

## CONTROL OF A REACTOR-FLASHER SYSTEM

Jong Ku Lee and Sun Won Park

Department of Chemical Engineering, KAIST, P. O. Box 150, Cheongryang, Seoul, Korea

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**Abstract**—This paper presents a case study in which several multivariable control strategies were tested for a reactor-flasher system of an industrial chemical process. This reactor-flasher system which has three manipulated variables and three controlled variables is open loop unstable. Since the system variables interact severely, controlling the system is very difficult with the traditional PID control. We examined various control strategies such as multiloop single variable control, modified single variable control with compensators, and PI control combined with Linear Quadratic Regulator (LQR), Linear Quadratic Gaussian(LQG)/Loop Transfer Recovery(LTR) and Dynamic Matrix Control (DMC) combined with LQR. DMC combined with LQR showed better control performance than the others while remaining robust in the face of modeling errors.

### INTRODUCTION

Multivariable control has been studied over the last two decades. However, single variable control with cascade and feedforward control is predominant in the process industries. Though it is simple to be implemented, there are many processes which are difficult to control using it.

A reactor-flasher system is shown in Figure 1. The flow rates of the feeds to the reactor are almost constant most of the time. The reactor product goes to the flasher where the vapor leaves the overhead to a subsequent distillation unit. The liquid accumulates in the flasher base and is recycled back to the reactor. In this paper we call this stream recycle 1 and the stream which is fed to the reactor from the distillation unit is recycle 2. The reactor pressure is controlled by the gas feed rate and it is not important in this study.

We focus our attention on the control of the reactor temperature and the liquid levels in the reactor and the flasher. To control the levels, the flow rates of the product and the recycles are manipulated. The reaction is exothermic and the heat generated in the reactor is removed primarily by product flashing. The temperatures of recycles 1 and 2 are much lower than that of the reactor. Therefore, changes of the recycle flow rates interact with the reactor temperature and in addition this system is open loop unstable.

In this study, we find the best control strategy by comparing the control performance of five control strat-

egies : multiloop single variable control, modified single variable control with compensators, PI control combined with Linear Quadratic Regulator(LQR), Linear Quadratic Gaussian(LQG)/Loop Transfer Recovery (LTR), and Dynamic Matrix Control(DMC) combined with LQR[10]. Here, we propose DMC combined with LQR as a new method to handle open loop unstable systems.

### MATHEMATICAL MODEL OF THE REACTOR-FLASHER PROCESS

The model equations of the reactor-flasher system are comprised of three equations : the mass balances of the reactor and flasher, the energy balance of the reactor. By simplifying the balance equations, we can get the mathematical model of the reactor-flasher system as follows[13] :

$$\frac{dx}{dt} = Ax + Bu \quad (1)$$

$$y_m = Cx + Du \quad (2)$$

$$\text{where } x = [x_1 \ x_2 \ x_3]^T$$

$$u = [u_1 \ u_2 \ u_3]^T$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2.2 \\ 0 & 0 & 13 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 1 & 1 \\ 0.67 & -1 & 0 \\ 0 & -0.66 & -1.6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



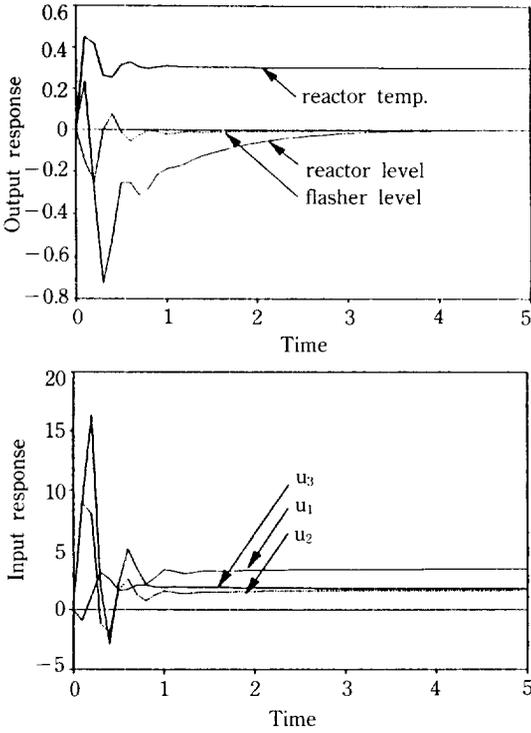


Fig. 4. The output and input responses of the reactor-flasher system in multiloop single variable control.

