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EMPIRICAL CORRELATIONS FOR THE MINIMUM TRANSPORT VELOCITY OF MULTIDISPERSE SLURRIES IN HORIZONTAL PIPES

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Abstract−Empirical methods were developed for the prediction of minimum transport velocity (MTV) of settling slurries through horizontal pipes. Multidisperse slurries of different particle sizes and densities were included in the development of correlation. The correlation is based on the separation of the general particle volume dependence and the limiting MTV at zero volume fraction. The latter was calculated assuming a monodisperse slurry of the least readily transportable group of particles in a hypothetical fluid. For pipe diameter less than 50 mm and total volume fraction of solid less than 0.15 the prediction showed satisfactory agreements with experimental values.

INTRODUCTION

Transportation of settling slurries through horizontal pipes has been subject to extensive research works. Two important problems in the transportation are the correlation of the minimum transport velocity and of the pressure drop. Complexity of particle-fluid behavior makes it difficult to approach these problems theoretically. Recent publication includes basic research works on flow behavior and pressure drop [1-3], but no fundamental works on the minimum transport velocity are known to authors.

Empirical methods by Durand [4], Sinclair [5], Thomas [6] and Oroskar and Turian [7] are examples developed for the correlation of the minimum transport velocity (MTV) of uniform slurries. Mixed slurries are studied by Condolius and Chapus [8]. In mixed slurry correlation an average drag coefficient or an effective diameter is usually employed. No satisfactory methods seem to be available yet for uniform slurries or for mixtures due to the inconsistency of the data set.

Recently Kim et al. [9] formulated a correlation in which the MTV is expressed as a product of the MTV at infinite dilution and a particle volume effect term. Instead of using experimental particle diameter they used the value derived from the experimental terminal settling velocity and satisfactory agreements with experimental data were obtained for uniform slurries. This method was extended to bidisperse slurries by Cho et al. [10].

In this study we develop a correlation for the calculation of the MTV of mixed slurries based on Kim et al.'s approach. This method will be a revision and an extension of Cho et al.'s method.

DEVELOPMENT OF CORRELATION

1. MTV for Uniform Slurries

We summarize steps in developing correlation for MTV of uniform particle slurries as given by Kim et al. [9]. Assuming the dimensional analysis is applicable, the correlation for MTV is put into the form,

$$V_c = V_0(1 + aC^b) \quad (1)$$

where V_c is the MTV, V_0 is the MTV at infinite dilution, and C is the volume fraction of solid. They found empirically that a and b are independent of particle properties. V_0 is given as follows.

$$V_0 = (2C_D)^{0.5}U_0/k_0 \quad (2)$$

Here U_0 is the terminal velocity at infinite dilution, and C_D is the drag coefficient and k_0 is a parameter which is roughly equivalent to the turbulent intensity.

$$C_D = \frac{4gd(s-1)}{3U_0^2} \quad (3)$$

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$$s=\rho_s/\rho_f \quad (4)$$

Replacing C_D in eq. (3) by eq. (3) we have

$$V_0=[8g(s-1)d/3]^{0.5}/k_0 \quad (5)$$

Empirical correlations are available for C_D as a function of the particle Reynolds number, $R_{ep}(=dU_0\rho_f/\mu_f)$, and we use expressions given in Bird et al.'s book [11]. Combining eq. (3) with these correlations we have expressions for d.

$$d=[18U_0\mu_f/g(\rho_s-\rho_f)]^{0.5} \quad \text{for } Re_p<2 \quad (6)$$

$$d=[5.175U_0^{0.875}\rho_f^{0.25}\mu_f^{0.375}]/[g^{0.625}(\rho_s-\rho_f)^{0.625}] \quad \text{for } 500>Re_p>2 \quad (7)$$

Eq. (6) is valid for $Re_p<0.1$ and is an approximation for $0.1<Re_p<2$ [10]. We can use d determined by above eqs. rather than geometrical diameters in eq. (5) to calculate minimum transport velocities. Use of d from eqs. (6) and (7) were found to improve the agreements between experimental and calculated minimum transport velocity.

2. MTV for Multidisperse Slurries

Cho et al. [10] found that eq. (1) is applicable to bidisperse slurries. To calculate V_0 by eq. (5) we need appropriate values for ρ_s, d and k_0.

