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TOWARD THE SYNTHESIS OF GLOBAL OPTIMUM HEAT EXCHANGER NETWORKS UNDER MULTIPLE-PERIODS OF OPERATION

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Abstract—An algorithmic-evolutionary synthesis procedure is proposed for flexible heat exchanger networks (HEN) under multiple-periods of operation. After a feasible network is synthesized at each period, they are combined to form a feasible super network structure which requires maximum energy recovery (MER) at each period and features minimum number of units (MNU). Beginning with the initial feasible super network structure, all the super network structures can be enumerated to generate the minimum cost super network structure. The key steps in the procedure are constituted of must-matches searches at each period and path tracing/list processing constructions that allow not only combination of networks of each period but also development of super network structures adjacent to the initial super network structure in some sense, while keeping maximum energy recovery at each period and minimum number of units. Then a trade-off between MER and MNU is performed to strictly reduce objective function values. The constructions and procedures are rigorously established and effectiveness of the composite algorithm is demonstrated via several test problems. These tests show that the proposed approach can find the optimum networks for the known standard problems, and new MNU/MER networks are identified which to date have not been reported in the literature.

INTRODUCTION

Chemical plants must often operate at different conditions over different time periods. For instance, a plant may process different types of feedstocks in a finite sequence of time periods or operate at various capacity levels depending upon the season. Thus, for the heat exchanger network involved in the chemical plants, target temperatures of some streams for a given network structure are often not met because of these variations of operating conditions. Therefore it is an important design problem from the practical point of view to synthesize the super network structures that are flexible enough to cope with the prescribed changes in flow rates, the inlet and target temperatures. The standard synthesis problem of heat exchanger network under multiple-periods of operation can be stated as follows:

Given are n_c cold streams, initially at specified inlet temperatures T_{ci}, which are to be heated to specified target temperatures T_{tc} and n_h hot streams at specified inlet temperatures T_{hi} which are to be cooled to specified target temperatures T_{th}. Flowrates, inlet and target temperatures vary for these streams at N_p periods of operation. The problem is then to determine the super network structure of heat exchangers, together with additional heaters and coolers, if required, which will be feasible for the N_p periods of operation and accomplish this task while minimizing the cost of plant (investment), steam and cooling water (utility).

This problem was first explicitly dealt with by Floudas and Grossmann [1]. They presented a systematic procedure using a mathematical formulation consisting of a multi-period mixed integer linear programming transshipment model. The procedure guaranteed minimum utility cost for each period of operation and the overall fewest number of units. But it could not guarantee the minimum number of units for the super network structure obtained by dividing a pinched problem into two unpinched subproblems each of which is synthesized independently and combined manually. Moreover, the super network structure obtained may not be the globally optimum one because there are many super network structures featuring minimum utility consumption and the fewest number of units.

In solving resilient network problems, Marselle and Morari [5] identified the worst operating conditions and designed networks for each of them. In principle their approach is thus equivalent to synthesizing networks with multiple operating periods. They proposed designing an MNU/MER network for each worst condition, and then combining manually these network structures to obtain the composite super network structure. However, no systematic procedure was given for the combination of these networks which can have units which are quite different from each other. Moreover, there is no guarantee that their combined network will feature the minimum number of units for the multiple-period problem.

Kotjabasakis and Linnhoff [2] applied the Pinch Design Method to multiperiod operation problems. After determining the pinch location and energy consumption for each operating case, they conducted the pinch region designs for each period, looking for common structures. Then they used the Sensitivity Tables to minimize the total annualized cost of plant. However, since the Pinch technology could be used in a number of ways to obtain alternative efficient designs, they could not claim global optimality. Therefore, there is a clear need for better ways to find the optimum network under multiple-periods of operation.

REVIEW OF SINGLE PERIOD PROBLEMS

From previous works on the synthesis of HEN, which corresponds to the synthesis of heat exchanger networks under the single-period of operation, the MNU/MER network is a prerequisite for the optimum network with respect to the objective function of annual investment and utility costs. Thus the minimum cost network is one of the feasible MNU/MER networks. This fact can also be applied to the synthesis of HEN under multiple-periods of operation.

For single-period of operation, the minimum utility requirements and identification of the pinch points are first computed using conventional methods [3]. From these pre-analysis results, they synthesize a MNU/MER network from the tick-off algorithm [4] which is a sufficient condition for MNU networks. Then they generate an adjacent MNU/MER networks starting this synthesized network using the following Theorem.

For any new unit (element) to be introduced in the synthesis matrix, there must exist a unique path leading from some element in the row (column) of the new unit to some element in the column (row) of the new unit.

This network modification is the basis of combination of two individual networks which are not superimposable each other without requiring extra units.

To reduce the size of enumeration of adjacent networks, they define must-matches which are required on thermodynamic or topological grounds. By definition such matches cannot be eliminated from a network by introducing new units.