We consider this control structure because the tuning parameters(controller gains, reset time) of PI controllers can be easily determined. Figure 8 shows the control structure of PI combined with LQR for the reactor-flasher system. In this figure, we include a state estimator to predict the states of the system. If C is an identity matrix, we do not have to use the state estimator.  $y$  denotes the selected controlled variables among the measured output variables, and the matrix E is introduced to select the controlled variables( $\bar{y} = E y_m$ ). In the case of the reactor-flasher process, E is an identity matrix. Since the open loop unstable system is stabilized by LQR, we can have less trouble determining the six tuning parameters. The final control structure is similar to LQR with integral action. Figure 9 shows configuration of PI combined with LQR for the reactor-flasher system. The signal into the final control element is the summation of the signals which come out from PI controllers and those from LQR. The gain matrix that is calculated from LQR is as follows:

$$K = \begin{bmatrix} -3.806 & 0.731 & -4.031 \\ -1.030 & -4.339 & -6.437 \\ -2.109 & 0.799 & -18.45 \end{bmatrix}$$

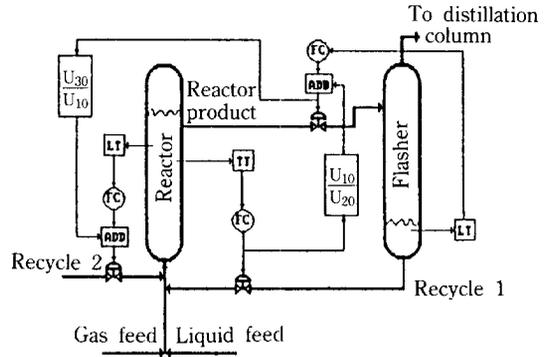


Fig. 5. The process configuration of modified single variable control with compensators.

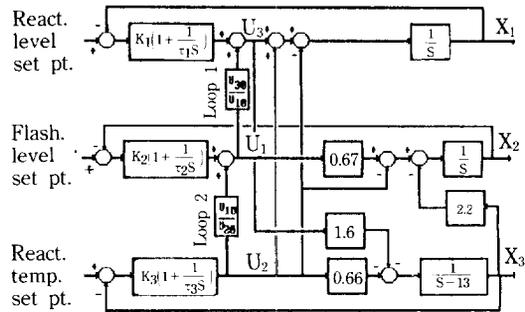


Fig. 6. The block diagram of modified single variable control with compensators.

The above gain matrix ( $K = (1/\rho)B^T P$ ) is calculated by solving the algebraic Riccati equation( $A^T P + PA + Q - (1/\rho)PB B^T P = 0$ ), where the matrix Q is an identity matrix and  $\rho$  is 0.05 for the cost function,  $J = \int_0^\infty (x^T Q x + \rho u^T u) dt$ . The PI controller parameters are  $K_1 = -8$ ,  $K_2 = -9$ ,  $K_3 = -11$ , and  $\tau_1 = \tau_2 = \tau_3 = 1$ . Figure 10 shows the output and input responses. The output response is better and manipulated variable changes are smaller than those of the previous two methods.

4. LQG-LTR

In early 1980's Doyle and Stein [11] developed the LQG/LTR as an eminent design method of linear multivariable control systems. The theory of LQG/LTR was developed from LQG optimal control theory. LQG/LTR and LQR have the same control structure as model based compensators. Only the design of controller parameters(control gain matrix and filter gain matrix) is different; LQG are designed in the respect of minimizing the least square errors, and LQG/LTR in the respect of loop formation. Those theories are well described in many literatures, so here we omit the detailed explanation.