A simple method is to use some average values for ρ_s and d. We found following averages give good results.

$$\rho_s=\Sigma C_i\rho_{si} \quad (8)$$

$$d=\Sigma C_i\rho_{si}d_i^4/\Sigma C_i\rho_{si}d_i^3 \quad (9)$$

Another method is to assume a hypothetical fluid medium which consists of fluid and all particle groups except for the least readily transportable group. Then we can treat mutidisperse slurries as uniform slurries in hypothetical fluid. Density and viscosity of the hypothetical fluid which consists of fluid and particle groups $i=1, 2, \cdots, k-1, k+1, \cdots$ is assumed as follows.

$$\rho_h=[\Sigma_{i\neq k}C_i\rho_{si}+(1-\Sigma_iC_i)\rho_f]/(1-\Sigma_{i\neq k}C_i) \quad (10)$$

$$\mu_h=\mu_f \quad (11)$$

To find the diameter of the particle group k in the hypothetical media d_{kh} we follow the method by Selim et al. [12] for sedimentation of mutidisperse particles. In particular, they suggested the terminal velocity of the least readily settling particle group k in the presence of other particles as given bellow.

$$U_{okh}=U_{ok}C_{Fk} \quad (12)$$

where

$$C_{Fk}=(\rho_{sk}-\rho_h)/(\rho_{sk}-\rho_f) \quad \text{for } Re_p<2 \quad (13)$$

$$C_{Fk}=(\rho_{sk}/\rho_h)^{0.29}[(\rho_{sk}-\rho_h)/(\rho_{sk}-\rho_f)]^{0.71} \quad \text{for } 500<Re_p<2 \quad (14)$$

Therfore from eqs. (5) and (6) we have

$$d_{kh}=d_kC_{Fk}^{0.5} \quad \text{for } Re_p<2 \quad (15)$$

$$d_{kh}=d_kC_{Fk}^{0.875} \quad \text{for } 500<Re_p<2 \quad (16)$$

In most cases the least readily settling group is the least readily transportable group, that is, the group with the largest V_0.

k_0 for uniform slurries through horizontal pipes were studied by Ryu et al. [13] and $k_0=0.20$ were determined for 25 mm pipes. For the range of pipe diameter between 16 mm and 50 mm, the following equation is suggested based on experimental data.

$$k_0=0.40/[\{(D-15)/10\}^{0.33}+1] \quad (17)$$

For multidisperse slurries we propose the following correlation.

$$k_{0k}=k_0(1-\Sigma_{i\neq k}C_i)^{0.937} \quad (18)$$

In this equation the particle volume dependent factor was determined by requiring that V_0 for uniform slurries be equivalent to those of multidisperse slurries of identical particles.

EXPERIMENT

The experimental system is a loop composed of pipe test section, a pump for transportation and an agitated slurry feed tank. The pipe line is 2.54 cm in inside diameter and 14 m in length. One meter section of the pipe is a glass tube where the MTV was determined visually. The other part of the pipe is of PVC. The schematic diagram of the apparatus is shown in Fig. 1.

Solid particles were put in the feed tank and suspended before transportation. Slurry flow rate was adjusted by bypass line. By raising or lowering the flow rates the MTV was determined where the particles began to slide along the bottom of the pipe or to resuspend. The determination was reproducible within five percent error.

Solid materials were heavy sand and cast iron powder whose particle sizes range from 89 micron to 230 micron. The maximum volumetric fraction of solid particles was 0.15.

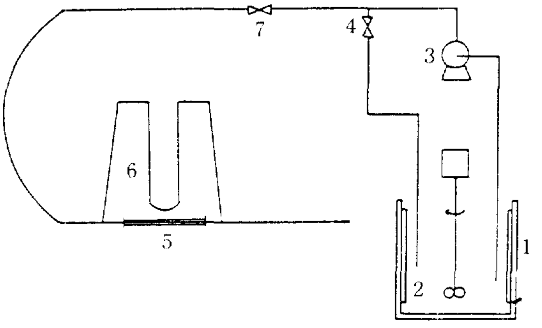

Fig. 1. Schematic diagram of apparatus for the measurement of minimum transport velocity.