For a super network structure to be optimum, the following necessary conditions are considered to be satisfied:

1. Maximum energy recovery at each period of operation.

2. Minimum number of heat exchanger units.

If a super network structure satisfies both the conditions, then, as reported by Floudas and Grossmann [1], such a network will be very close to the optimal solution.

Next it will be shown how the single-period synthesis procedure can be extended for synthesizing super HEN's under multiple-periods of operation. The extension of the single-period pre-analysis (MER at each period) to multiple-period is straightforward. The minimum utility requirements are simply computed for each period separately and the pinch points at each period of operation are located.

The extension of MNU from single-period to multiple-period has not been thought to be easy because of the need of manual combination of network structures synthesized for each period of operation. Furthermore, the minimum number of units of a super network structure has not been clearly defined. Floudas and Grossmann [1], who used mathematical programming for synthesis of networks under multiple-periods of operation, claimed to synthesize super network structures with the *fewest* number of units. However, it turns out that these networks sometimes have more units than the minimum number of units, specifically, in case of pinched problems.

Therefore, to find the optimum super network structure we might proceed as follows:

1. Define the minimum number of units for the multiple-periods of operation.

2. Develop a systematic procedure to combine the feasible networks for each period, which does not require considerable efforts since in principle these networks can be easily modified to be superimposable upon one another. This possible superimposition becomes a basis for synthesizing an initial MNU/MER super network structure.

3. Enumerate all super network structures or evolve

the initial super network structure to find the optimum one among the MNU/MER super network structures, depending on the problem size.

4. Improve the super network structures by a trade-off between MER and MNU.

Multiple-period problems can thus again be attacked by the following four steps: pre-analysis, network invention, enumeration or evolution, and trade-off.

PRE-ANALYSIS

For a specified minimum temperature approach ΔT_m for heat exchange, the heating and cooling requirements and the pinch point T^* in the network are determined at each period as in a single-period problem. After the minimum number of units at each period is computed, the must-matches at each period are found to prevent generation of impossible super network structures [3]. It should be noted that the more shifts of pinch points, the more must-matches exist because every pinch point requires must-matches [3], resulting in the reduction in size of the enumeration problem. Then at each period, the following equation is formulated.

$$\mathrm{MNU} = \mathrm{NUMM}(\in \mathrm{U(must\text{-}matches)}) + \mathrm{NRM}(\in \mathrm{U(must\text{-}matches)}) \qquad (1)$$

where NUMM is the number of matches in the union of must-matches obtained from all the periods and NRM is the number of remaining matches.

The minimum number of heat exchanger units under multiple-periods of operation is first defined as follows.

$$N_{min} = N_{(must\text{-}matches)} + \max\ (\mathrm{NRM}) \qquad (2)$$

where N_{min} is the minimum number of units and $N_{(must\text{-}matches)}$ is the number of must-matches. This formula assumes that any two single-period networks can be modified with keeping the number of units unchanged so that the network with the smaller number of units can be totally superimposed on the other. This implies that the modified network with the smaller number of units is structurally a subset of the modified larger network. Thus it is better to start from the network of the period with the most remaining matches. As a property of super network structure, the network structure of each period should be a subnetwork of the super network structure.

In the relatively rare case in which networks cannot be superimposed even with network modification using this procedure, then we just combine the two networks. In this case, the minimum number of units will be increased by the number of units that cannot be superimposed. Thus, the number of units of such a super network structure is not guaranteed to be the real minimum number of units. However extra units which are not required, will be eliminated in the refinement step to reduce the number of units.

Floudas and Grossmann [1] reported the failure of separate heuristic synthesis for each period, and then manual combination into a final network. Without any modification of a network, a combination of networks derived for each period usually requires more units than the minimum number since any given network may not be superimposed fully on the others.

NETWORK SYNTHESIS

Using the must-matches, an initial feasible network structure is synthesized at each period by existing methods, including the methodology suggested in [3]. This structure features the maximum energy recovery and the minimum number of units. While keeping the number of units in the network constant, the network structure can be easily modified by introducing a new unit and manipulating heat loads. With this modification, an initial super network structure is synthesized as follows.

(1) Find all the must-matches at each period.

(2) First, construct a super network structure consisting of all of these must-matches.

(3) Synthesize an initial MNU/MER network for each period.

(4) Compute the number of remaining matches at each period from Eq. (1).

(5) Synthesize a feasible network for the period of the most remaining matches, using as many must-matches found in (1) as possible.

(6) Combine the network of (5) to the network of (2).

(7) Synthesize and superimpose the network for the period of the second most remaining matches on the network of (6). Repeat this procedure until the feasible networks of all the periods are superimposed.

ENUMERATION OR EVOLUTION

Since MER is always satisfied at each period, only the annual cost of heat exchange equipment needs to be considered in the objective function. Once an initial super network structure is synthesized, an enumeration step is needed to find all the feasible super network structures, so that eventually the optimum super network structure can be found. But, if the number of must-matches is not large, the evolutionary step of three phases is preferred [3].

Before enumeration, a refinement step is required to reduce the number of units in a network if the final combined super network structure does not have the minimum number of units. In this step, from the heat load redistribution, the units which are neither must-matches nor used for all the periods are tested to determine whether they can be removed without creating any new non-existent units.