Fig. 11 shows the block diagram of LQG/LTR. The

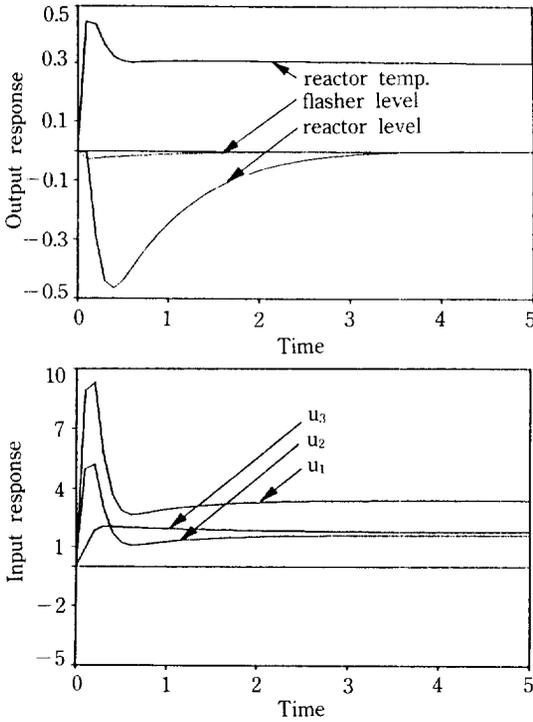


Fig. 7. The output and input responses of the reactor-flash-er system in modified single variable control with compensators.

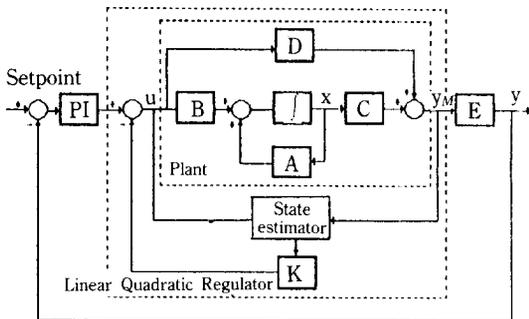


Fig. 8. The block diagram of PI control combined with LQR.

control structure of LQR/LTR in the reactor-flasher system is similar as the following DMC combined with LQR structure. Here, we have to design the filter gain matrix(H) and control gain matrix(G). First, we add integral elements into the model equation to remove the steady state off-set. Next, we use the design method of Kalman filter to get the filter gain matrix(H) by solving Ricatti equation. Then, we can use LTR method to get the control gain matrix(G). The gain

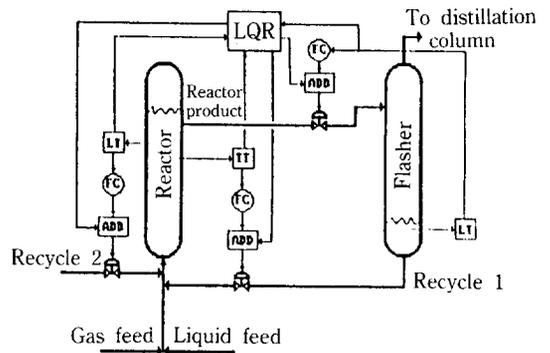


Fig. 9. The process configuration of PI control combined with LQR.

