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MODEL PREDICTIVE CONTROL FOR MULTIVARIABLE UNSTABLE PROCESSES WITH CONSTRAINTS ON MANIPULATED VARIABLES

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Abstract – The original MPC(Model Predictive Control) algorithm cannot be applied to open loop unstable systems, because the step responses of the open loop unstable system never reach steady states. So when we apply MPC to the open loop unstable systems, first we have to stabilize them by state feedback or output feedback. Then the stabilized systems can be controlled by MPC. But problems such as valve saturation may occur because the manipulated input is the summation of the state feedback output and the MPC output. Therefore, we propose Quadratic Dynamic Matrix Control(QDMC) combined with state feedback as a new method to handle the constraints on manipulated variables for multivariable unstable processes. We applied this control method to a single-input-single-output unstable nonlinear system and a multi-input-multi-output unstable system. The results show that this method is robust and can handle the input constraints explicitly and also its control performance is better than that of others such as well tuned PI control, Linear Quadratic Regulator (LQR) with integral action.

INTRODUCTION

The universal drive for more efficient use of energy in the chemical and allied industries has resulted in the imposition of stricter demands on control systems. For effective control, the control system must cope with the problems caused by time delays, interactions among system variables, and inherent system nonlinearities. In addition, it must be capable of handling constraints in the input variables as well as in the output variables while remaining robust in the face of modeling errors, measurement noise, and unmodeled disturbances.

The steps taken by the Shell Oil Company (U.S.A.) towards solving the above problems led to the development of the Dynamic Matrix Control (DMC) technique which first appeared in the open literature in Cutler and Ramaker (1979) after having been applied with notable success on industrial processes since 1973.

In 1986, an extended method for the solution of the DMC problem was introduced. The method denoted as QDMC (Quadratic Dynamic Matrix Control) consists of the on-line solution of a quadratic programming (QP) problem which minimizes the sum of squared deviations of controlled variable projections from their setpoints subject to maintaining projections of constrained variables within bounds (Morshedi, et al., 1986).

For open loop unstable systems, the discrete representation of the dynamics is impossible because open loop unstable systems can never reach steady state. Therefore, the original DMC algorithm cannot handle the open loop unstable systems. In 1990, model predictive control combined with coordinate control strategies was proposed to accomplish the control of a nonlinear, open-loop unstable process (P. M. Hidalgo and C. B. Brosilow [9]). This algorithm computes the manipulated variables so that the model output exactly tracks the desired model output at the next time horizon by solving a Newton's algorithm. But this method cannot prevent the saturation of manipulated variables explicitly and would be unstable in the case of severe plant/model mismatch though large filter time constant of the reference trajectory equation can be used to stabilize the system.

In this study, we propose QDMC combined with state feedback as a new method to handle constraints of controlled variables and manipulated variables for open loop unstable systems. This method has a similar structure to original QDMC except for constraint equations that include state feedback information. We compare the performance of the proposed method with that of Linear Quadratic Regulator (LQR) with integral action.

THEORETICAL BACKGROUND

1. QDMC Combined with State Feedback

The philosophy of Quadratic Dynamic Matrix Control (QDMC) is well described by Garcia and Morshedi [7]. Here, we express a simple quadratic solution of DMC and next derive the QDMC constraint equation combined with state feedback. One can express the least-squares solution of the DMC equations as the following quadratic minimization problem:

$$\min_{u(\bar{k})} S=\frac{1}{2}[Au(\bar{k})-e(\bar{k}+1)]^T\Gamma^T\Gamma[Au(\bar{k})-e(\bar{k}+1)] + \frac{1}{2}u(\bar{k})^T\Lambda^T\Lambda u(\bar{k}) \qquad (1)$$

where A : dynamic matrix
$u(\bar{k})$: manipulated input vector at present time, $[\Delta I(\bar{k})\cdots\Delta I(\bar{k}+l)]$, l = control horizon
$e(\bar{k}+1)$: predicted error vector
Γ : weighting matrix of controlled variables
Λ : weighting matrix of manipulated variables