If a super network structure has more units than the theoretical MNU, there always exist heat load loops [3]. For some units in those loops which are not used for all the periods, an optimization problem is formulated which serves to assign some heat loads during the period for which those units are never used.

For MNU networks, the number of new units which can be introduced is

$$N_{min} = (N_{source} - 1)(N_{sink} - 1) - \varepsilon \tag{3}$$

where N_{source} and N_{sink} are the number of source and sink streams, respectively, and ε is 0 but 1 if both steam and cooling water are required, for the match between steam and cooling water is nonsense. Moreover, if a unit which not only has the smallest heat load and but also is a must-match exists at the even position in the paths of a new unit, the new unit cannot be introduced [3]. These impossible matches reduces the number of new units which it is possible to introduce.

Since the number of units for multiple-periods of operation is usually larger than $(N_{streams} - 1)$, the number of new units to be introduced can be reduced further. Therefore, the search for all possible networks sometimes does not need much computation. This is why we use the enumeration method to guarantee the optimum network for multiple-periods of operation.

To attain the optimum network, we can use the following objective function for comparison.

$$\min F = \sum_{i=1}^{N_{unit}} f_{ai} + \sum_{k=1}^{N_p} \sigma^k (f_s^k + f_w^k) \tag{4}$$

where f_a is the cost of equipment unit, while f_s and f_w are the cost of stream and cooling water, respectively. The second term is constant because MER is guaranteed at each period.

EXAMPLES

1. Example 1

For the problem shown in Table 1, solved by Floudas and Grossmann [1], in which the flow rates and temperatures of two hot and two cold streams are varied in three periods of operation, must-matches are first found at each period [3].

Period 1: H=338.4, C=432.154, T*=239-249 and must-matches [3] are

- S-C2 (rule 1 in AP, only one cold stream C2 exists in AP.)
- H2-C2 (rule 1 in AP, only one cold stream C2 exists AP.) or (rule 10 in AP, streams H2 and C2 pass through the pinch point of 239-249 in AP, while $C_{H2} < C_{C2}$ (12.66<15.).)
- H1-C2 (rule 10 in BP, streams H1, H2 and C2 pass through the pinch temperature of 239-249 in BP. Match H2-C2 exists already. To satisfy pinch condition C2 must be split into two streams, one branch for H1 and the other for H2.)
- H1-W (rule 4, $T_{t_{H1}}$ (100) is lower than $T_{i_{C1}} + \Delta T_m$ (96+10) of the coldest stream.)
- MNU=5 (=6−1)

Period 2: H=1602.13, C=0., No Pinch and must-matches [3] are

- S-C2 (rule 2, $T_{t_{C2}}$ (270) is higher than $T_{i_{H2}} - \Delta T_m$ (239-10) of the hottest stream.)
- H2-C2 and H1-C2 (Both HC_{H1} and HC_{H2} are greater than HC_{C1} with no cooling requirement, thus matches H1-C2 and H2-C2 are unavoidable.)
- NMU=4 (=5−1)

Period 3: H=10., C=1793.15, T*=249-259 and must-

Table 1. Stream data for example 1 (3 Periods)

Period 1

Stream	T_i [c]	T_t [c]	c [kW/c]	HC [kW]
H1	249	100	10.550	1571.950
H2	259	128	12.660	1658.460
C1	96	170	9.144	676.656
C2	106	270	15.000	2460.000

Period 2

Stream	T_i [c]	T_t [c]	c [kW/c]	HC [kW]
H1	229	120	7.032	766.488
H2	239	148	8.440	768.040
C1	96	170	9.144	676.656
C2	106	270	15.000	2460.000

Period 3

Stream	T_i [c]	T_t [c]	c [kW/c]	HC [kW]
H1	249	100	10.550	1571.950
H2	259	128	12.660	1658.460
C1	116	150	6.096	207.264
C2	126	250	10.000	1240.000

$\Delta T_m = 10$ for all the periods

SN-1

	S	H2	H1
C2	$	$	$
C1	*		#
W	%	$	$

SN-2

	S	H2	H1
C2	$	$	$
C1		*	#
W	%	$	$

$: must match
%: infeasible match
*: impossible match
#: possible match

Fig. 1. Two possible super network structure matrices of example 1.

matches [3] are

- S-C2 (rule 1 in AP, only one cold stream C2 and only steam exist in AP.)
- H2-C2 or H2-C1 (rule 10 in BP, streams H1, H2 and C2 pass through the pinch point, while $C_{C2}<C_{C1}$ (10.<10.55) and $C_{C2}<C_{H2}$ (10.<12.66).)
- H1-W and H2-W (rule 8, the cooling requirement (1793.15) is greater than the largest heat content of hot stream H2 (1658.46).)
- NMU=5 (=6−1)

Note that the change of the pinch points and large variation of utility requirements result in many must-matches. Thus the super network structures should consist of five must-matches, S-C2, H2-C2, H1-C2, H1-W, and H2-W. Based on these must-matches, the remaining matches are computed at each period. Then NUMM and NRM are computed at each period.

