangle$
for T=0.288 m: $Kg \in \langle 0.014; 0.071\rangle$; $We \in \langle 590; 2{,}156\rangle$;
for T=0.634 m: $Kg \in \langle 0.01; 0.099\rangle$; $We \in \langle 60; 2{,}088\rangle$.

number Kg. For a stirred tank with a diameter of T=0.634 m, assuming Q_{GV}=const, comparable values of the gas hold-up that were obtained in a tank with a diameter of T=0.288 m were obtained by increasing the gas flow number by more than two times in the stirred tank with the Rushton turbine stirrer, and almost two times in the stirred tank with A 315 stirrer.

Assuming constant values of Kg, the gas hold-up increased with increasing values of the volumetric gas flow rate Q_{GV}. A more significant increase in the gas hold-up (from 2 to 3 times), assuming constant values of Kg, with an increase in the volumetric gas flow rate Q_{GV} was obtained for a stirred tank with diameter D=0.634 m. For a stirred tank with a diameter of T=0.288 m, the influence was between 1.2 and 2 times. This influence depended on the applied stirrer.

The effects of tank scale, gas flow number, Kg, Weber number We, the concentration of aqueous sucrose solution c, and concentration of yeast suspension y_s on the gas hold-up for a gas-liquid and gas-biophase-liquid systems were developed as the relationship (equation developed using Statistica 13.3):

$$\varphi = x_1\cdot Kg^{x_2}\cdot We^{x_3}\cdot(1+c)^{x_4}\cdot(1+x_5\cdot y_s) \tag{18}$$

The functions x_1, x_2, x_3, x_4, x_5 in Eq. (18) are listed in Table 6.

The results of calculations of the residence time of gas bubbles in geometrically similar stirred tanks are compared in Fig. 8 in the form of the relationship $t_R=f(\varphi)$, and Fig. 9 in the form of the relationship $t_R=f(P_G/V_L)$. The residence time t_R of gas bubbles increased with increasing gas hold-up and decreased with increasing the volumetric gas flow rate Q_{GV}. The higher the value of the volumetric gas flow rate Q_{GV}, the smaller the effect of the gas hold-up on the residence time t_R of gas bubbles. Assuming a constant value of the volumetric gas flow rate Q_{GV}, the residence time t_R of gas bubbles did not depend on the tank scale.

The effect of the tank scale on the residence time t_R of gas bubbles was revealed when the residence time t_R of gas bubbles in both tanks was compared as a function of the specific power consumption P_G/V_L (Fig. 9). The residence time of the gas in the liquid increased with increasing unit power consumption, P_G/V_L. Assuming a constant value of the specific power consumption, 2.5 times (gas-liquid system) and 1.5 times (gas-biophase-liquid system) higher values of the residence time t_R of gas bubbles were obtained for the stirred tank with diameter T=0.634 m compared to the results obtained for the tank with diameter T=0.288 m.

For a stirred tank with diameter T=0.634 m, assuming Kg= const, increasing the concentration of aqueous sucrose solution from 1% wt. to 10% wt. caused an almost two-fold (for lower values of the volumetric gas flow rate Q_{GV} and stirred tank with Rushton stirrer) increase in the value of the gas hold-up. This effect decreased with the increasing value of the volumetric gas flow rate Q_{GV}. Assuming a constant value of Kg=const, higher values of the gas hold-up were obtained for the stirred tank with Rushton turbine stirrer