Here, dynamic matrix A is comprised of the step response coefficients for the inner closed loop system. The above quadratic problem can be simplified as following:

$$\min S=\frac{1}{2}u(\bar{k})^T H u(\bar{k})-g(\bar{k}+1)^T u(\bar{k}) \qquad (2)$$

$$\text{s.t. } Cu(\bar{k})\geq c(\bar{k}+1)$$
$$u_{min}\leq u(\bar{k})\leq u_{max}$$

where

$H=A^T\Gamma^T\Gamma A-\Lambda^T\Lambda$ (the QP Hessian matrix)

$g(\bar{k}+1)=A^T\Gamma^T\Gamma e(\bar{k}+1)$ (the QP gradient vector)

Next thing we have to do is to make the linear inequality constraints matrix. In the following we show how the inequalities are formulated for each of the constrained variables for a s-input r-output system.

1-1. Manipulated Variables

In QDMC, the vector $u(\bar{k})$ contains not only the present moves to be implemented but also predictions of the future moves. But if we want to bound the range of manipulated variables, we do not have to limit all the elements of $u(\bar{k})$, because only $\Delta I_i(\bar{k})$ is implemented and $u(\bar{k})$ is recalculated at the next time. In the case of QDMC combined with state feedback, final control action is the summation of $I_i(\bar{k})$ and state feedback, that is $I_i(\bar{k})-K_i x(\bar{k})$ where K_i is the i[th] row vector of the inner loop state feedback gain and $x(\bar{k})$ is the vector of the present state value. LQR can be used to calculate the state feedback gain matrix K. So one can bound the predicted level of the i[th] manipulated variable as follows:

$$I_{i\,min}\leq I_i(\bar{k})+\Delta I_i(\bar{k})-K_i x(\bar{k})\leq I_{i\,max} \qquad (3)$$

where $I_i(\bar{k})$ is the present value of the i[th] manipulated variable; and $I_{i\,min}$, $I_{i\,max}$ are the lower and upper limits respectively. In matrix form, these constraints are expressed as:

$$\begin{bmatrix} -1 & & \\ & \ddots & \\ & & -1 \\ 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} u(k)\geq \begin{bmatrix} [I_1(\bar{k})-I_{1\,max}]-K_1x(\bar{k}) \\ \vdots \\ [I_s(\bar{k})-I_{s\,max}]-K_sx(\bar{k}) \\ [I_{1\,min}-I_1(\bar{k})]+K_1x(\bar{k}) \\ \vdots \\ [I_{s\,min}-I_s(\bar{k})]+K_sx(\bar{k}) \end{bmatrix} \qquad (4)$$

1-2. Controlled Variables

Dynamic Matrix is made of the step response coefficients of the inner closed loop system. So the constraint equations of controlled variables are the same as before. For a single output system, with respective maximum and minimum limits O_{max}, O_{min}, the constraint equations are formulated as:

$$\begin{bmatrix} -A \\ A \end{bmatrix} u(k)\geq \begin{bmatrix} (O_s-O_{max})\mathbf{1}-e(\bar{k}+1) \\ (O_{min}-O_s)\mathbf{1}+e(\bar{k}+1) \end{bmatrix} \qquad (5)$$

where $\mathbf{1}=[1\ 1\cdots 1]$

Extension to the multiple-output case is straightforward.

$$\begin{bmatrix} -\underline{A} \\ \\ \underline{A} \end{bmatrix} \begin{bmatrix} u_1(\bar{k}) \\ \vdots \\ u_s(\bar{k}) \end{bmatrix} \geq \begin{bmatrix} (O_{1s}-O_{1\,max})\mathbf{1}-e_1(\bar{k}+1) \\ \vdots \\ (O_{rs}-O_{r\,max})\mathbf{1}-e_r(\bar{k}+1) \\ (O_{1\,min}-O_{1s})\mathbf{1}+e_1(k+1) \\ \vdots \\ (O_{r\,min}-O_{rs})\mathbf{1}+e_r(k+1) \end{bmatrix} \qquad (6)$$

where O_{rs} is the value of the r-th controlled variable setpoint; $O_{r\,max}$ and $O_{r\,min}$ are the maximum and the minimum limit of the r-th controlled variable respectively.