Period 1: NUMM=4 and NRM=1
Period 2: NUMM=3 and NRM=1
Period 3: NUMM=4 and NRM=1

The minimum number of units of the super network structure will be 6 (=5+1). Since there is no match for C1, the H1-C1 or H2-C1 match should exist in the design. This results in only two possible super network structures for this problem as shown in Fig. 1. However, since stream H2 has to be used for the H2-C2 match under the pinch to feature MNU, only H1-C1 match is possible. The corresponding matches and the heat exchanged at each unit in each period of operation are shown in Table 2 for SN-1.

From the information in Table 2, the super network structure configuration is derived manually as shown in Fig. 2 [3]. Since MNU is 6, which is one more than theoretical MNU of single period network, a heat loop exists among H2-C2, H1-C2, H1-W, and H2-W matches. Thus heat loads can be reassigned among these specified matches, but super network structure of Fig. 2 turns out to be optimal after optimization technique (Box method) is applied on two variables of heat load of H2-W at period 1 and of H1-C2 at period 3.

Compared with Floudas and Grossmann's results [1], the above solution contains one less unit than

Table 2. Matches and heat exchanged of example 1

(unit: kW)

Unit	Match	Period 1	Period 2	Period 3
1	S-C2	338.400	1602.128*	10.000
2	H2-C2	1658.460*	768.040	1230.000
3	H1-C2	463.140*	89.832*	0.000
4	H1-C1	676.656	676.656*	207.264
5	H1-W	432.154	0.000	1364.686*
6	H2-W	0.000	0.000	428.460*

*: largest area for a given match

their network because they divided this problem into two unpinched subproblems at T*, synthesized them independently, and combined them manually. Furthermore their arbitrary values of the two heat loads variables are 354.794 and 200.0 instead of 0.0 and 0.0.

2. Example 2

For this problem shown in Table 3, where only flow rates of four hot and three cold streams are varied in three periods of operation, pre-analysis results and the must-matches at each period are obtained first.

Period 1: H=11., C=1531.96, T*=239-249 and must-matches [3] are

- S-C3 (rule 1 in AP, only one cold stream C3 exists in AP.)
- H4-C3 (rule 1 in AP or 10 in BP, only one cold stream C3 exists in AP or streams H2, H4 and C3 pass through the pinch point of 239-249 in BP, while $C_{H4}<C_{C3}$ (7.<15.). To satisfy the feasibility criterion at T* [3], stream C3 must be split for stream H4.)
- H2-C3 (rule 10 in BP, H2, H4 and C3 cross the pinch point in BP, while $C_{H2}<C_{C3}$ (10.55<15.). Stream C3 has to be split for stream H2.)
- H3-W (rule 5, $T_{t_{H3}}$ (106) is equal to $T_{t_{C1}}$ (96+10) of the coldest stream and C_{H3} is greater than C_{C1} (14.77>7.62).)
- MNU=9 (=9−1+1−0) (C3 must be split at T*)

Period 2: H=100.32, C=391.384, T*=217-227 and must-matches [3] are

- S-C3 (rule 1 in AP, only one cold stream C3 exists in AP.)
- H4-C3 and H2-C3 (rule 1 in AP or 10 in AP, only one cold stream C3 exists in AP or streams H2, H4 and C3 pass through the pinch point of 217-227 in AP, while $C_{H2}<C_{C3}$ (8.44<18.) and $C_{H4}<C_{C3}$ (7.<18.).)
- H3-C3 (rule 10 in BP, streams H2, H3, H4 C2 and C3 cross the pinch in BP. From the feasibility criterion at T*, stream C3 must split into three branches for H2, H4 (which already exist as must-matches in AP) and H3.)
- H3-C2 (rule 10 in BP, streams H2, H3, H4, C2 and

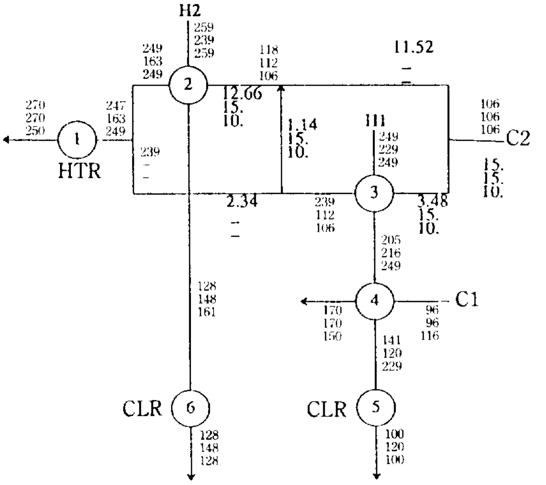

Fig. 2. A super network structure of example 1.