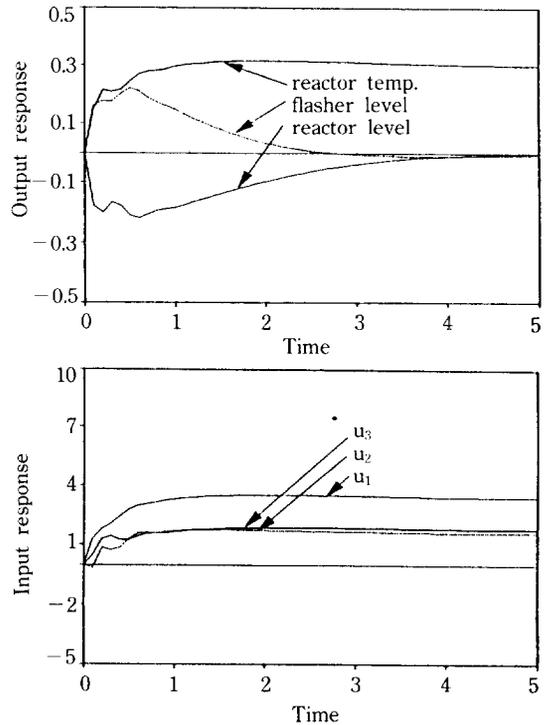


Fig. 10. The output and input responses of the reactor-flash-er system in PI control combined with LQR.

matrices H and G that are solved in this way are as follows:

$$H = \begin{bmatrix} -158.1 & -422.6 & -710.4 \\ -68.1 & -693.4 & -1039.2 \\ 64.1 & 234.2 & 187.8 \\ 142.5 & -0.2 & -0.7 \\ -0.2 & 144.3 & 1.0 \\ -0.7 & 1.0 & 157.7 \end{bmatrix}$$

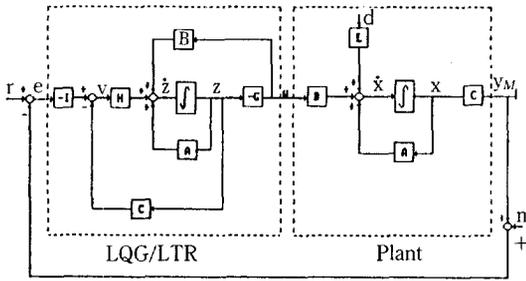


Fig. 11. The block diagram of LQG/LTR.

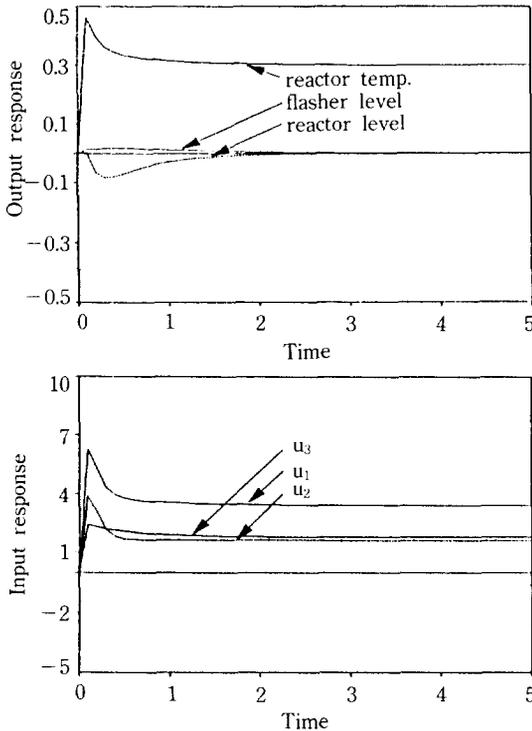


Fig. 12. The output and input responses of LQG/LTR.

$$G = \begin{bmatrix} 130.7 & -48.7 & -2.1 & -9174.4 & 835.5 & -5089.6 \\ -48.7 & 145.3 & 40.7 & 612.4 & -936.4 & -3924.5 \\ -2.1 & 40.7 & 194.5 & 3931.2 & 3408.5 & -9884.6 \end{bmatrix}$$

The output and input responses are shown in Figure 12. We can see the control performance is far better than the PI control combined with LQR.

**5. DMC Combined with LQR**

Open loop unstable systems can be stabilized by constant state feedback gains which are given by LQR, and then we can make the discrete representation of the inner closed loop system (the area surrounded by the outer dotted-line in Figure 13) that is stable.