The simplified block diagram of QDMC combined with state feedback is shown in Fig. 1. In the figure, we include a state estimator to predict the states of the system and simply represent it as a state estimator block to avoid complexity. If Kalman filter is used as the state estimator, then the stability and robustness of the system would be improved. But here, the state estimator is not important. So we do not mention

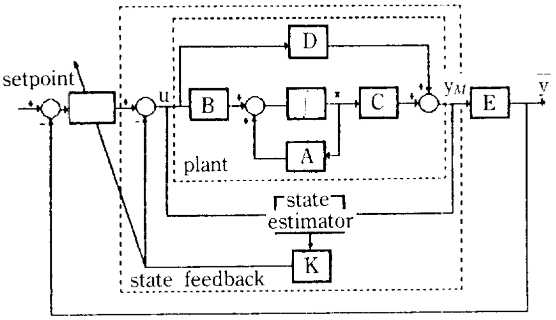

Fig. 1. The block diagram of QDMC combined with state feedback.

about it further. $\bar{y}$ denotes the selected controlled variables among the measured output variables, and the matrix E is introduced to select the controlled variables ($\bar{y}=Ey_m$). Since we stabilize the open loop unstable systems, we can get the step response coefficients of the closed loop system from the model equation. Therefore, it makes QDMC control the closed loop system as a master controller. The resulting control structure has two control loop; one comes from state feedback and the other from QDMC. This structure is a little different from cascade form. Because the output from QDMC is not the setpoint of feedback controller. In Fig. 1, the line passing though the QDMC block denotes that the state feedback information is used in QDMC for handling input constraints. Therefore QDMC gives the input action so that the final manipulated action should not violate the input constraints considering the state feedback action.

2. Linear Quadratic Regulator (LQR) with Integral Action

Another method of controlling multivariable unstable processes is LQR with integral action. We briefly discuss the basic LQR and LQR with integral action because, in the later simulation, we compare the performance between this method and QDMC combined with state feedback.

The basic LQR problem uses the following state equations, cost function, and full state control law:

$$\dot{x}=Ax+Bu \tag{7}$$

$$y=Cx+Du \tag{8}$$

$$J=\int_0^\infty (x^TQx+u^TRu)\ dt,\ Q=Q^T\geq 0,\ R=R^T>0 \tag{9}$$

$$u=-Kx \tag{10}$$

The objective is to find K which minimizes the cost function J. The solution is $K=R^{-1}B^TP$, where $P=P^T\geq 0$ is the unique positive semi-definite solution to the algebraic Riccati equation:

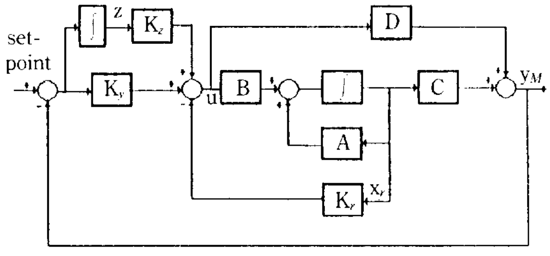

Fig. 2. The block diagram of LQR with integral action.

$$A^TP+PA+Q-PBR^{-1}B^TP=0 \tag{11}$$

The conditions for existence of a solution are that (A, B) are stabilizable and the solution lies in a stable closed loop system matrix (A-BK).