C3 pass through the pinch in BP. From must-matches of H2-C3,

- H3-C3 and H4-C3 which are already determined, the remaining split branch H3 must be matched with stream C2 in BP, while $C_{H3} > C_{C2}$ (11.816−2.56 >7.296).)
- H3-C1 (rule 9, from must-match H3-C2 and the sufficient condition for MNU, a hidden pinch appears at 150-160. From the above-the-hidden-pinch region, H3-C1 is required splitting of H3 for stream H1.)
- H1-C1 (rule 9, from must-match H3-C2 and the tick-off rule for MNU, a hidden pinch appears at 150-160. From the below-the-hidden-pinch region, H1-C1 becomes a must-match. Stream C1 must be split for stream H1.)
- H3-W (rule 5, $T_{t_{H3}}$ (106) is equal to $T_{t_{C1}} + \Delta T_m$ (96+10) of the coldest stream and C_{H3} is greater than C_{C1} (11.816>9.144).)
- MNU=10 (=9−1+3−1) (Stream H3 must be split into two streams and C3 into three streams at T*. Then H3-C3 can be matched perfectly. Later, C1 will be also split because of a hidden pinch point but H1-C1 will become a perfect match.)

Period 3: H=0., C=2925.86, No Pinch and must-matches [3] are

- H4-C3 (rule 3, $T_{t_{C3}}$ (250) is lower than $T_{i_{H4}} - \Delta T_m$ (271-10) but higher than $T_{i_{H2}} - \Delta T_m$ (249-10) of the second hottest stream H2.)
- H2-C3 (rule 9, from the match H4-C3 and the tick-off rule for MNU, ΔT_m violation occurs starting at 224.33-233.33, which becomes an incomplete must-match. Therefore, H2-C3 match is required with splitting C3 for streams H4 and H2.)

Table 3. Stream data for example 2 (3 Periods)

Stream	T_i [c]	T_t [c]	c [kW/c]	HC [kW]
H1	160	110	8.790	439.500
H2	249	138	10.550	1171.050
H3	227	106	14.770	1787.170
H4	271	146	7.000	875.000
C1	96	160	7.620	487.680
C2	116	217	6.080	614.080
C3	140	250	15.000	1650.000
H1	160	110	7.032	351.600
H2	249	138	8.440	936.840
H3	227	106	11.816	1429.936
H4	271	146	7.000	875.000
C1	96	160	9.144	585.216
C2	116	217	7.296	736.896
C3	140	250	18.000	1980.000
H1	160	110	10.548	527.400
H2	249	138	12.660	1405.260
H3	227	106	17.724	2144.604
H4	271	146	8.400	1050.000
C1	96	160	6.096	390.144
C2	116	217	4.864	491.264
C3	140	250	12.000	1320.000

$\Delta T_m = 10$ for all the periods

- H3-W (rule 5, $T_{t_{H3}}$ (106) is equal to $T_{t_{C1}} + \Delta T_m$ (96+10) of the coldest stream and C_{H3} is greater than C_{C1} (17.724>6.096).)
- H1-W (rule 5, $T_{t_{H1}}$ (110) is just above than $T_{t_{C1}} + \Delta T_m$ (96+10) of the coldest stream C1 and C_{H1} is much greater than C_{C1} (10.548>6.096). If streams H1 and C1 are matched, the maximum quantity of heat exchanged is only 57.77, violating the tick-off rule.)
- H2-W and H4-W (rule 8, $HC_{H3} + HC_{H1}$ (2144.604+527.4) is less than the minimum cooling requirement (2925.86). Thus another cooler is required.)
- NMU=8 (=8−1+1−0) (The H4-C3 must-match is not complete.)

Note that the pinch points are different in three periods and utility requirements change drastically, resulting in many must-matches. Thus super network structures are made up of ten must-matches, S-C3, H4-C3, H2-C3, H3-C3, H3-C2, H3-C1, H1-C1, H3-W, H2-W, and H1-W. Based on these must-matches, feasible networks are synthesized at each period. Then NUMM and NRM are computed at each period.

Period 1: NUMM=8 and NRM=1

Period 2: NUMM=9 and NRM=1

Period 3: NUMM=7 and NRM=1

Thus the minimum number of units of a super network structure will be 11. Only three networks are possible for this problem as shown in Fig. 3.

SN-1

	S	H4	H2	H3	H1
C3	$	$	$	$	*
C2	*		*	$	*
C1	*		*	$	$
W	%	#	$	$	$

SN-2

	S	H4	H2	H3	H1
C3	$	$	$	$	*
C2	*		*	$	*
C1	*	#	*	$	$
W	%		$	$	$

SN-3

	S	H4	H2	H3	H1
C3	$	$	$	$	*
C2	*	#	*	$	*
C1	*		*	$	$
W	%		$	$	$

$: must match
*: impossible match
%: infeasible match
#: possible match

Fig. 3. Three possible super network structure matrices of example 2.