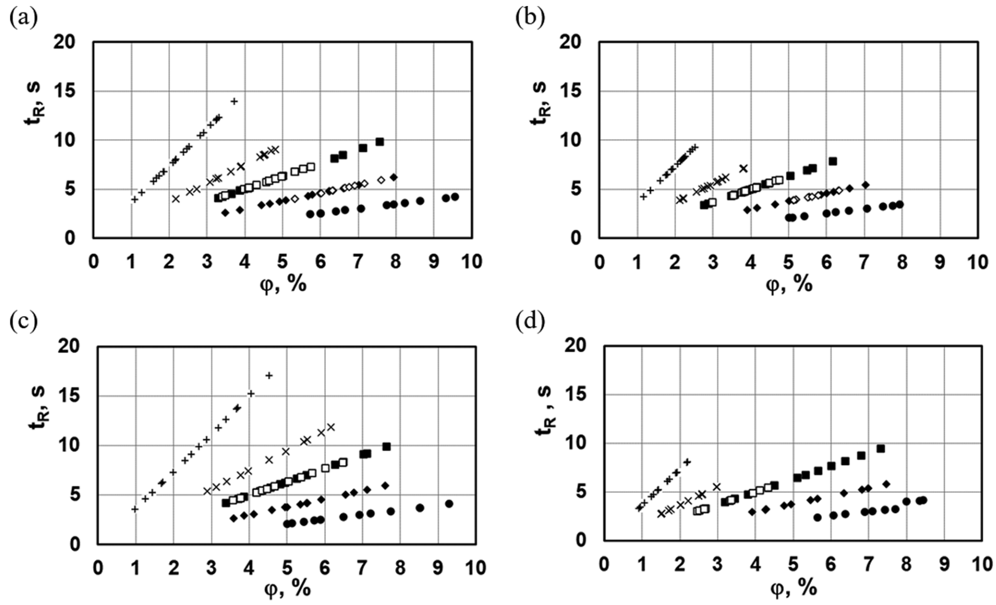

Fig. 8. The dependence $t_R=f(\varphi)$; (a) RT, c=2.5%; y_s=0%; (b) A315, c=2.5%; v=0%; (c) RT, c=2.5%; y_s=1%; (d) A 315, c=2.5%; y_s=1%; ■, ◆, ▲, ● - T=0.288 m; +, x, □ - T=0.634 m; + - Q_{GV}=0.17 vvm; x - Q_{GV}=0.33 vvm; ■, □ - Q_{GV}=0.5 vvm; ◆, ◇ - Q_{GV}=0.83 vvm; ● - Q_{GV}= 1.5 vvm.

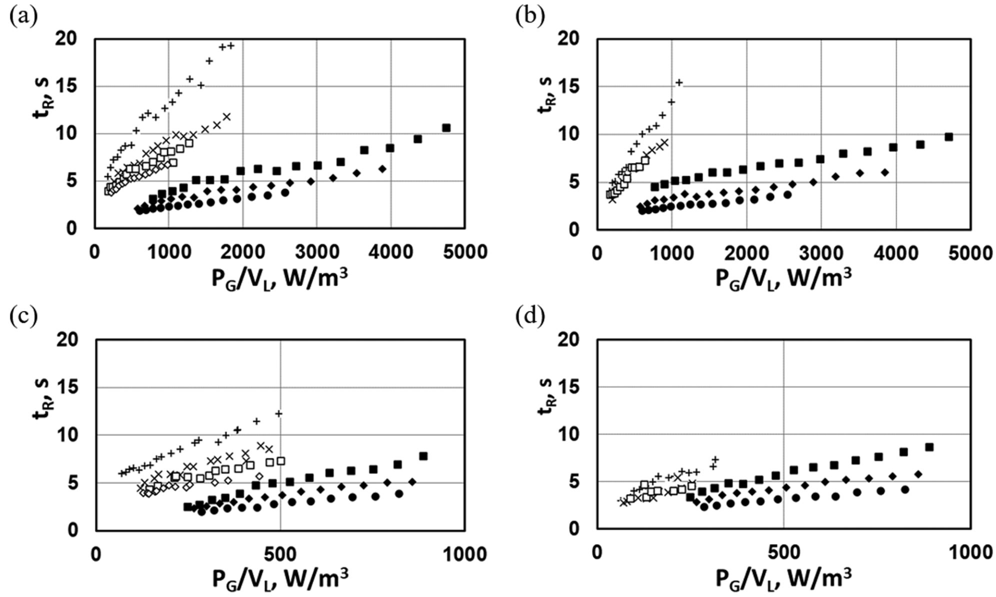

Fig. 9. The dependence t_R=f(P_G/V_L); (a) RT, c=5%; y_s=0%; (b) RT, c=5%; y_s=1%; (c) A315, c=5%; y_s=0%; (d) A 315 c=5%; y_s=1%; ■, ◆, ▲, ● - T=0.288 m; +, x, □ - T=0.634 m; + - Q_{GV}=0.17 vvm; x - Q_{GV}=0.33 vvm; ■, □ - Q_{GV}=0.5 vvm; ◆, ◇ - Q_{GV}=0.83 vvm; ● - Q_{GV}= 1.5 vvm.