But the linear quadratic formulation produces a proportional state feedback controller. From classical control theory one recognizes that proportional controllers lead to offset when there are setpoint changes or load changes in the process; thus, it would be desirable to formulate the optimal feedback control problem so as to allow integral control action which would eliminate these offsets. One method of incorporating integral action into the controller is to augment the state variables to include new variables z(t) where $\dot{z}=Ix$ are those state variables for which integral action is desired. Thus the new state and manipulated variables are

$$\hat{x}=\begin{bmatrix} z \\ \hline x \end{bmatrix},\ x=\begin{bmatrix} y_m \\ \hline x_r \end{bmatrix},\ \hat{u}=\begin{bmatrix} 0 \\ \hline u \end{bmatrix}$$

where y_m are measured output variables and x_r are remaining state variables except measured output variables, and process dynamics for the new state variables is as follows:

$$\dot{\hat{x}}=P\hat{x}+Q\hat{u} \tag{12}$$

$$y_m=V\hat{x}+W\hat{u} \tag{13}$$

where $P=\begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}$, $Q=\begin{bmatrix} 0 \\ B \end{bmatrix}$, $V=[0\ C]$, $W=[0\ D]$

When the objective function is also modified to accommodate the new state variables, i.e.,

$$J=\int_0^\infty (\hat{x}^TQ\hat{x}+\hat{u}^TR\hat{u})\ dt \tag{14}$$

the linear quadratic optimal control law takes the form

$$
\begin{aligned}
\hat{u} &= -K\hat{x} \\
&= -K_z z - K_y y_m - K_r x_r \\
&= -K_z \int y dt - K_y y_m - K_r x_r
\end{aligned} \tag{15}
$$

where K_z = state feedback gains for integral element
K_y = state feedback gains for measured output variables
K_r = state feedback gains for remaining state variables except output variables

which naturally includes integral action. So those offsets can be eliminated. Fig. 2 shows the block diagram of LQR with integral action.

APPLICATION EXAMPLES

1. A SISO Nonlinear Unstable System

We consider a jacketed CSTR in series with separator and recycle [8]. An exothermic, second-order, autocatalytic, irreversible reaction takes place in the reactor, A+B→2B. The system is described by three nonlinear differential equations as following:

• material balance for substance A

$$V_1 dc_{A1}/dt = Fc_{A0} + rFc_{A3} - (1+r)Fc_{A1} - R_1 V_1 \tag{16}$$

• energy balance for the reactor

$$V_1 \rho c_p dT_1/dt = F\rho c_p T_0 + rF\rho c_p T_3 + V_1(-\Delta H)R_1 - (1+r)F\rho c_p T_1 - U_1 A_1 (T_1 - T_{c1}) \tag{17}$$

• energy balance for the jacket

$$V_{J1} \rho_c c_{pc} dT_{c1}/dt = F_{c1} \rho_c c_{pc} T_{c0} - F_{c1} \rho_c c_{pc} T_{c1} + U_1 A_1 (T_1 - T_{c1}) \tag{18}$$

• reaction rate

$$R_1 = k_0 c_{A1}(10 - c_{A1}) \exp\left[\frac{-E}{R_u T_1}\right] \tag{19}$$

and Table 1 shows design specifications of the system. By linearization of these equations around the steady state, a third-order linear model may be obtained in the form:

$$dx/dt = Ax + bu \tag{20}$$

$$y_m = Cx \tag{21}$$

$$\bar{y} = Ey_m \tag{22}$$

where $x^T = [c_{A1} \ T_1 \ T_{c1}]$
$y_m^T = [c_{A1} \ T_1 \ T_{c1}]$
$\bar{y} = [T_1]$