Table 4. Matches and heat exchanged of example 2

(unit: kW)

Unit	Match	Period 1	Period 2	Period 3
1	S-C3	11.00	100.320	0.000
2	H4-C3	847.00	847.000	660.000
3	H2-C3	792.00	835.560	660.000
4	H3-C3	0.00	197.120	0.000
5	H3-C2	614.08	736.896	491.264
6	H3-C1	487.68	233.616	390.144
7	H1-C1	0.00	351.600	0.000
8	H4-W	28.00	28.000	390.000
9	H2-W	379.05	101.280	745.260
10	H3-W	685.41	262.120	1263.196
11	H1-W	439.50	0.000	527.400

For SN-1, the corresponding matches and the amount of heat exchanged at each unit in each period of operation are shown in Table 4. From the information in Table 4, a configuration of the super heat exchanger network is derived manually as shown in Fig. 4. Since MNU is 11, which is three more than the theoretical MNU, there are 6 heat load loops. Thus heat loads in these loops can be reassigned among the specified matches as done in example 1.

Compared with Flodas and Grossmann's results [1], this super network structure has three less units than their network because they synthesized two unpinched subnetworks independently after dividing this pinched problem at T*. Their values of H, C, and T* at the second period are 231.36, 347.424, and 140-150 instead of 100.32, 391.384, and 217-227.

There may be some computational errors in their work, because the heat balance equation between H and C at the second period is not satisfied. These errors eliminated the must-matches, H3-C2 and H1-C1, from their network.

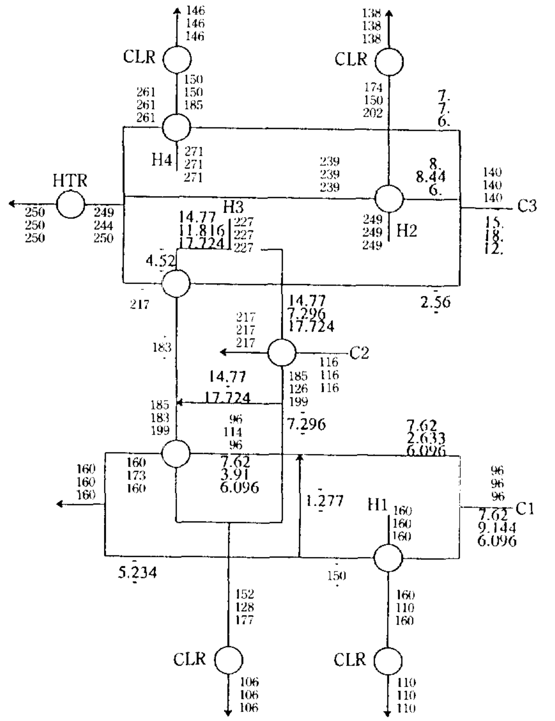

Fig. 4. A super network structure of example 2.

TRADE-OFF BETWEEN MER AND MNU

After a super network structure is synthesized, we can reduce the objective function value further by a trade-off between MER and MNU. This network improvement can be approached by three evolutionary methods. First of all, in the multiple-periods of operation problems, heat exchanger units are installed over the whole periods, while utilities are only consumed for certain periods. Depending on the weight factor σ of Eq. (4), the maximum energy recovery of a certain period *k* can be sacrificed to reduce the objective function value by saving equipment cost. Especially, a heat exchanger unit in the network is not used for all the operating periods, this unit can be eliminated to reduce the equipment cost while increasing the utility cost. To ensure this elimination, the following equation from Eq. (4) is checked.

$$\sum_{k=1}^{N_p} \sigma^k (f_s^k + f_u^k) - f_u < 0 \qquad (5)$$

From this equation, the reduction of objective function value is realized if the eliminating units is only for fewer periods of operation. It should also be noted

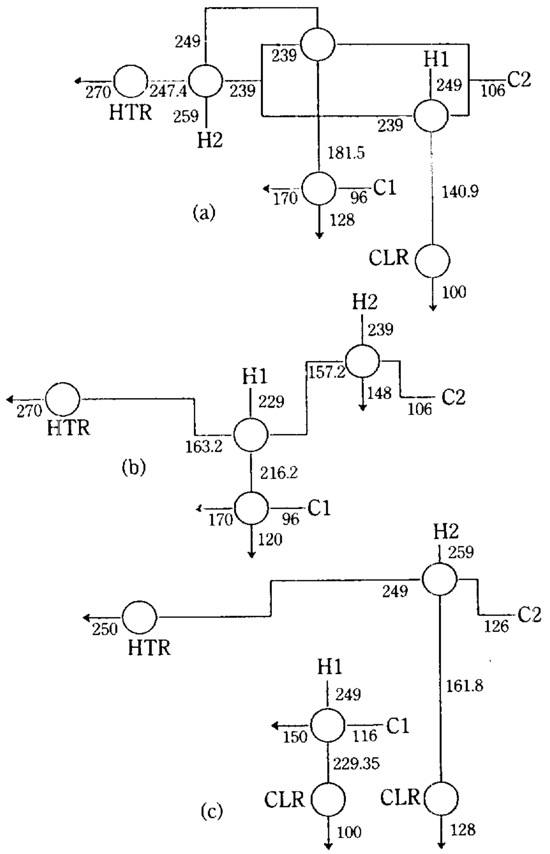

Fig. 5. Individual network structure of each period.