1. Resevoir tank
2. Baffle
3. Centrifugal pump
4. Stop valve
5. Glass tube
6. Manometer
7. Ball valve

Physical properties of particle groups are listed in Table 1. Also shown in Table 1 are the measured terminal velocities. The terminal velocities were determined using a glass tube 1 meter in length and 2.54 cm in diameter from the measurement of hindered settling velocities by extrapolating to zero volume fraction.

RESULTS AND DISCUSSION

Mixed slurry calculations cannot be better than those for uniform slurries. There are wide discrepancies among various correlations so far published and comparison of a single set of data with calculated values by any one correlation method can easily lead to an error of more than fifty percent for the minimum transport velocity (MTV) of uniform slurries [14]. Correlation by Kim et al. which is the basis for the present mixture calculation is no exception. When compared with the data sets which are the basis of the correlation and are of different particle size, density and pipe diameter, the agreement is better than ten percent. Comparison with other data set [15] or other method of correlation, for example Durand's method [4] gives error of about twenty percent. We think that the pipe diameter dependent part of k_0 in eq. (17) is probably not reliable. We expect better correlation if this dependence is adjusted using experimental MTV data for each diameter.

We also observe for MTV of uniform slurries; (1) The MTV increases with solid volume fraction up to the value of 0.15 above which it tends to level off, (2) The particle size affects the MTV up to the diameter of about 0.35 mm above which the effect of particle size is negligible [8], (3) Numerical values of the constants in eq. (1) are a=2.745 and b=0.2180. These observations are assumed for mixed slurries.

In the development of correlation three methods are proposed for the prediction of the MTV. They are as follows;

1) use average solid density and particle diameter as defined by eqs. (8) and (9) and proceed as for uniform slurries,

2) use experimental particle diameter determined from particle size analysis and find V_{0k}, which is the maximum among V_{0i} (i=1, 2, ⋯),

3) or use the particle diameter from experimental free fall velocity and proceed as in 2).

Properties of the hypothetical fluid is rather peculiar in that the viscosity is that of fluid while the density is for the mixture. Such a choice is not inconsistent with the theory of sedimentation and probably reflects that there is relative motion between solids and fluid in the hypothetical fluid. In fluid with very fine particles or neutrally buoyant particles viscosities are known to be modified.

The results of calculation are summarized in Table

Table 1. Properties of particles

Material	Mean particle size (μm)	Sieve size (mesh)	Density (g/cm^3)	Terminal velocity exp'tl	Terminal velocity calc'd	Re_p
Heavy	230	−60/+70	2.634	2.44**	2.67**	4.85
sand	163	−80/+100	2.634	1.92	1.91	2.92
	137	−100/+120	2.634	1.37	1.56	1.71
	115	−120/+150*	2.634	.98	.99	1.02
	89	−150/+200*	2.634	.56	.56	0.46
Cast	194	−70/+80	5.233	4.84	4.56	8.35
iron	163	−80/+100	5.183	3.75	3.69	5.48
	137	−100/+120	5.235	3.10	2.98	3.80
	95	−140/+170	5.189	1.95	2.09	1.66

*U.S. Tyler sieves, Others ASTM; **units are in cm/sec

Table 2. Summary of comparison between calculated and experimental results

System	No. of sets	No. of points	Relative RMS deviation (%)		
			method[1]	method[2]	method[3]
Heavy sand-heavy sand	9	57	3.00	5.57	5.28
Heavy sand-cast iron	3	28	11.32	5.12	5.56
Tridisperse heavy sand	1	8	8.21	13.26	7.56

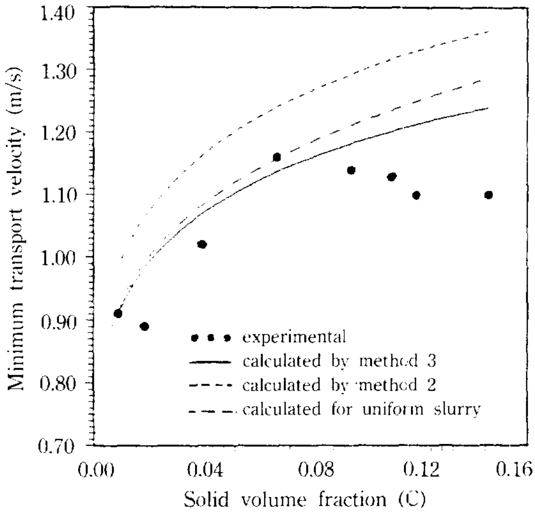

Fig. 2. Comparison of experimental data with calculated values for tridisperse heavy sand slurry (Volume ratio of −60/+70, −80/+100, −120/+140 groups is 1:2:1).