Table 1. Design specifications of the system

System parameters	Value
concentration of A in the feed, c_{A0}, kmol/m^3	10
feed flow, F, m^3/min	0.21
feed temperature, T_0, K	294
activation energy, E, kJ/kmol	92,973
preexponential term, k_0, 1/(min kmol)	4.25×10^{12}
reaction heat, $-\Delta H$, kJ/kmol	6×10^4
recycle, r(F_3/F)	3
reactor volume, V_1, m^3	18.925
overall heat-transfer coefficient, U_1, kJ/(m^2 K min)	68
temperature of the reactor, T_1, K	320.33
concentration of the reactor, c_{A1}, kmol/m^3	2.678
temperature of the recycle, T_3, K	325.58
concentration f the recycle, c_{A3}, kmol/m^3	1.611
jacket volume, V_{J1}, m^3	5.0
coolant flow of the reactor, F_{c1}(m^3/min)	3.21
coolant feed temperature, T_{c0}, K	294
coolant exit temperature of the reactor, T_{c1}, K	298
specific heat of reactants, c_p, kJ/(kg K)	3.14
specific heat of coolant, c_{pc}, kJ/(kg K)	4.18
density of reactants, ρ, kg/m^3	900
density of coolant, ρ_c, kg/m^3	1000
universal gas constant, R_u, kJ/(kmol K)	8.314

$$u = [F_{c1}]$$

$$A = \begin{bmatrix} -0.0469 & -0.0063 & 0.0 \\ 0.2891 & 0.0525 & 0.0472 \\ 0.0 & 0.1209 & -0.7629 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0 \\ 0.0 \\ -0.8 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E = [0 \ 1 \ 0]$$

The eigenvalues of the system are -0.7698, -0.0259, 0.0384. Therefore, the above system is an open loop unstable system. If QDMC combined with LQR is applied to this system, the control configuration can be constructed as Fig. 3. First we get the constant state feedback gain vector ($K = [-7.8927 \ -2.7984 \ -0.5430]$) from LQR in which the weighting matrices (Q and R) are all identity matrices. Next we get the step response coefficients for the closed loop system with the above state feedback gains to make the Dynamic Matrix.

Fig. 4 shows the comparison of the process output and input responses among well-tuned PI control (Kc $= -3$, $\tau_I = 9.5$), LQR with integral action ($K_z = -3.33$, $K_y = -27.91$, $K_r = [-23.79 \ -2.96]$), and QDMC combined with state feedback (move suppression factor $= 0.01$) to the reactor temperature setpoint change

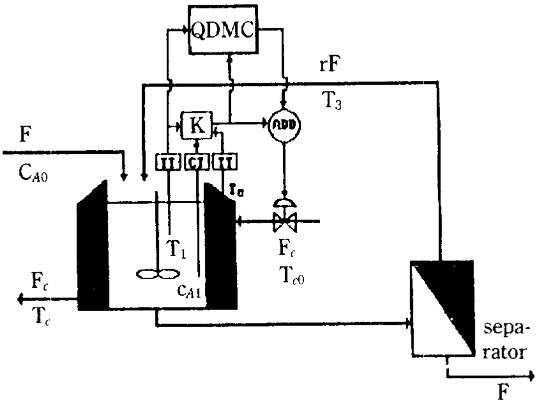

Fig. 3. The control configuration of QDMC combined with state feedback to the unstable jacketed CSTR system.