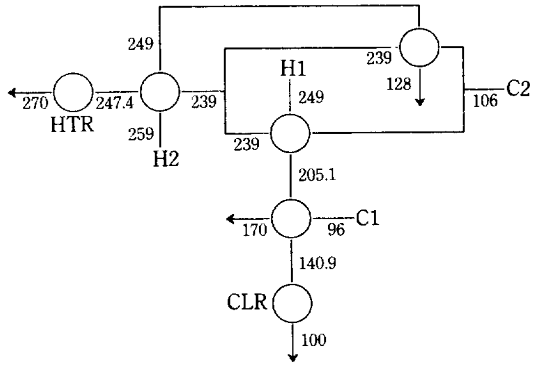

Fig. 6. A modified network structure for period 1.

that the elimination results in heat load increments of both heating and cooling requirements, as computed using heat load path by Linhoff and Hindmarsh [4].

Before describing the other methods, we first define the determining unit as follows.

1. Definition

1-1. Determining unit

A unit is said to be determining if its area is fully utilized among the multiple operating periods. That period can be called a determining period. The areas of a super network structure consist of those of the determining units [2].

Another way to reduce the objective function value is to redistribute heat loads in heat load loops (HLL). A super network structure usually has a heat load loop because networks synthesized at each period are not always superimposable.

The areas of units with larger Decision Index [3] can be reduced by redistributing their heat loads. However, the concurrent area increments of the other units in the same HLL does not affect the objective function if they are not determining units.

A third way to improve a super network structure is to relax MER at a certain period to reduce the area of the determining unit of that period. The heat loads along the heat load path among the heater, cooler and determining unit are redistributed as shown by Linnhoff and Hindmarsh [4].

In summary, the procedure consists of the following steps for a synthesized super network structure:

(1) Identify the unit in the heat load path which is used fewest and not checked before.

(2) Check the network improvement by deleting it with increases of heating and cooling loads. If the super network structure is improved, go to step (1).

(3) If the super network structure has a HLL, find the optimum redistribution of heat loads in the HLL to improve the super network structure.

(4) For all determining units, check the network improvement by relaxing MER at the determining period using heat load path in order to reduce those areas until they are not determining. The following illustration demonstrates how to improve a synthesized network from a trade-off between MER and MNU.

1-2. Illustration

We revisit Example 1 to improve the network by trade-off between MER and MNU. Floudas and Grossmann [1] manually derived network configuration for each period separately as shown in Fig. 5 (a), (b) and (c) and combined them only to obtain a network with eight matches. But, if the network of Fig. 5 (a) is modified as shown in Fig. 6 and then combined with others to the network of Fig. 7 (called SNS-1), the number of units is only seven, as they solved using the MILP method. Or, if the networks of Fig. 5 (b) and (c) are modified to those of Fig. 8 (a) and (b), respectively, the combined network (called SNS-2) has also seven

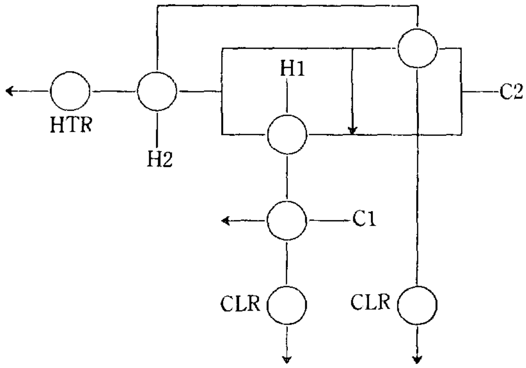

Fig. 7. A combined super network structure.

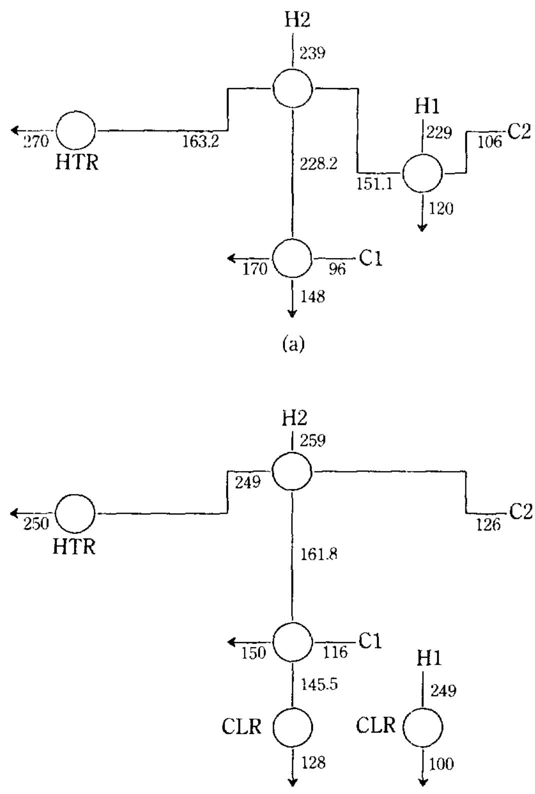

Fig. 8. Modified network structures for period 2 and 3.

units. Table 5 shows comparison of these networks with other networks published so far.