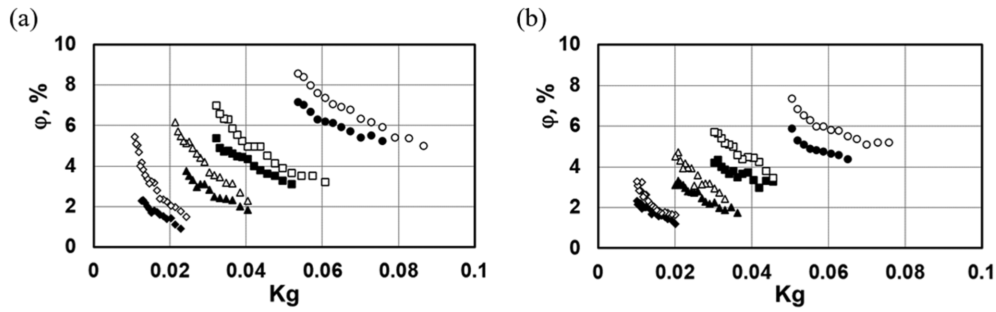

Fig. 10. The dependence φ=f(Kg); T=0.634 m; (a) RT, (b) A315; full - c=1%; y_s=0%; empty - c=5%; y_s=0%; ◆, ◇ - Q_{GV}=0.17 vvm; ▲, △ - Q_{GV}= 0.33 vvm; ■, □ - Q_{GV}=0.5 vvm; ●, ○ - Q_{GV}=0.83 vvm.

compared to the results obtained for the stirred tank with A 315 stirrer (Fig. 10).

CONCLUSIONS

A significant problem that arises in the design of stirred tanks is the answer to the question of to what extent test results, which are usually obtained in the laboratory on small measuring stations, can be used for design calculations of a large apparatus without the risk of error. The present work attempts to answer this question. For this purpose, about 3000 measurement points were made, based on which the influence of many parameters, including the scale of the tank on the gas hold-up, the power consumption, or the residence time t_R of gas bubbles, was determined. The selected range of scale change included two stirred tanks differing ten times in liquid volume. The values of the gas hold-up, the power consumption, or the residence time t_R of gas bubbles depend differently on the stirrer speed, the value of the volumetric gas flow rate Q_{GV}, and the concentration of sucrose c in the aqueous solution, the concentration of yeast suspension y_s.

Based on the experimental studies conducted, it was found that:

1. The gas hold-up φ depends on the scale of the tank. A comparable value of the gas hold-up φ in both stirred tanks was obtained by increasing the stirrer speed by more than two times in the tank with a diameter T=0.288 m compared to the stirred tank with a diameter T=0.634 m.

2. A significantly greater effect of both stirrer speed and volumetric gas flow rate Q_{GV} on the gas hold-up φ was observed in a stirred tank with diameter T=0.634 m than in a tank with diameter T=0.288 m.

3. As the stirrer speed increases, the gas hold-up increases up to three times. In most cases, the influence of the stirrer speed on the gas hold-up decreases with an increase in the volumetric gas flow rate Q_{GV}.

4. Increasing the volumetric gas flow rate Q_{GV} increases the gas hold-up by about 1.2 to 3 times. The most significant effect of volumetric gas flow rate Q_{GV} on the gas hold-up was found for the stirred tank with a diameter T=0.634 m. This influence decreased

with increasing stirrer speed and increasing sucrose concentration in the two- and three-phase system.

5. A comparable P_G/V_L value for both stirred tanks (T=0.288 m or T=0.634 m) is achieved when the speed in the small stirred tank (T=0.288 m) is 1.5 times higher compared to the large tank T=0.634 m (this applies to Rushton turbine stirrer).