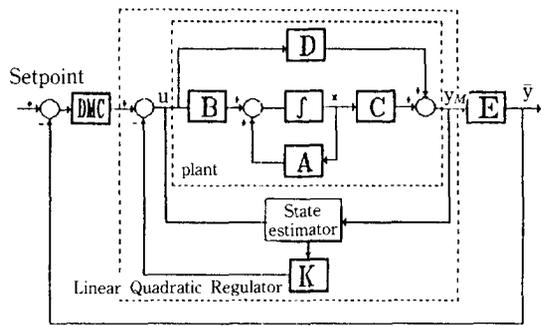


Fig. 13. The block diagram of DMC combined with LQR.

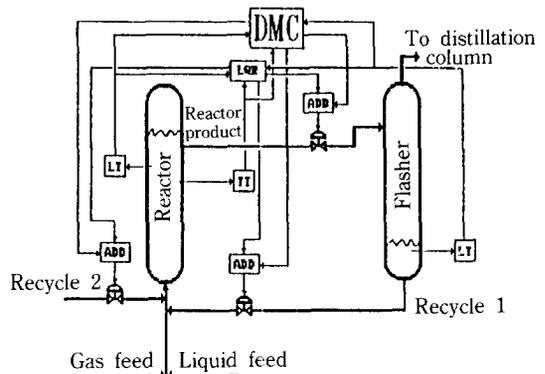


Fig. 14. The process configuration of DMC combined with LQR.

Since we stabilize the open loop unstable system, we can get the step response coefficients of the closed loop system. Therefore, Dynamic Matrix Controller controls the closed loop system as a master controller. The resulting control structure is one of cascade forms. The process is stabilized by LQR(inner loop) and the outer loop is comprised of DMC. We used the same gain matrix in the case of PI control combined with LQR.

Since we got the constant gain matrix from LQR, we can get the step response coefficients of the closed loop system with the state feedback gains to make a Dynamic Matrix. Then the DMC combined with LQR is completed. The control configuration of DMC combined with LQR is shown in Figure 14. From the output and input responses in Figure 15, we can see that the performance of DMC combined with LQR (move suppression factors are all 0.1) is far better than those of the others.

**6. Model/Plant Mismatch**

In order to investigate how robust DMC combined with LQR is, we assume that DMC is based on the

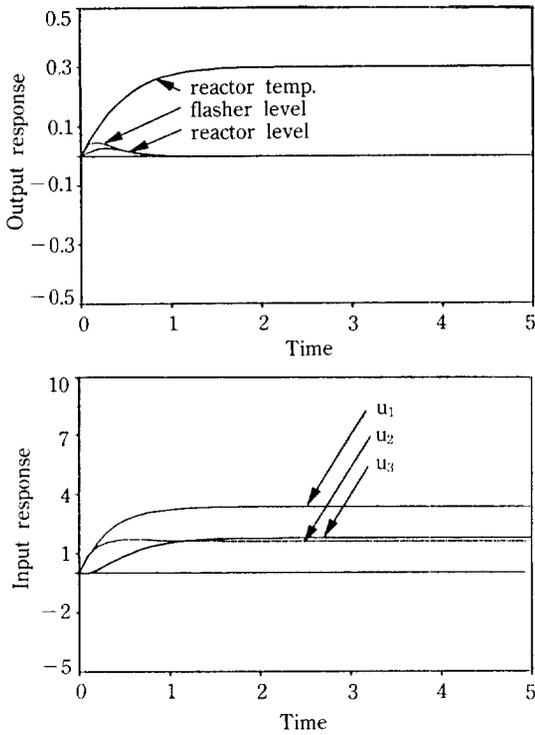


Fig. 15. The output and input responses of the reactor-flasher system in DMC combined with LQR.