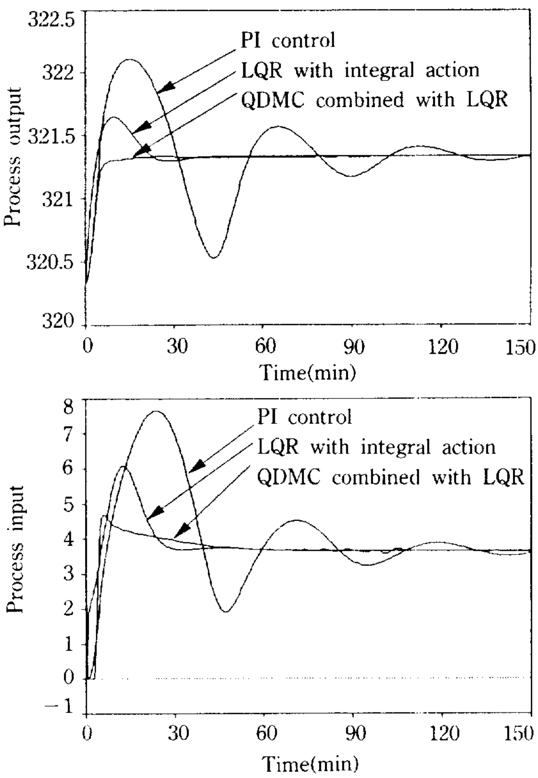

Fig. 4. The comparison of the process output and input responses among well-tuned PI control, LQR with integral action and QDMC combined with state feedback.

of 1 degree. Here, coolant flow rate cannot flow conversely, that is, F_c always has positive sign, which is a constraint of the manipulated variable. The tuning

Table 2. Design specifications of the second reactor

System parameters	Value
second reactor volume, V_2, m^3	18.925
overall heat-transfer coefficient, U_2, kJ/(m^2 K min)	68
temperature of the second reactor, T_2, K	325.58
concentration of the second reactor, c_{A2}, kmol/m^3	1.128
jacket volume of the second reactor, V_{j2}, m^3	5.0
coolant flow of the second reactor, F_{c2}(m^3/min)	0.9743
coolant exit temperature of the second reactor, T_{c2}, K	306

parameters of PI control, LQR with integral action were determined so that the manipulated variables are positive. But, for QDMC combined with state feedback, we made the constraint equation for F_c to be greater than zero. So we could use small move suppression factor to get the fast process output response. The performance of QDMC combined with state feedback is better than that of PI control and that of LQR with integral action. Also we can know the fact that QDMC combined with state feedback can handle even the system with mild nonlinearity.

2. A MIMO Nonlinear Unstable System

We consider two jacketed CSTR in series with separator and recycle as a MIMO unstable system. As the previous example, the same reaction takes place in each reactor. Three nonlinear differential equations are added to describe the second reactor as following:

Second Reactor:

• material balance for substance A

$$V_2 dc_{A2}/dt = (1+r)Fc_{A1} - (1-r)Fc_{A2} - R_2 V_2 \tag{23}$$

• energy balance for the reactor

$$V_2 \rho c_p dT_2/dt = (1+r)F\rho c_p T_1 - (1+r)F\rho c_p T_2 + V_2(-\Delta H)R_2 - U_2 A_2 (T_2 - T_{c2}) \tag{24}$$

• energy balance for the jacket

$$V_{j2}\rho_c c_{pc} dT_{c2}/dt = F_{c2}\rho_c c_{pc} T_{c0} - F_{c2}\rho_c c_{pc} T_{c2} + U_2 A_2 (T_2 - T_{c2}) \tag{25}$$

• reaction rate

$$R_2 = k_0 c_{A2}(10 - c_{A2})\exp\left[\frac{-E}{R_u T_2}\right] \tag{26}$$

Table 2 shows design specifications of the above system. The model equations of the first reactor are same as the previous SISO unstable system. After we linearize these equations around the steady state, we can get a sixth-order linear model as following.

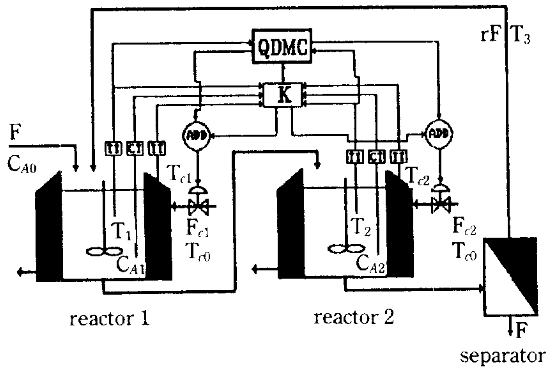

Fig. 5. The control configuration of QDMC combined with state feedback to the unstable two jacketed CSTR system.