2. The table shows that all three methods give results which are not significantly different and that all three methods are acceptable. They give error percentages which are approximately same as those for uniform slurries. The method 3) gives best results and is particularly good when the density difference is large.

Comparison for a tridisperse system is shown in Fig. 2. In this case no density difference exist and method 1 and 3 give the similar results while method 2 is inferior. In this case, a large difference exists in particle diameters of the controlling group which are measured values in method 2 and are determined from terminal velocity in method 3. Also shown is the MTV curve for uniform slurry of the controlling group. Although not shown in the figure data points for uniform slurry closely follow the calculated curve.

Real multidisperse slurries are expected to have a continuous size distribution with possible density variation. In this case the results varies depending on how you discretize the distribution. We have done some model calculation to see this effect. Starting from 1:2:1 volume fraction ratio tridisperse mixture we divided the largest group into two groups keeping the total volume fraction and averge size constant. The difference turned out to be about one percent. This procedure was repeated once more and we obtained about one percent more average difference. Although these figures are only indicative, we may say that as long as we keep the size of largest particle group not too small the results are not very sensitive to how we discretize the distribution.

The method 1) is not quite good when the density difference is significant. The method 2) is generally inferior to method 3), which indicates that the diameter calculated from terminal velocity is somewhat better for the purpose. The difference between the calculated and experimental terminal velocity are as large as ten percent as shown in Table 1.

To apply the method 3), we need to determine the terminal velocity of each group of particles which constitute multidisperse slurries. This value is experimentally determined by extrapolation from finite volume fraction data. In practice we need not to determine U_0 for all groups of particles but for one or two groups which are likely to control the transport. When densities are nearly indentical, the group with the largest particle diameter is the controlling one. Following equation is generally accepted and may be used for extrapolation to find U_0 from measured V_c.

$$U_c = U_0(1-C)^{n-1} \quad (19)$$

Here C is the volume fraction of solids and n is an index which is determined experimentally.

It is difficult to compare the results with other investigators. For example Condolios and Chapus [8] proposed a mixing rule as

$$C_D^{0.5} = \Sigma w_i C_{D_i}^{0.5} \quad (20)$$

where w_i is the weight fraction of the ith group. They used this equation for head loss calculation. For present application mixing rules for other properties are also required. Of average diameters which may be generalized as

$$d = \Sigma C_i \rho_{si} d_i^N / \Sigma C_i \rho_{si} d_i^{N-1} \quad (21)$$

the best results were obtained with N=4. With this value eq. (21) reduces to eq. (9).

CONCLUSION

Three methods are proposed for the calculation of the minimum transport velocity of multidisperse settling slurries in horizontal pipes. The methods are based on a uniform slurry correlation for particle size up to 230 μm, pipe diameter up to 50 mm, and the maximum volume fraction of solid 0.15. In one method an average particle size was used while in the other two methods, concept of hypothetical fluid was assumed in which least transportable group of particles forms uniform slurry. In all three methods the agreements with experimental data are comparable with those of uniform slurry correlation. Recommendations are also discussed for the use of each method.

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NOMENCLATURE

a, b : regression constants in eq. (1)
C : volumetric fraction of settling slurry [−]
C_D : drag coefficient of a particle in quiescent fluid [−]
C_f : correction factor [−]
d : particle diameter [−]
D : pipe diameter [m]
g : gravitational constant [m/sec^2]
k_s : turbulent intensity factor at infinite dilution [−]
N : flow behavior index [−]
Re_p : particle Reynolds number [d U_∞ ρ_f/μ_f]
s : density ratio of particle to fluid [−]
U_∞ : terminal falling velocity of a particle [m/sec]
V_0 : V_c at infinite dilution of slurry [m/sec]
V_c : minimum transport velocity [m/sec]

Greek Letters

μ_f : viscosity of water [kg/m^3]
ρ_f : density of water [kg/m^3]
ρ_s : density of particle [kg/m^3]
ρ_h : density of hypothetical fluid [kg/m^3]
ρ : density of suspension [kg/m^3]

Subscripts

i : particle group i
k : particle group k which determines the transport condition
kh : property of particle group k in hypothetical fluid

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