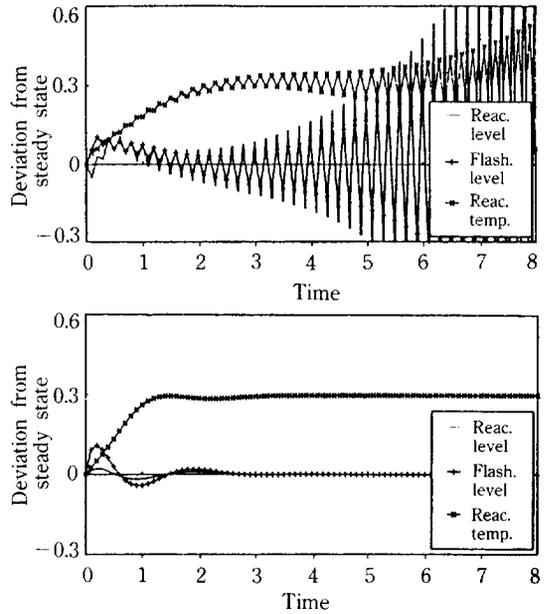


Fig. 17. The output response in the case of model/plant mismatch [ $\pi_2 = -6$  (above) and  $\pi_2 = 2$  (below)].

mathematical model of eq. (1) and the plant model can be changed to eq. (3):

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \pi_1 & 0 & 0 \\ 0 & \pi_2 & -2.2 \\ 0 & 0 & \pi_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -1 & 1 & 1 \\ 0.67 & -1 & 0 \\ 0 & -0.66 & -1.6 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (3)$$

where  $\pi_1$ ,  $\pi_2$  and  $\pi_3$  denote the eigenvalues of the system matrix. Therefore, by changing the eigenvalues different from those of eq. (1), we can test the robustness of DMC combined with LQR (same tuning parameters with 3-4) in the case of model/plant mismatch. We change only one eigenvalue at a time while the others are fixed.

Figure 16 shows the output responses in the case of model/plant mismatch ( $\pi_1 = -6$  and  $2$ ). We can see that the control may be unstable, as  $\pi_1$  is shifted toward the right half side of the  $s$ -plane. But we can guarantee the robustness of the control in the case of  $\pi_1$  shifting toward the left half side of the  $s$ -plane. Figure 17 shows the responses of controlled variables with the same condition as above except  $\pi_2$  instead of  $\pi_1$ . The results are similar to the above case. But from Figure 18, we find that the changes of eigenvalue  $\pi_3$  ( $\pi_3 = 7$  and  $20$ ) from  $13$  do not significantly affect the responses of the controlled variables. Even when

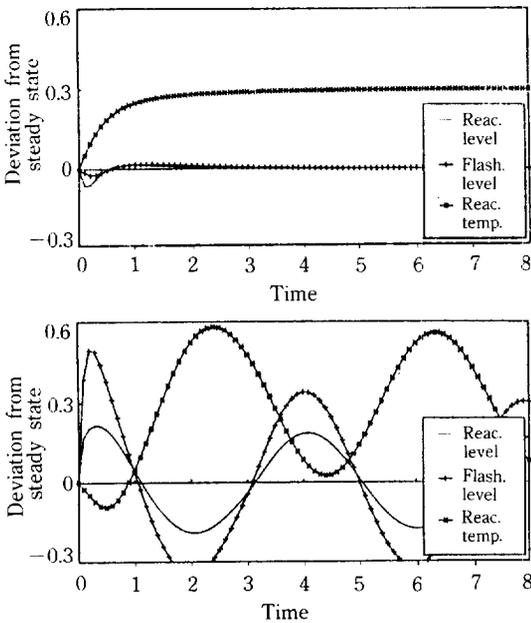


Fig. 16. The output response in the case of model/plant mismatch [ $\pi_1 = -6$  (above) and  $\pi_1 = 2$  (below)].