6. The effect of stirrer type on specific power consumption increases with increasing stirrer speed. Comparing the results obtained for the tanks with diameter T=0.288 m or 0.634 m, it was found that the influence of the stirrer type on the specific power consumption significantly decreased with the increase in the stirred tank diameter.

7. The gas hold-up φ and the residence time t_R of gas bubbles increased with increasing specific power consumption. A more significant effect of specific power consumption, ranging from 1.5 to 3 times, on the gas hold-up φ was found for a stirred tank with a diameter T=0.634 m. This effect depends on the type of stirrer used.

8. The residence time t_R of gas bubbles increases with an increase in the gas hold-up φ and decreases with an increase in the volumetric gas flow rate Q_{GV}. The residence time of gas bubbles does not depend on the scale of the tank, assuming a constant value of the volumetric gas flow rate Q_{GV}, but it does depend on the scale of the tank assuming a constant value of the specific power consumption P_G/V_L.

9. Regardless of the tank scale, a greater impact of operating and physical parameters (stirrer speed, volumetric gas flow rate, sucrose concentration in aqueous solution, and yeast suspension concentration) on the values of hydrodynamic parameters (gas hold-up or power consumption) in gas-liquid and biophase-gas-liquid systems was observed for the Rushton stirrer.

10. Note that the A315 stirrer causes axial-radial fluid circulation. The geometrical construction of this stirrer is allowed to produce low shear stress (it is advantageous for biological systems). For constant values of operational and physical parameters (stirrer speed, volumetric gas flow rate, sucrose concentration in aqueous solution, and yeast suspension concentration), the obtained value of the mixing power is much lower than for the stirred tank with Rushton's turbine stirrer. However, it was found that assuming a constant value of the mixing power, the hydrodynamic conditions in the stirred tanks are much worse when the A 315 stirrer is mounted on the shaft compared to the Rushton turbine stirrer. Obtaining comparable hydrodynamic conditions for the A315 stirrer (to the conditions obtained for the stirred tank with Rushton's turbine stirrer), e.g., the gas hold-up, requires a significant increase in the stirrer speed and, consequently, also the mixing power. Due to hardware capability, it is often impossible to increase the stirrer's rotation speed significantly.

SYMBOLS

a : length of stirrer blade [m]
B : width of the baffle [m]
b : width of stirrer blade [m]
c : sucrose concentration [% mass.]
D : diameter of the stirrer [m]
d_d : sparger diameter [m]
e : off-bottom clearance of gas sparger [m]
H : liquid height in the stirred tank [m]
h : distance between stirrer and bottom of the stirred tank [m]
h_{g-c} : the height of a gas-liquid mixture in the stirred tank [m]
i : number of stirrers
J : number of baffles
n : stirrer speed [1/s]
P_G/V_L : specific power consumption [W/m^3]
Q_{GV} : volumetric gas flow rate [(m^3/min)/m^3=vvm]
T : inner diameter of the stirred tank [m]
V_L : volume of the liquid in the stirred tank [m^3]
V_G : gas flow rate [m^3/s]
w_{og} : superficial gas velocity, $=\frac{4\dot{V}_g}{\pi D^2}$ [m/s]
y_s : yeast concentration [% mass.]
Z : number of stirrer blades

Greek Symbols

β : pitch of the stirrer blade [deg]
η : dynamic viscosity of the liquid [Pas]
φ : gas hold-up
ν : kinematic viscosity of the liquid [m^2/s]
ρ : density of the liquid [kg/m^3]
ρ_{Leff} : effective density of the liquid [kg/m^3]
σ : surface tension [N/m]

Subscripts

b : refers to a biophase
G : refers to a gas phase
L : refers to a liquid phase
S : refers to a solid phase
W : refers to water

Dimensionless Numbers

$Kg=\frac{\dot{V}_g}{nD^3}$: gas flow number

$Fr=\frac{n^2D}{g}$: Froude number

$Fr_g=\frac{w_{og}^2}{Dg}$: Froude number

$Re=\frac{nD^2\rho_L}{\eta_L}$: Reynolds number

$We=\frac{n^2D^3\rho_L}{\sigma_L}$: Weber number

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