$$dx/dt = Ax + Bu \tag{27}$$

$$y_m = Cx \tag{28}$$

$$\bar{y} = Ey_m \tag{29}$$

where $x^T = [c_{A1}\ T_1\ T_{c1}\ c_{A2}\ T_2\ T_{c2}]$
$y_m = [c_{A1}\ T_1\ T_{c1}\ c_{A2}\ T_2\ T_{c2}]$
$\bar{y} = [T_1\ T_2]$
$u = [F_{c1}\ F_{c2}]$

$$A = \begin{bmatrix} -0.047 & -0.006 & 0 & 0 & 0 & 0 \\ 0.289 & 0.052 & 0.047 & 0 & 0 & 0 \\ 0 & 0.121 & -0.763 & 0 & 0 & 0 \\ 0.033 & 0 & 0 & -0.073 & -0.005 & 0 \\ 0 & 0.033 & 0 & 0.846 & 0.035 & 0.047 \\ 0 & 0 & 0 & 0 & 0.121 & -0.316 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.8 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -2.4 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Since one of the system poles is located in the right half plane, the above linearized system is also open loop unstable.

Fig. 5 shows the control configuration of QDMC combined with state feedback which is applied to this system. As shown in Fig. 5, the signal into the final control element is the summation of the signal which comes from QDMC and that from state feedback. Here, the state feedback gain matrix is calculated from

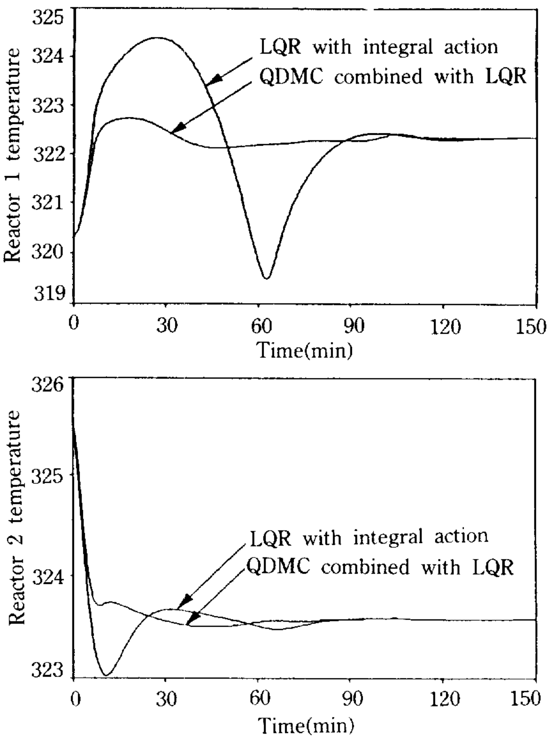

Fig. 6. The comparison of the process output responses between QDMC combined with state feedback and LQR with integral action in the unstable two jacketed CSTR system.

LQR:

$$K = \begin{bmatrix} -7.84 & -2.92 & -0.55 & -1.43 & -0.20 & -0.003 \\ -1.59 & -0.31 & -0.01 & -6.41 & -1.08 & -0.90 \end{bmatrix}$$

K is calculated by solving Riccati equation when R is an identity matrix.

Fig. 6 shows the comparison of the output responses between LQR with integral action and DMC combined with state feedback when we change the setpoints of the first reactor temperature and the second one to 322.33 and 323.58 respectively. The feedback gains of the LQR with integral action are as following:

$$K_z = \begin{bmatrix} -0.577 & -0.016 \\ 0.156 & -0.577 \end{bmatrix}, \quad K_y = \begin{bmatrix} -7.614 & -0.156 \\ -0.409 & -4.632 \end{bmatrix},$$

$$K_r = \begin{bmatrix} -8.447 & -0.510 & -0.261 & -0.005 \\ -0.735 & -0.014 & -11.831 & -0.598 \end{bmatrix}$$