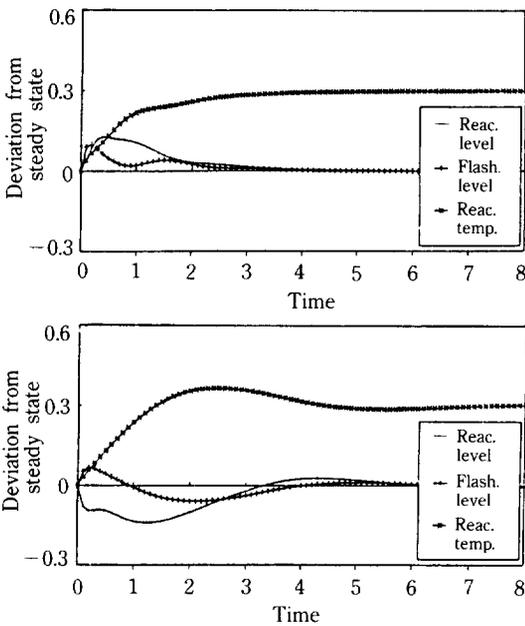


Fig. 18. The output response in the case of model/plant mismatch [ $\pi_3=7$ (above) and  $\pi_3=20$ (below)].

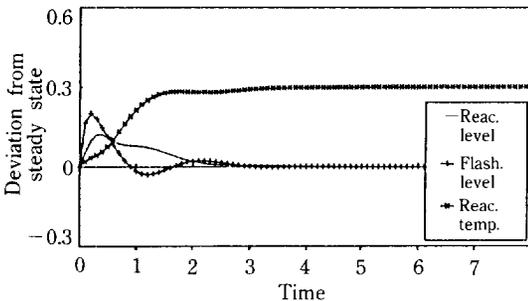


Fig. 19. The output response in the case of model/plant mismatch [ $\pi_1=1$ ,  $\pi_2=2$ ,  $\pi_3=20$ ].

we change all the eigenvalues ( $\pi_1=1$ ,  $\pi_2=1$ ,  $\pi_3=20$ ), the control shows robustness (Fig. 19). Therefore, from the above examples, we can see that DMC combined with LQR is quite robust in the case of model/plant mismatch. LQR/LTR also shows good robustness even though we do not show the result of the robustness study due to space limitation.

## CONCLUSIONS

The study was undertaken to learn what control strategy is most satisfactory for the unstable reactor-flasher system. The following conclusions have emerged :

1. Multiloop single variable control is easy to understand and implement. But tuning the controller parameters is very difficult and its performance is very poor.

2. Modified single variable control with compensators shows better performance than the previous one. However, difficult tuning remains and the operability may fall as a result of the complex control structure.

3. PI control combined with LQR shows better performance than the other two. Since the close loop system is stabilized by LQR, the control parameter tuning is much easier and the operability can be improved.

4. LQG/LTR shows far better performance than the above methods and shows good robustness.

5. DMC combined with LQR shows the best performance while remaining robust in the face of modeling errors.

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## NOMENCLATURE

- A, B, C, D : matrices used in the state space model
- ADD : summation block
- E : matrix to select the controlled variables
- FC : flow controller
- G : control gain matrix
- II : filter gain matrix
- LT : level transmitter
- TT : temperature transmitter
- $K_i$  : proportional control gain,  $i=1, 2, 3$
- LC : level controller
- s : Laplace variable
- $u_1$  : flow rate of the reactor product
- $u_2$  : flow rate of recycle 1
- $u_3$  : flow rate of recycle 2
- $u_{10}$  : a steady state condition of  $u_1$
- $u_{20}$  : a steady state condition of  $u_2$
- $u_{30}$  : a steady state condition of  $u_3$
- x : state vector
- $x_1$  : mass of liquid in the reactor
- $x_2$  : mass of liquid in the flasher
- $x_3$  : reactor temperature
- $y_m$  : measured output vector
- $\bar{y}$  : selected output vector
- $\tau_i$  : reset time,  $i=1, 2, 3$

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