For the network of Fig. 7 whose objective function is 352,200 \$/yr (refer to Ref. 3 for the calculation basis), we can apply the trade-off procedures. First of all, since the H2-C2 match in AP is used only for period 1, it is eliminated. By redistributing heat loads in the heat load path of S-C2, H2-C2 (AP) and H2

Table 5. Comparison among networks as given in literature

Item	Ref. (1)	Ref. (2)	SNS-1	SNS-2	SN-1
Area [m^2]					
S-C2	28.45	28.6	28.5	28.5	28.5
H2-C2(AP)	11.765	11.8	11.8	11.8	0.0
H1-C2	20.15	66.4	11.9	57.2	11.9
H2-C2(BP)	123.44	23.7	100.6	60.8	165.8
H2-C1	54.84	27.2	0.0	33.8	0.0
H1-C1	0.0	0.0	20.0	0.0	20.0
H1-W	26.7	22.6	29.4	31.8	29.4
H2-W	18.67	24.6	13.6	7.6	13.6
ΣA [m^2]	284.02	204.9	215.8	231.5	269.2
N_{unit}	7	7	7	7	6
f [\$/yr]	402,300	356,200	352,200	369,300	357,200
f_a [\$/yr]	269,400	223,300	219,300	236,400	224,300
f_s [\$/yr]	94,700	94,700	94,700	94,700	94,700
f_u [\$/yr]	38,200	38,200	38,200	38,200	38,200

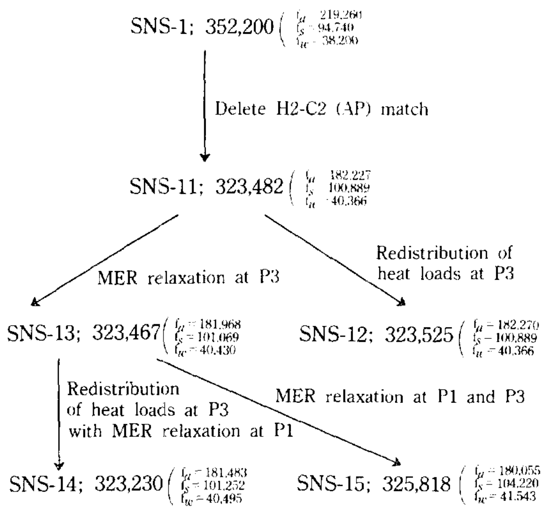

Fig. 9. Applied improvement procedure for illustration.

-W, we can reduce the objective function value to 323,482. Since the largest Decision Index value occurs at the H2-C2 match of period 3 among the determining units, we first redistribute heat loads of the heat load loop of H2-C2, H2-W, H1-W and H1-C2 at period 3 until the unit of H2-C2 match of period 1 becomes determining. In this case, we obtain a network with higher objective function value of 323,525. Thus we relax MER at period 3 by determining the optimal redistribution of heat loads in the heat load path of S-C2, H2-C2 and H2-W. Then we can improve the network whose objective function is 323,467. Now that the unit of H2-C2 match is determining for both period 1 and 3, we can redistribute the heat loads of the heat load loop of H2-C2, H2-W, H1-W and H1-C2 at period 3 while relaxing MER at period 1. Then the objective function value becomes 323,230. Even though we can relax MER's at both period 1 and 3 to improve the network, we find the network with lower objective function value of 325,818. This results from double increments of utility cost at period 1 and 3 with an decrement of area of the H2-C2 match. Then no further improved network can be found. The improvement procedure for this illustration is summarized in Fig. 9.

SUMMARY AND DISCUSSION

A systematic procedure has been proposed for the heat exchanger network synthesis with multiple-periods of operation. Must-matches are first found and a feasible network is synthesized at each period. The union of must-matches forms the basis of super network structures. The networks synthesized at each period are modified to be superimposable on each other by redistributing heat loads. Then these networks are combined to form a feasible super network structure, which insures the maximum energy recovery at each period and can feature the minimum number of units. Finally, three evolutionary procedure are applied for a trade-off between MER and MNU.

No theoretical guarantee of minimum number of units or optimality can be provided. However, for two example problems, networks with fewer number of units or lower objective function value than those reported in the literature are found.

NOMENCLATURE

C : cooling requirement
c : heat capacity flow rate
F : objective function
f : objective function value
H : heating requirement
HC : heat content of stream
N : number
n : number
S : steam
T : temperature
T* : pinch point
ΔT_m : minimum allowable temperature approach
W : cooling water
σ : weight factor

Subscripts

a : area
c : cold stream
h : hot stream
i : inlet condition
min : minimum
o : outlet condition
P : period
s : steam
t : target condition
w : cooling water

Superscript

k : period number

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