We can see that the performance of DMC combined with state feedback is far better than that of LQR with integral action. Here, we also limit F_{c1}, F_{c2} to be greater than zero. So in the case of LQR with integral action, if negative manipulated action is given, we treat it

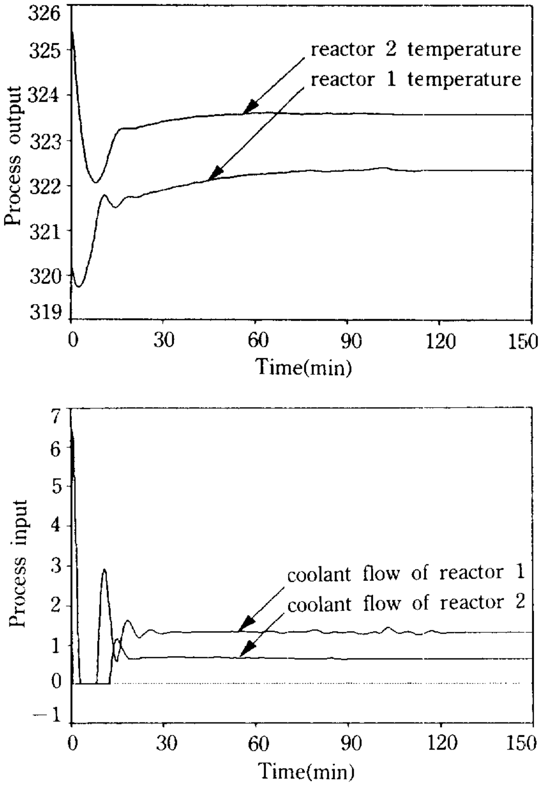

Fig. 7. The process output and input responses of QDMC combined with state feedback in the case of model/plant mismatch.

to be zero. Fig. 7 shows the process output and input of QDMC combined with state feedback for the model/plant mismatch. In this simulation, we changed 68 to 100 for the overall heat transfer coefficient. From the figure, we can see that QDMC combined with state feedback shows robustness.

CONCLUSIONS

We propose a new approach combining QDMC with state feedback to control open loop unstable systems with constraints on manipulated variables. We applied this control method to a SISO nonlinear unstable system and the results show that it is better than the others such as well tuned PI control, LQR with integral action. Moreover, when we expanded it to a MIMO unstable system, its performance showed far better than that of LQR with integral action. Also QDMC combined with state feedback shows robustness for the model/plant mismatch. Therefore, QDMC combined with state feedback is an effective method of controlling multivariable unstable systems with constraints on manipulated variables.

NOMENCLATURE

A	: dynamic matrix of controlled variable step response coefficients
CT	: composition transmitter
$e(\bar{k}+1)$	: controlled variable projected setpoint error vector
$e_i(\bar{k}+1)$	: i^{th} controlled variable projected setpoint error vector
$I(k)$	: system manipulated variable at time k
$\Delta I(k)$	: move of manipulated variable at time k
k	: discrete time
$\bar{k}$	: present time
K	: gain matrix
l	: control horizon
O_r	: r-th controlled variable
O_{rs}	: r-th controlled variable setpoint
O_s	: controlled variable setpoint
Q	: weighting matrix of controlled variables
R	: weighting matrix of manipulated variables
R_u	: universal gas constant
r	: number of manipulated variables
s	: number of controlled variables
TT	: temperature transmitter
$u(\bar{k})$	: vector of present and future moves of manipulated variables, $\Delta I(\bar{k})$
$u_i(\bar{k})$	: i^{th} manipulated variable moves
x	: state variables
x_r	: uncontrolled variables
y_m	: measured output variables
$\underline{y}$	: selected controlled variables
z	: augmented variables

Superscript

*	: projection based on moves up to present time $\bar{k}$

Subscripts

m	: feedback measurement
max	: maximum
min	: minimum
r	: index for uncontrolled variables
y	: index for controlled variables
z	: index for augmented variables

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trolled System: Theory and Design", Prentice-Hall (1984).
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